Isogeometric Analysis and Shape Optimization in Fluid Mechanics

Peter Nørtoft

DTU Compute

Joint work with Jens Gravesen, Allan R. Gersborg, Niels L. Pedersen,

Morten Willatzen, Anton Evgrafov, Dang Manh Nguyen, and Tor Dokken

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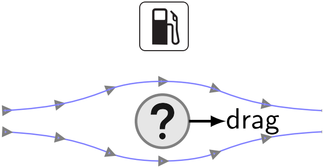
Technical University of Denmark

The aim is to analyze and optimize flows using isogeometric analysis

Shape Optimization



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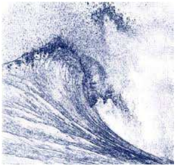
Navier-Stokes Flow Model



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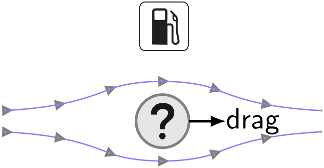

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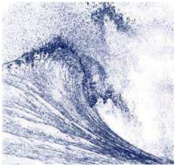

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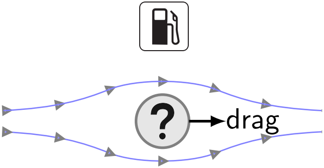
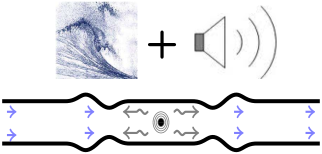


Flow Acoustics Model



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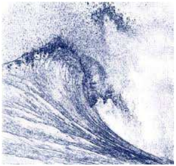
Navier-Stokes Flow Model



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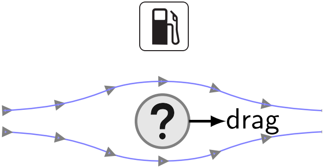
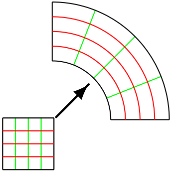


Isogeometric Analysis



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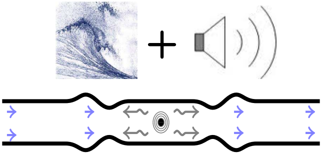
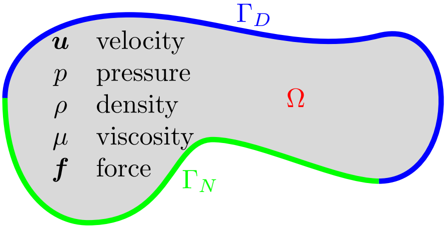


Flow problems are governed by a boundary value problem



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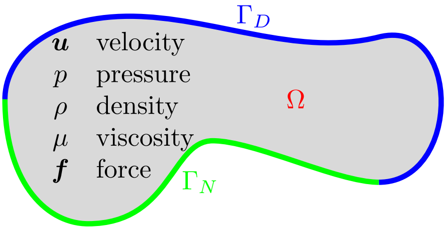

2D steady-state incompressible Navier-Stokes flow

$$\rho(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla p - \mu\nabla^2\boldsymbol{u} = \rho\boldsymbol{f} \quad \text{in } \Omega$$

$$\nabla\cdot\boldsymbol{u} = 0 \quad \text{in } \Omega$$

$$\boldsymbol{u} = \boldsymbol{u}^* \quad \text{on } \Gamma_D$$

$$(\,\mu\nabla u_i - p\,\boldsymbol{e}_i\,)\cdot\boldsymbol{n} = 0 \quad \text{on } \Gamma_N$$

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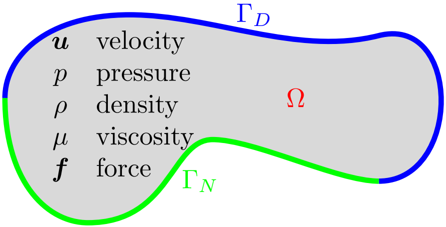

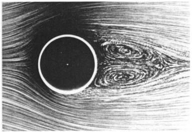

[M. Van Dyke]

$\mathrm{Re} = \frac{\rho U L}{\mu} \lesssim 10^3$

- viscous fluid
- slow flow
- small scale

2D steady-state incompressible Navier-Stokes flow

$$
\begin{aligned}
\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p - \mu \nabla^2 \boldsymbol{u} &= \rho \boldsymbol{f} && \text{in } \Omega \\
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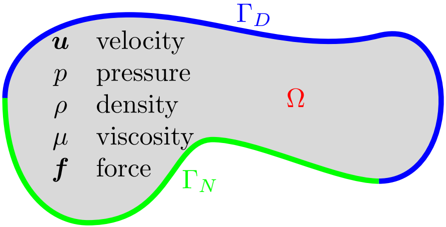

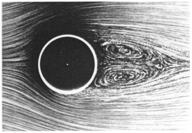
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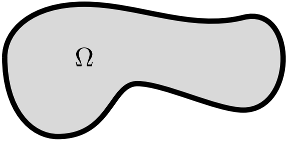
Challenge: solve this using isogeometric analysis

[Bazilevs et al., 2006b; Bazilevs & Hughes, 2008; Akkerman et al., 2010; . . .]

