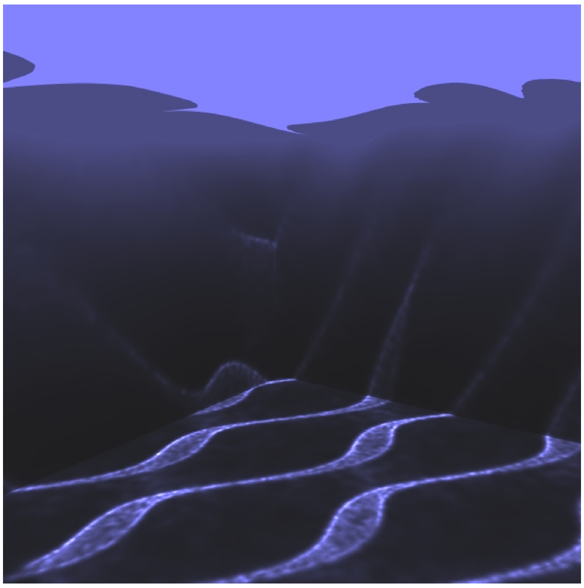
Photon Differentials

Adaptive Anisotropic Density Estimation in Photon Mapping

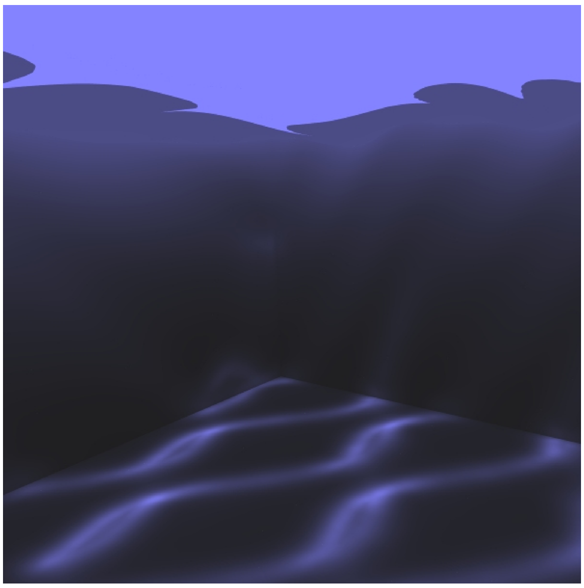
Jeppe Revall Frisvad
Technical University of Denmark

Trade-off problem in photon mapping

- Effect of changing bandwidth (no. of photons in estimates):



Low

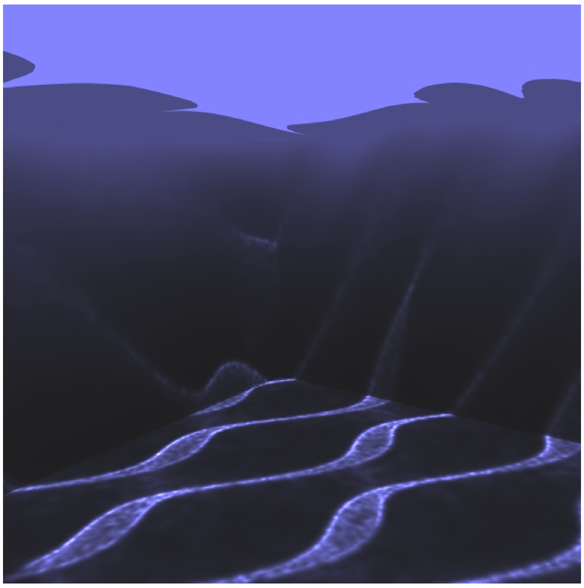


High

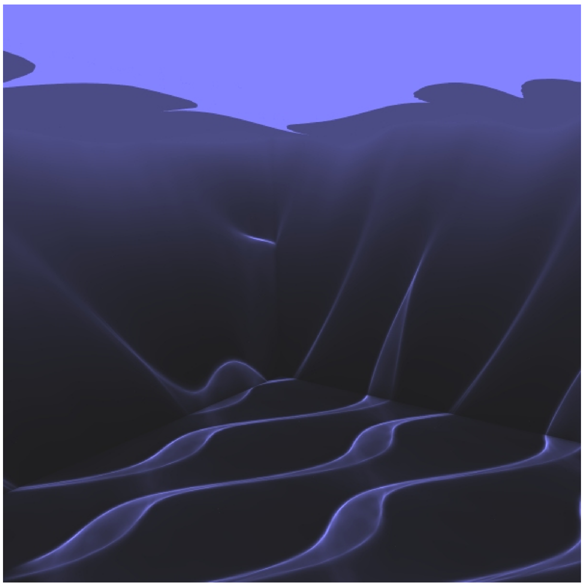
- The trade-off is between noise and blur.

