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Room acoustic simulations using the spectral element method

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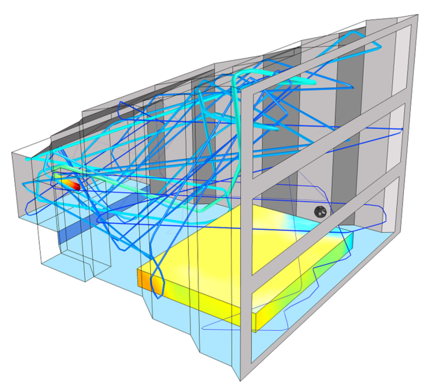
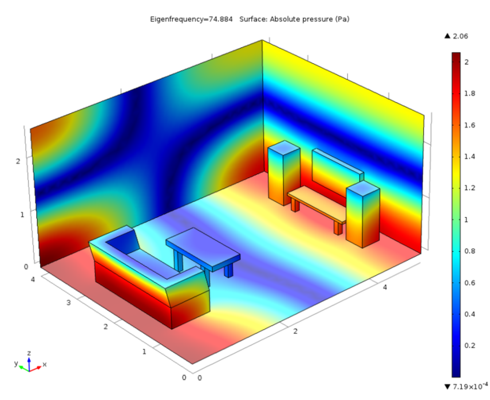
Introduction & motivation

This poster describes the work being carried out in a cross-disciplinary research project, where the aim is to develop a new way of simulating sound in spaces ("room acoustics"), with unprecedented accuracy and efficiency. The areas of application are many-fold, including architecture, virtual reality, computer game audio, music, hearing research and entertainment. The team consists of researchers from the Acoustic Technology Group and the Scientific Computing Section at DTU and from the Danish architectural firm Henning Larsen.

Historically, room acoustic simulations have been carried out by means of so-called "geometrical acoustics" (GA) methods, such as ray-tracing. In GA, several simplifying approximations regarding sound propagation are made, which make the computational task more manageable, but at the cost of significantly reduced accuracy. It is desirable to use numerical methods instead of GA methods, due to their inherent accuracy. However, numerical simulation of room acoustics is a particularly challenging problem, due to the large 3D domains, complex geometries and a wide frequency range of interest. Thus, there is a need for a highly efficient numerical method that simultaneously can handle complex geometries.