Both geometry, velocity and pressure are parametrized by B-splines



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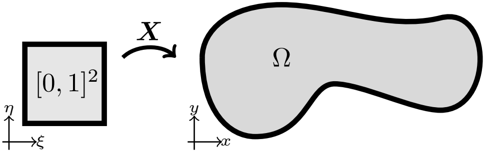
$$\boldsymbol{X}(\xi,\eta) = \sum_{i=1}^{N^g} \bar{\boldsymbol{x}}_i \mathcal{R}_i^g(\xi,\eta)$$



$\mathcal{R}_i^g$ Bivariate NURBS
$\bar{\boldsymbol{x}}_i$ control point

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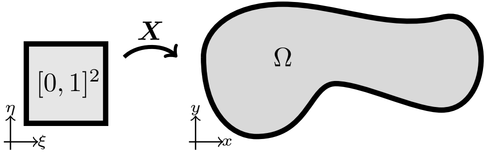

$\mathcal{R}_i^{\boldsymbol{u}}, \mathcal{R}_i^p, \mathcal{R}_i^g$ Bivariate NURBS/B-spline

$\bar{\boldsymbol{u}}_i, \bar{p}_i, \bar{\boldsymbol{x}}_i$ control point/variable

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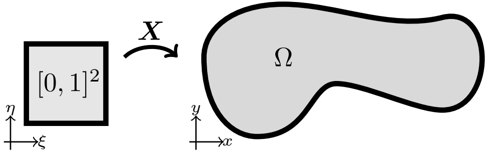

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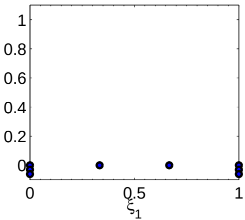

Parameter domain $[0,1]^2$

Univariate B-spline:

- knot vector
- polynomial degree

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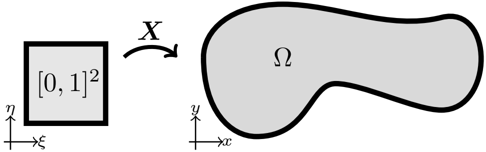

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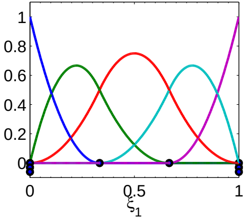

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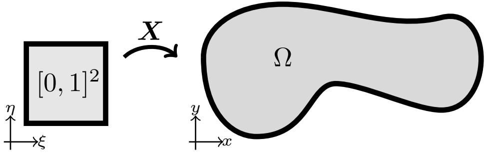

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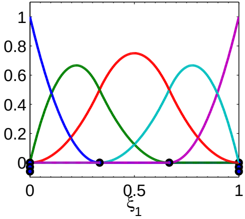

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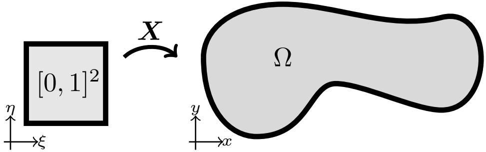

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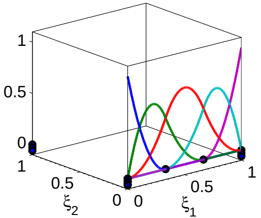

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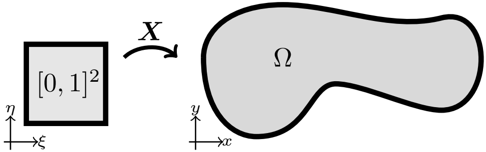

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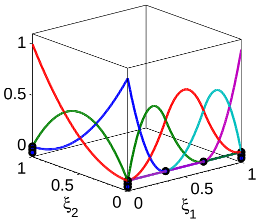

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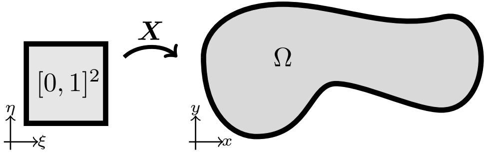