Why photon differentials?

- Using the same number of photons in the map:



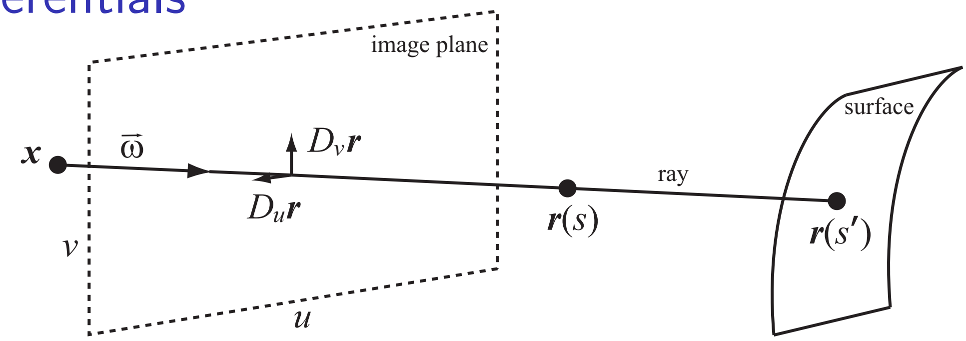
Standard PM



Photon Differentials

- Ray differentials improve texture filtering.
- Photon differentials improve photon flux density estimation.

Ray differentials



- A ray is modelled by the parametrisation of a straight line:

$$\mathbf{r}(s) = \mathbf{x} + s\,\vec{\omega} \quad , \quad s \in [0, \infty[\quad , \quad |\vec{\omega}| = 1 \ .$$

- Suppose we let
  - u and v parameterise the image plane
  - s' be the distance to the first intersection along the ray

  then $\mathbf{r}(s') \mapsto \mathbf{r}(u, v)$, and the ray differential [Igehy 1999]

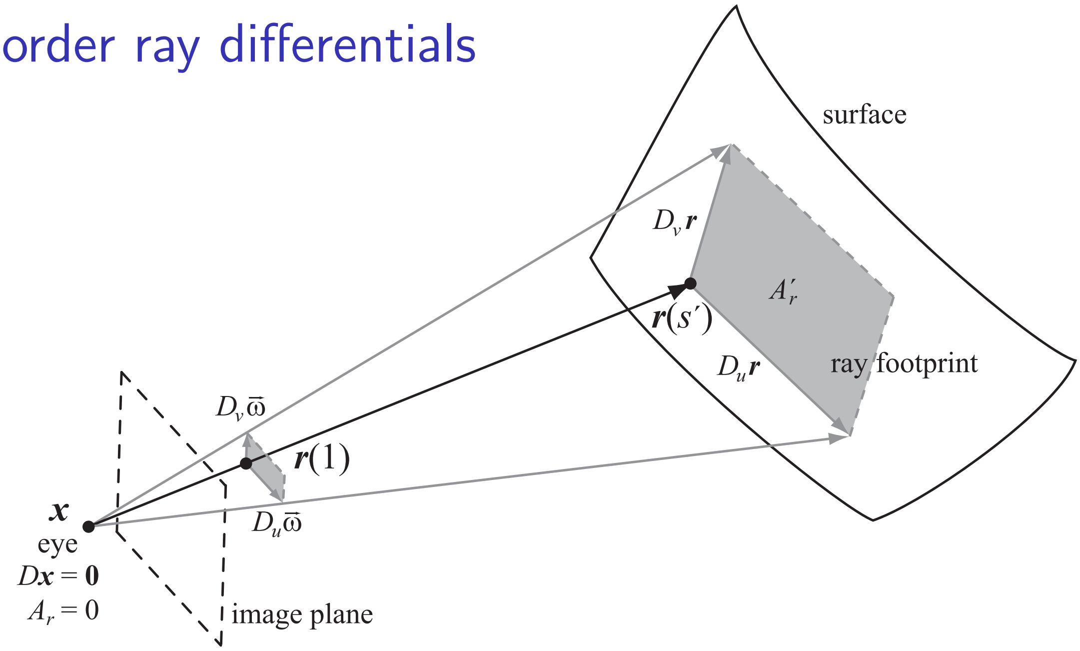
$$D\mathbf{r} = \begin{bmatrix} D_u\mathbf{r} & D_v\mathbf{r} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{r}}{\partial u} & \frac{\partial \mathbf{r}}{\partial v} \end{bmatrix}$$

  tells where a ray would end up if slightly offset in uv-space.