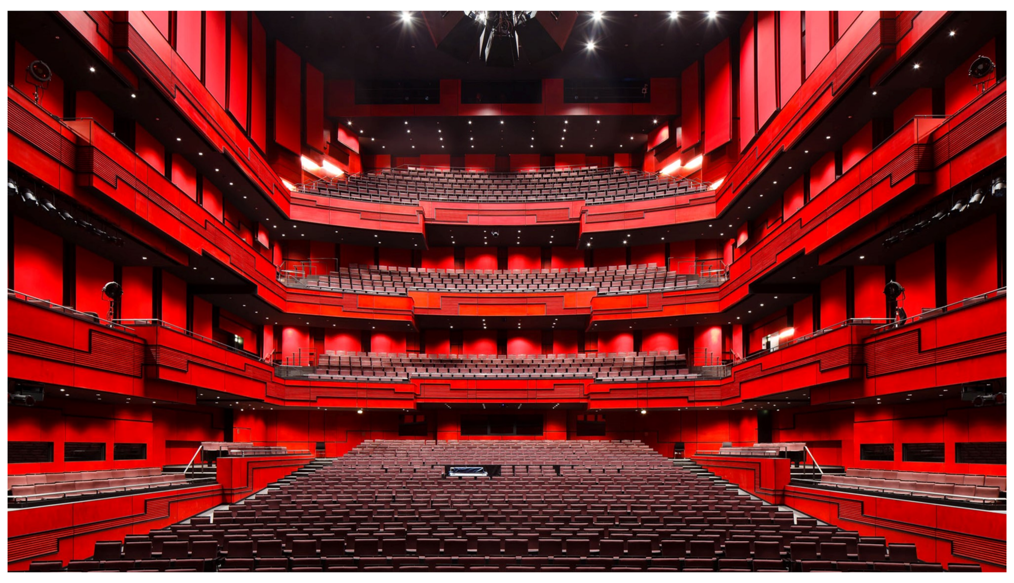


Geometrical acoustics



Numerical acoustics

Here, a numerical scheme based on a high-order spatial discretization using a spectral element method (SEM), coupled with an explicit high-order Runge-Kutta temporal integration method, is presented. The scheme supports the use of adaptive unstructured meshes, high-order polynomial basis functions and curvilinear mesh elements, for accurate representation of acoustic wave propagation in complex geometries. Various numerical experiments reveal the accuracy and efficiency of the proposed numerical scheme and highlight the need for high-order methods with high geometric flexibility for accurate and cost-effective simulations of room acoustics.



Numerical discretization

The sound field in a room can be described by the linearized Euler equations with zero mean flow

$$\begin{aligned} \mathbf{v}_t &= -\frac{1}{\rho}\nabla p, \\ p_t &= -\rho c^2 \nabla\cdot\mathbf{v}, \end{aligned} \qquad \text{in } \Omega, \tag{1}$$

where p is the acoustic pressure, v is the particle velocity, ρ is the density of the medium and c is the speed of sound. This can be discretized in the standard spectral element method approach, yielding the following semi-discrete system

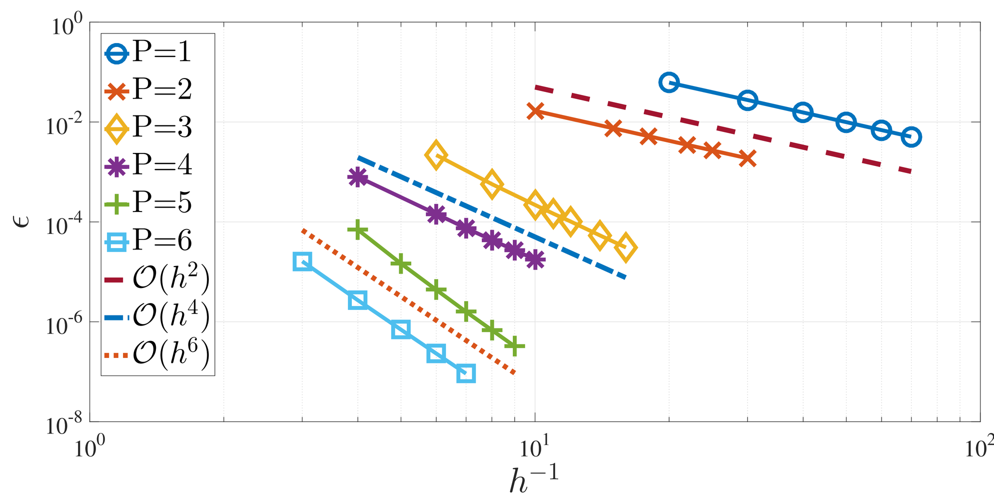
$$\begin{aligned} M\hat{u}' &= -\frac{1}{\rho}S_x\hat{p}, & M\hat{v}' &= -\frac{1}{\rho}S_y\hat{p}, \\ M\hat{w}' &= -\frac{1}{\rho}S_z\hat{p}, & M\hat{p}' &= \rho c^2\left(S_x^T\hat{u} + S_y^T\hat{v} + S_z^T\hat{w}\right), \end{aligned} \tag{2}$$

where u, v, w represent the x, y, z components of the particle velocity, M is the global mass matrix and S are the global stiffness matrices and. These matrix operators are constructed through the assembly of local nodal element basis functions, defined on standard elements and transformed to represent arbitrary elements on the mesh. In the derivation of the local element basis functions, the orthonormality of modal basis functions is exploited to avoid the use of numerical quadrature rules.

