Control of PDE's - The HUM method of J.L.Lions

We consider a hyperbolic bvp: $u'' + Au = 0$ on a bounded domain Ω, with initial conditions (u_0, u_1) in $L^2 \times H^{-1}$, and subject to homogeneous Dirichlet bc on the boundary $\Gamma = \partial\Omega$. The task is now to determine a function (control) $g \in L^2(\Gamma)$ such that the solution of the problem, now with the bc $\gamma u = g$ instead, satisfies $u(T, x) = u'(T, x) = 0$, for some prescribed time T. Also, instead of $g \in L^2(\Gamma)$ we would like to take $g \in L^2(\Gamma_0)$, where $\Gamma_0 \subset \Gamma$, and we would like g to be of minimal norm. The problem was (partially) solved by Lions in the late '80s and early '90s and after 2000 it has become a very powerful tool in the analysis of PDE's. It is today clear that the key to the method is the analysis of two operators of a pseudo-differential nature, actually a factorization of a time dependent version of the classical Dirichlet-to-Neumann operator from elliptic theory.

Michael Pedersen