References

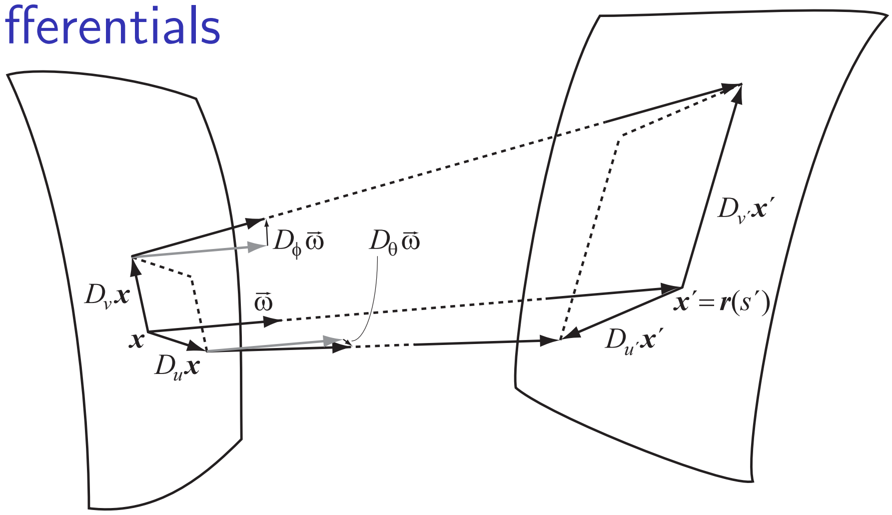
- Igehy, H. Tracing ray differentials. In *Proceedings of ACM SIGGRAPH 1999*, A. Rockwood, Ed., ACM/Addison-Wesley, pp. 179–186.

First-order ray differentials



- In the first order Taylor approximation, a ray differential is given by two pairs of differential vectors.
  - Positional differential vectors: $D\mathbf{x} = \begin{bmatrix} D_u\mathbf{x} & D_v\mathbf{x} \end{bmatrix}$
  - Directional differential vectors: $D\vec{\omega} = \begin{bmatrix} D_u\vec{\omega} & D_v\vec{\omega} \end{bmatrix}$.
- The differential vectors span parallelograms which define ray footprint ($D\mathbf{x}$) and beam spread ($D\vec{\omega}$).

Photon differentials

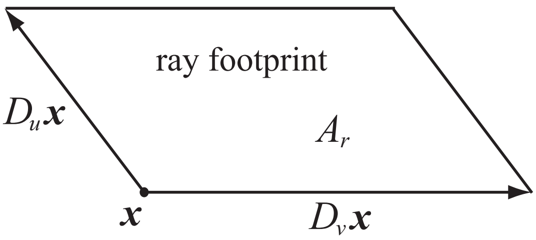


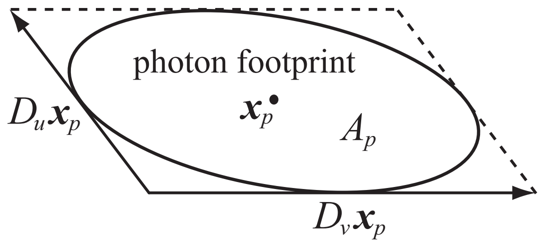
- No camera: we need different local coordinate systems.
  - u and v parameterise the light source surface.
  - θ and ϕ parameterise the emission solid angle.
- Now $\mathbf{r}(s') \mapsto \mathbf{r}(u, v; \theta, \phi) = \mathbf{x}(u, v) + s'(u, v; \theta, \phi)\,\vec{\omega}(\theta, \phi)$.
- Photon differential: $D\mathbf{r} = (\begin{bmatrix} D_u & D_v \end{bmatrix} + \begin{bmatrix} D_\theta & D_\phi \end{bmatrix})\mathbf{r}$.
- Photon differential vectors:
  - Positional differential vectors: $D\mathbf{x} = \begin{bmatrix} D_u\mathbf{x} & D_v\mathbf{x} \end{bmatrix}$
  - Directional differential vectors: $D\vec{\omega} = \begin{bmatrix} D_\theta\vec{\omega} & D_\phi\vec{\omega} \end{bmatrix}$

  define light ray footprint ($D\mathbf{x}$) and beam spread ($D\vec{\omega}$).