Properties of the numerical scheme

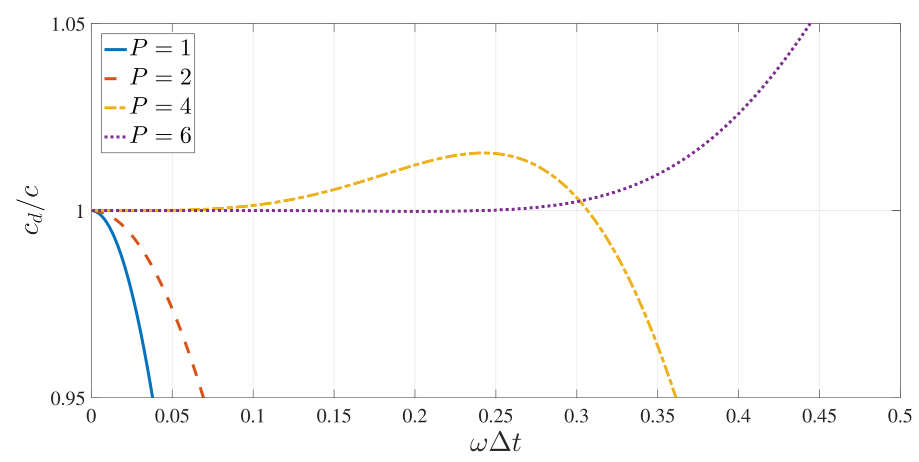
Convergence

A convergence test is presented using a periodic 3D domain. The figure reveals the favorable numerical error properties when using high-order polynomial basis functions.



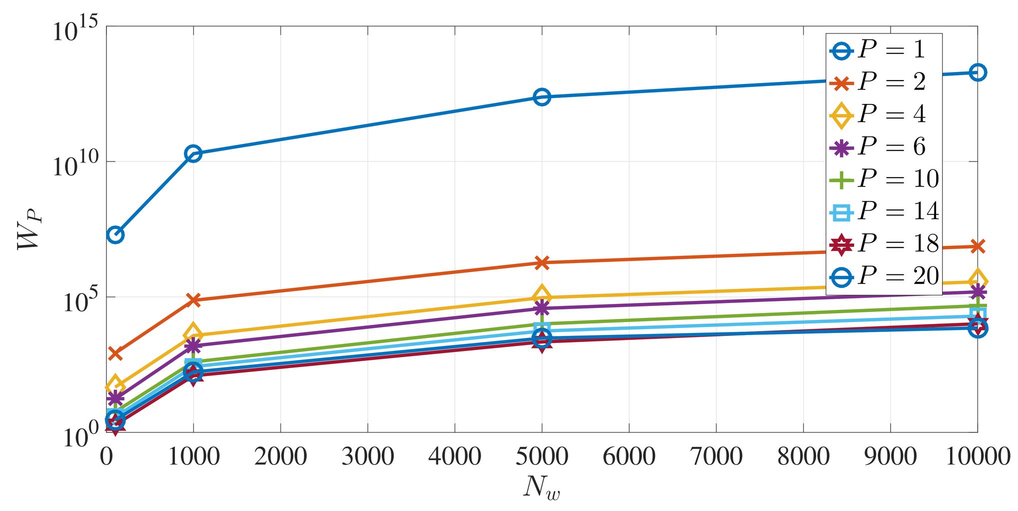
Dispersion analysis

The objective of the analysis is to understand the relationship between the exact physical wave speed c and the discrete wave speed c_d, which will vary with frequency, causing dispersion errors. A general, multi-modal method for analyzing these properties has been developed in this project. This method takes both the spatial and the temporal discretization into account. An example of a dispersion relationship, for a given spatio-temporal resolution, is shown below. It is clear that the high-order discretization results in a wider frequency range where the numerical wave speed accurately captures the physical wave speed.