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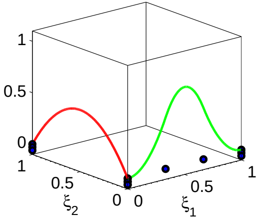

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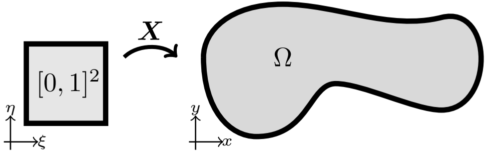

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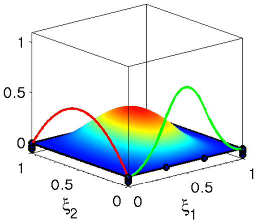

Parameter domain $[0,1]^2$

Univariate B-spline: $\mathcal{N}_i(\xi_1)$, $\mathcal{M}_j(\xi_2)$
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Bivariate Tensor Product B-spline:
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- 2 polynomial degrees
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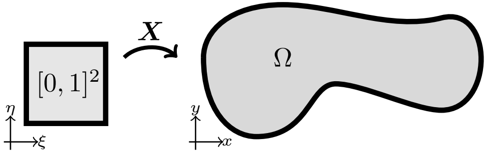

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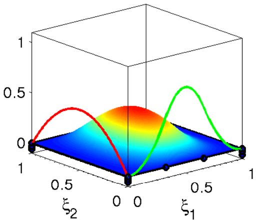

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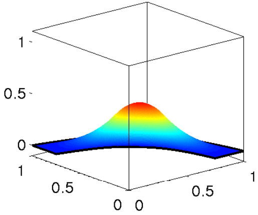

Physical domain Ω

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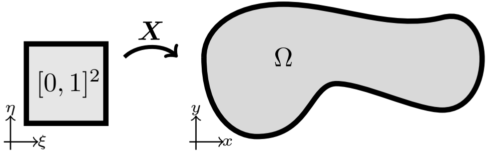

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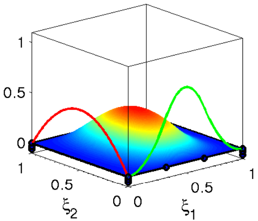
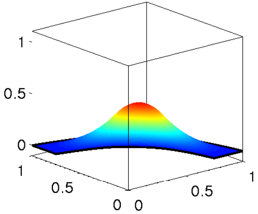


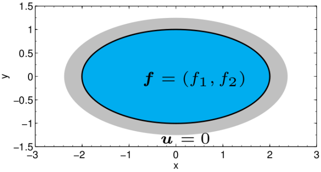
Parameter domain $[0,1]^2$

$$\boldsymbol{M}(\boldsymbol{U})\boldsymbol{U} = \boldsymbol{F}$$



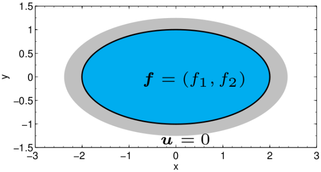
Physical domain Ω

Test of error convergence: flow with analytical solution



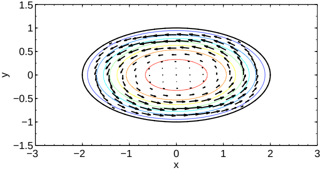
$$
\begin{aligned}
f_1 &= f_1(x,y) \\
f_2 &= f_2(x,y) \\
\boldsymbol{u}|_\Gamma &= \mathbf{0}
\end{aligned}
$$

Test of error convergence: flow with analytical solution



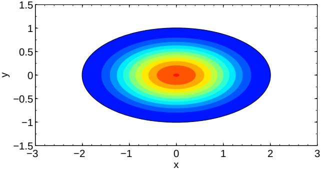
$$
\begin{aligned}
f_1 &= f_1(x,y) \\
f_2 &= f_2(x,y) \\
\boldsymbol{u}|_\Gamma &= \boldsymbol{0}
\end{aligned}
$$

$\boldsymbol{u}^\star$

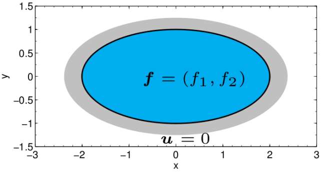


$$
\begin{aligned}
u_1^\star &= -U \, \sin(\pi \tilde{r}^2) y \\
u_2^\star &= U/4 \, \sin(\pi \tilde{r}^2) x \\
p^\star &= 4/\pi^2 + \cos(\pi \tilde{r}) \\
\tilde{r} &= \sqrt{(x/2)^2 + y^2}
\end{aligned}
$$