Photon footprint

- The parallelogram spanned by the positional differential vectors is the *ray footprint*.



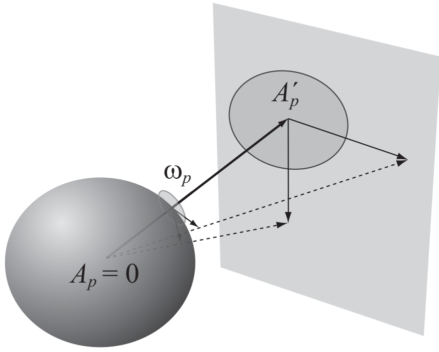


- The max area ellipse inscribed in the parallelogram with centre in the photon position $\mathbf{x}_p$ is the *photon footprint*.
- The area of the photon footprint is then

$$A_p = \frac{\pi}{4} A_\mathbf{r} = \frac{\pi}{4} |D_u \mathbf{x}_p \times D_v \mathbf{x}_p| \ ,$$

- and, by analogy, the photon solid angle is

$$\omega_p = \frac{\pi}{4} |D_\theta \vec{\omega}_p \times D_\phi \vec{\omega}_p| \ .$$



Emitting photon differentials

- A light source emits photons from points $\mathbf{x}_e$ across an area A_e and in directions $\vec{\omega}_e$ within a solid angle ω_e.
- The initial differential vectors of an emitted photon are
  - $\begin{bmatrix} D_u \mathbf{x}_e & D_v \mathbf{x}_e \end{bmatrix}$ an orthogonal basis of the tangent plane at $\mathbf{x}_e$.
  - $\begin{bmatrix} D_\theta \vec{\omega}_e & D_\phi \vec{\omega}_e \end{bmatrix}$ an orthogonal basis of the plane normal to $\vec{\omega}_e$.
- To ensure $\sum_p A_p = A_e$ and $\sum_p \omega_p = \omega_e$, we set the initial lengths of the vectors to

$$|D_u \mathbf{x}_e| = |D_v \mathbf{x}_e| = 2\sqrt{\frac{A_e}{\pi n_e}}$$
$$|D_\theta \vec{\omega}_e| = |D_\phi \vec{\omega}_e| = 2\sqrt{\frac{\omega_e}{\pi n_e}} \ ,$$

  where n_e is the number of photons emitted from the source.
- Point lights emit photons with $D_u \mathbf{x}_e = D_v \mathbf{x}_e = \mathbf{0}$.
- Collimated lights emit photons with $D_\theta \vec{\omega}_e = D_\phi \vec{\omega}_e = \mathbf{0}$.

Photon tracing

- Emitted flux is confined by the solid angle of the ray.
- Flux carried by a ray changes like radiance upon reflection and refraction.
- Tracing photons is like tracing ordinary rays.
- Whenever the photon is traced to a non-specular surface:
  - It is stored in a kd-tree.
  - Position is stored.
  - Direction from where it came is stored.
  - Flux (Φ_p) is stored.
- Russian roulette is used to stop the recursive tracing.

Tracing photon differentials

- Emitted flux is confined by the cone which is spanned by the photon differential.
- Photon differentials change like ray differentials upon reflection and refraction.
- Tracing photon differentials is like tracing ordinary ray differentials.
- Whenever the photon is traced to a non-specular surface:
  - It is stored in a kd-tree.
  - Position is stored.
  - Direction from where it came is stored.
  - Irradiance ($E_p = \Phi_p / A'_p$) is stored (instead of flux).
  - Positional differential vectors $D_{u'}\mathbf{x}'$ and $D_{v'}\mathbf{x}'$ are stored.
- Russian roulette is used to stop the recursive tracing.