Computational work effort

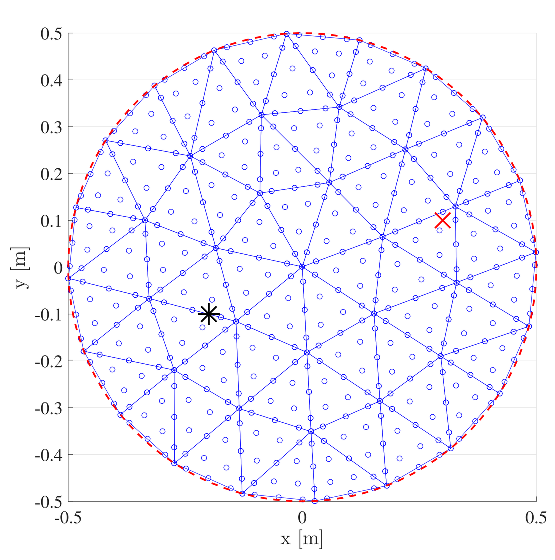
Due to the increased accuracy of the high-order discretization, as shown above, a coarser spatial discretization can be used when using the high-order basis functions. This can improve the computational efficiency of simulation drastically. The figure below shows the estimated computational work effort needed to propagate a 3D wave, as a function of simulation duration, measured in wave periods.



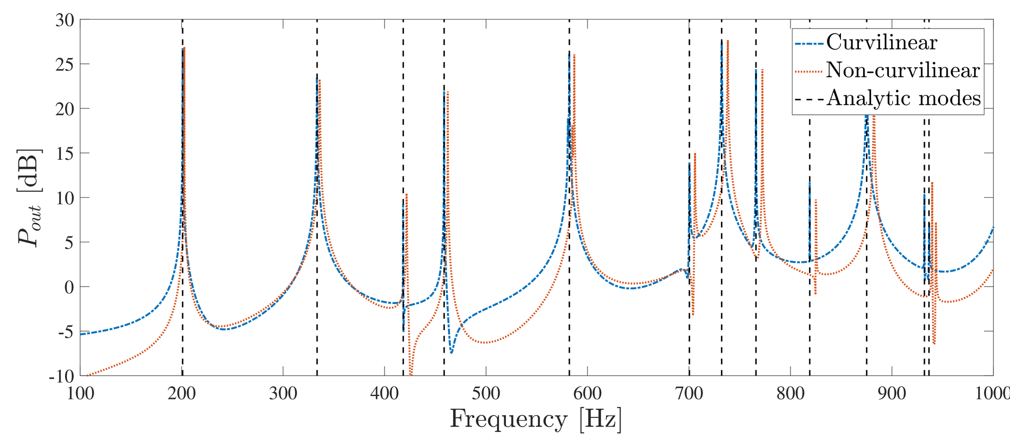
Simulation results

2D disc

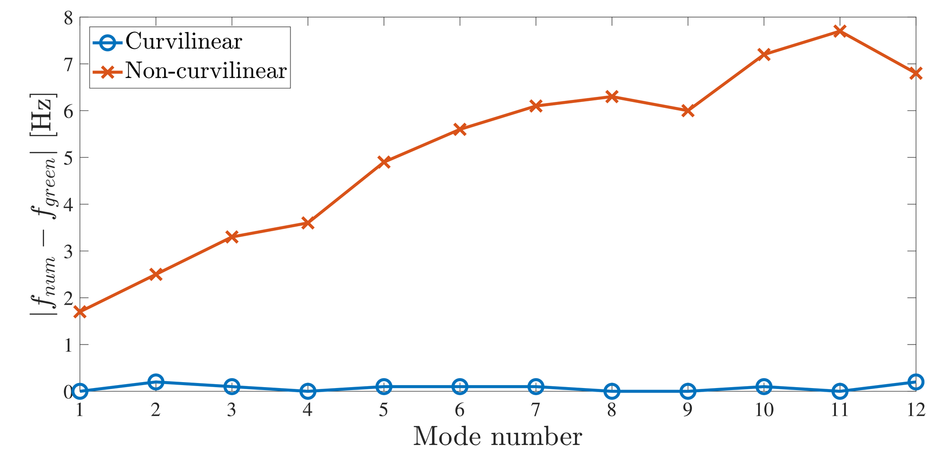
This test case is chosen to illustrate the geometric flexibility of the SEM. When curved boundaries occur, using straight-sided mesh elements will introduce errors the response, unless an extremely fine mesh is used to capture the geometry, which destroys performance. This can be mitigated by using curvilinear mesh elements.



The acoustic response of the disc is simulated with and without the use of curvilinear boundary elements, in both cases using $P = 4$ basis functions. The resulting responses are shown below.

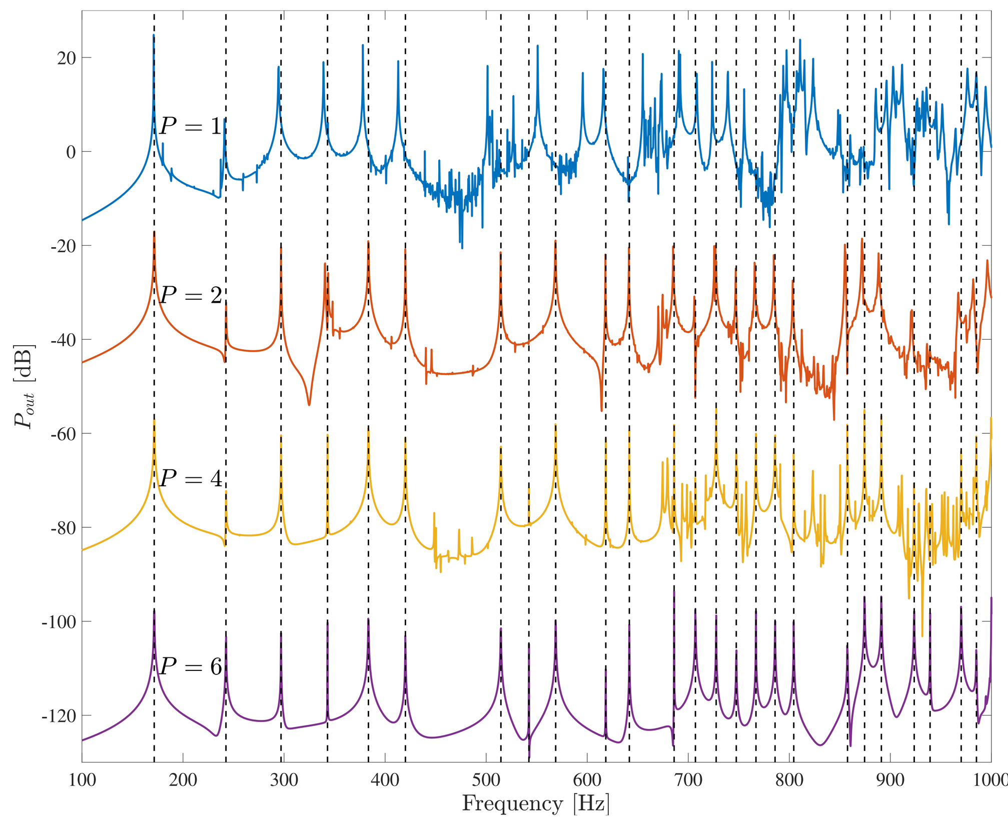


Clearly, the use of curvilinear boundary elements mitigates the apparent mistuning of the modal frequencies of the disc, which arise when the straight sided elements are used. The error is further analyzed in the figure below.



3D room

The acoustic response of a 3D cube shaped room is shown below, simulated using different basis function orders. In all cases the same "fineness" in spatial discretization is used, i.e. the same number of DOF's on the mesh. When high-order basis functions are used, the frequency range of the simulation effectively extends.



Conclusions

The SEM appears to be a particularly suitable candidate for numerical room acoustic simulations, due to its accuracy, efficiency and flexibility. The usage of these methods, once fully matured, will allow for much more realistic virtual acoustics, and thereby improved immersion into the virtual space, than is known today.

Interested? Want to collaborate?

We are always looking for talented students to collaborate with, e.g. by creating special courses or doing master thesis'. There are many interesting challenges that remain unsolved, such as

- Implementation of a matrix-free solver
- Implementation on modern many-core hardware (GPU's)
- Improved meshing techniques
- Reduced order techniques
- Adding viscothermal losses to the medium
- Modelling sound sources (loudspeakers and the human voice)
- Comparison of different algorithms
- etc, etc, etc

Please contact us if you are interested!