Radiance estimation using photon differentials

- Irradiance of a projected photon differential

$$E_p = \Phi_p / A'_p$$

- Reflected radiance

$$L_r(\mathbf{x}, \vec{\omega}) = \int_{2\pi} f_r(\mathbf{x}, \vec{\omega}', \vec{\omega}) \, \mathrm{d}E(\mathbf{x}, \vec{\omega})$$

- Radiance estimate

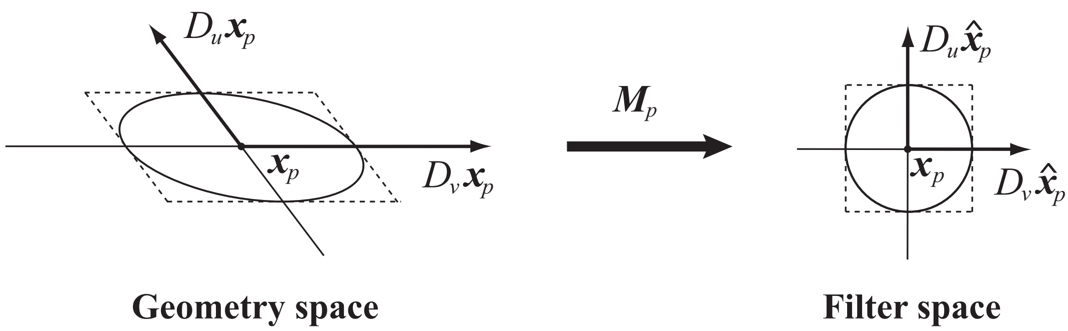
$$L_r(\mathbf{x}, \boldsymbol{\omega}) \approx \widehat{L}_r(\mathbf{x}, \vec{\omega}) = \sum_{p=1}^{n} f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \Delta E_p(\mathbf{x}, \vec{\omega}_p)$$

- To ensure that no energy is lost in the estimate, we must find all the n photons with footprints that overlap a surface point.
- We can induce smoothing by scaling all photon footprints.

Adaptive anisotropic kernel density estimation

- Transform by $\mathbf{M}_p = \left[\frac{1}{2} D_u \mathbf{x}_p \quad \frac{1}{2} D_v \mathbf{x}_p \quad \vec{n}_p\right]^{-1}$,

  where $\vec{n}_p = \frac{D_u \mathbf{x}_p \times D_v \mathbf{x}_p}{|D_u \mathbf{x}_p \times D_v \mathbf{x}_p|}$ is the surface normal at $\mathbf{x}_p$.

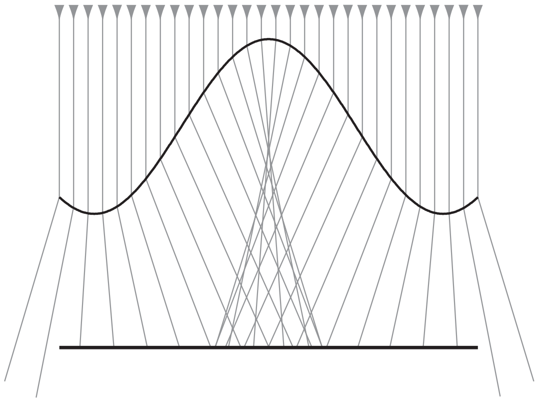


- Radiance estimate with filtering

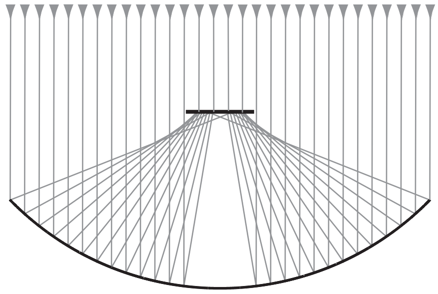
$$\widehat{L}_r(\mathbf{x}, \boldsymbol{\omega}) = \sum_{p=1}^{n} \pi K\left(|\mathbf{M}_p(\mathbf{x} - \mathbf{x}_p)|^2\right) f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) E_p$$

Case studies

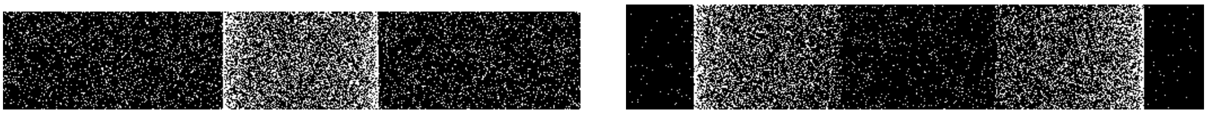
Refraction



Reflection



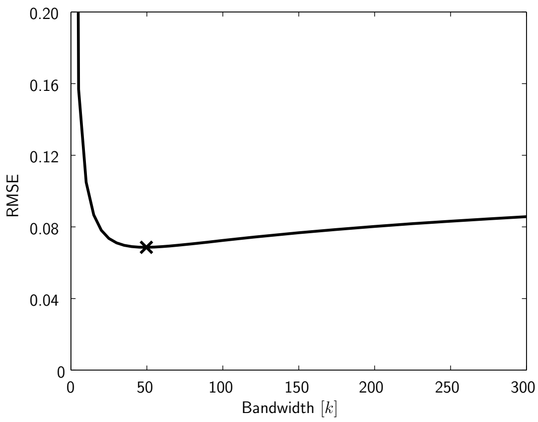
Photon distribution in the map

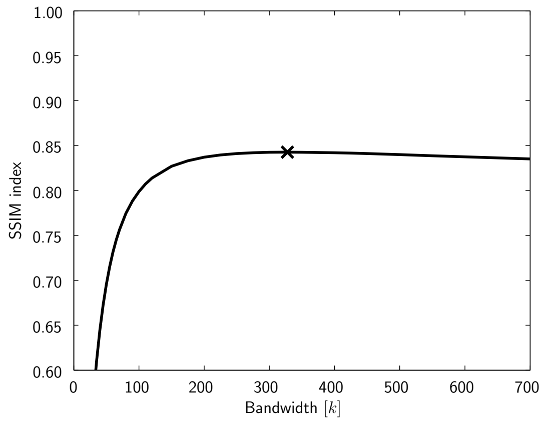


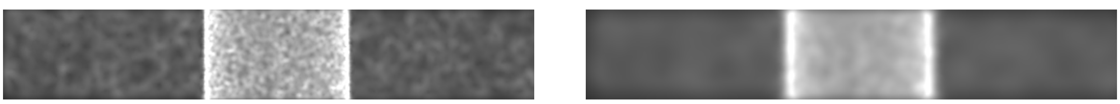
Rendered reference images

Optimal bandwidth - *k*nn photon mapping

- Finding the optimal bandwidth using image quality measures:
  - RMSE: **r**oot **m**ean **s**quare **e**rror.
  - SSIM: **s**tructural **sim**ilarity index.

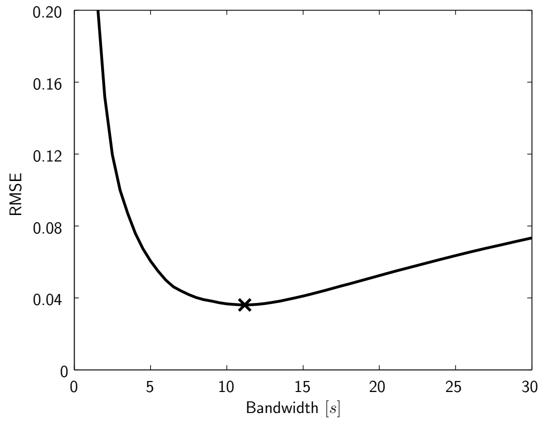


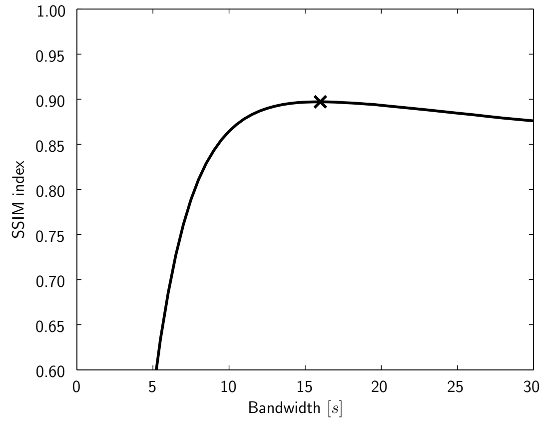




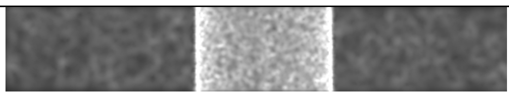
Optimal bandwidth - photon differentials

- Finding the optimal bandwidth using image quality measures:
  - RMSE: **r**oot **m**ean **s**quare **e**rror.
  - SSIM: **s**tructural **sim**ilarity index.





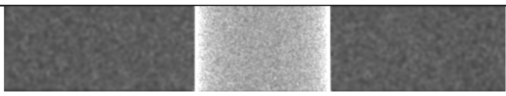
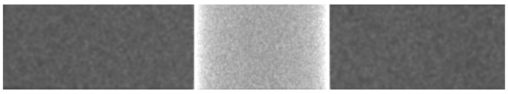
Refraction - equal number of photons comparison

Method	RMSE-optimal bandwidth	SSIM-optimal bandwidth
***k*nn**	 RMSE $= 0.0686$	 SSIM $= 0.8426$
pd	 RMSE $= 0.0361$	 SSIM $= 0.8972$

- Using 20,000 photons in the map.
- Comparing

*k*nn *k*-nearest neighbours photon mapping.
pd photon differentials.

Refraction - equal quality comparison

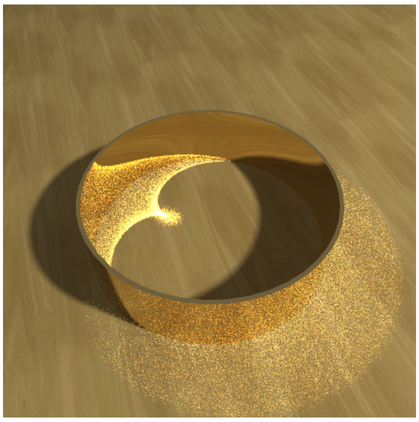
Method	RMSE-optimal bandwidth	SSIM-optimal bandwidth
k**nn**		
	n = 200,000, RMSE = 0.0363	n = 200,000, SSIM = 0.8776
		
	n = 500,000, RMSE = 0.0250	n = 500,000, SSIM = 0.8973
pd		
	n = 20,000, RMSE = 0.0361	n = 20,000, SSIM = 0.8972

Reflection - comparison

Method	RMSE-optimal bandwidth	SSIM-optimal bandwidth
k**nn**		
	n = 20,000, RMSE = 0.0740	n = 20,000, SSIM = 0.8207
		
	n = 75,000, RMSE = 0.0505	n = 75,000, SSIM = 0.8513
		
	n = 420,000, RMSE = 0.0262	n = 420,000, SSIM = 0.8919
pd		
	n = 20,000, RMSE = 0.0508	n = 20,000, SSIM = 0.8921

The gold ring cardioid caustic - equal time comparison

path traced reference (20 h)



RMSE=0.085

SSIM=0.79

path tracing ($\frac{20}{250}$ h)

RMSE=0.044

SSIM=0.95

standard photon mapping

RMSE=0.030

SSIM=0.96

photon differen-tials

References on photon differentials and more applications

- Photon differentials

- Schjøth, L., Frisvad, J. R., Erleben, K., and Sporring, J. Photon differentials. In *Proceedings of GRAPHITE 2007*, pp. 179–186, ACM, 2007.

- Photon differentials for diffuse interreflections.

- Fabianowski, B., and Dingliana, J. Interactive global photon mapping. *Computer Graphics Forum (Proceedings of EGSR 2009) 28*, 4 (June-July), pp. 1151–1159, 2009.

- Photon differentials for temporal blur.

- Schjøth, L., Frisvad, J. R., Erleben, K., and Sporring, J. Photon differentials in space and time. In *Computer Vision, Imaging and Computer Graphics: Theory and Applications*, P. Richard and J. Braz, eds., Communications in Computer and Information Science 229, pp. 274–286, December 2011.

- Photon differentials for participating media.

- Schjøth, L. Anisotropic Density Estimation in Global Illumination, PhD thesis, University of Copenhagen, Faculty of Science, 2009.
- Jarosz, W., Nowrouzezahrai, D., Sadeghi, I., and Jensen, H. W. A comprehensive theory of volumetric radiance estimation using photon points and beams. *ACM Transactions on Graphics 30*(1), pp. 5:1–5:19, January 2011.