$p^\star$

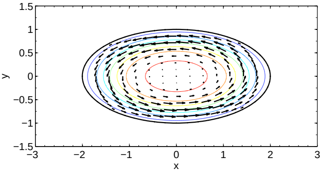


Test of error convergence: flow with analytical solution



$$f_1 = f_1(x,y)$$
$$f_2 = f_2(x,y)$$
$$\boldsymbol{u}|_\Gamma = \boldsymbol{0}$$

$\boldsymbol{u}^\star$



$$u_1^\star = -U \sin(\pi\tilde{r}^2)y$$
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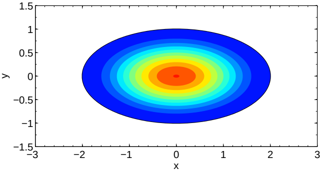
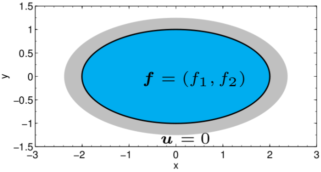
$p^\star$



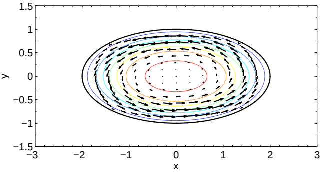
$$\text{Re} = 200$$

Test of error convergence: flow with analytical solution



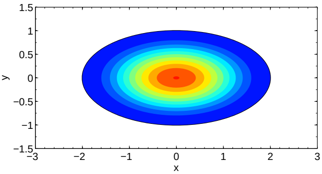
$$\begin{aligned} f_1 &= f_1(x,y) \\ f_2 &= f_2(x,y) \\ \boldsymbol{u}|_\Gamma &= \boldsymbol{0} \end{aligned}$$

$\boldsymbol{u}^\star$



$$\begin{aligned} u_1^\star &= -U \sin(\pi\tilde{r}^2)y \\ u_2^\star &= U/4 \sin(\pi\tilde{r}^2)x \\ p^\star &= 4/\pi^2 + \cos(\pi\tilde{r}) \\ \tilde{r} &= \sqrt{(x/2)^2 + y^2} \end{aligned}$$

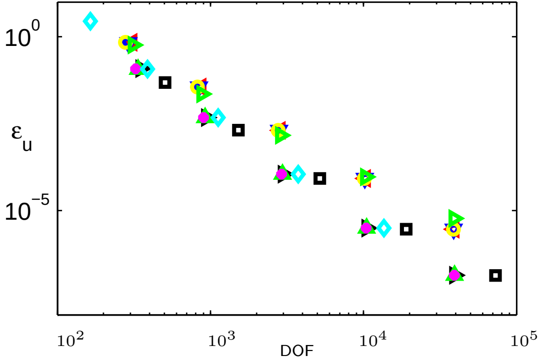
$p^\star$



$$\mathsf{Re} = 200$$

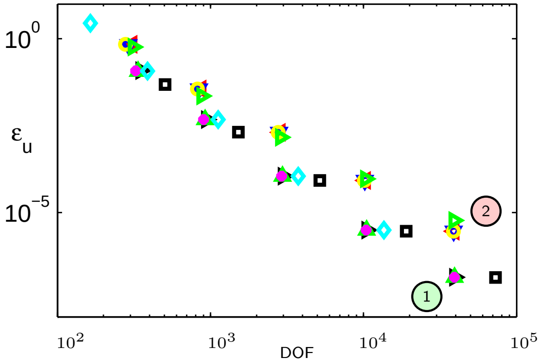
$$\epsilon_u^2 = \iint_\Omega \|\boldsymbol{u}(x,y) - \boldsymbol{u}^\star(x,y)\|^2 \, \mathrm{d}x \, \mathrm{d}y$$

Discretizations with higher regularity perform better



a	$u4_1^14_1^1\ v4_1^14_1^1\ p4_0^14_0^1$
b	$u4_0^24_0^2\ v4_0^24_0^2\ p4_0^14_0^1$
c	$u4_1^14_1^1\ v4_1^14_1^1\ p3_0^13_0^1$
d	$u4_0^24_0^2\ v4_0^24_0^2\ p3_0^13_0^1$
e	$u4_1^14_1^1\ v4_1^14_1^1\ p2_0^12_0^1$
f	$u4_0^24_0^2\ v4_0^24_0^2\ p2_0^12_0^1$

Discretizations with higher regularity perform better

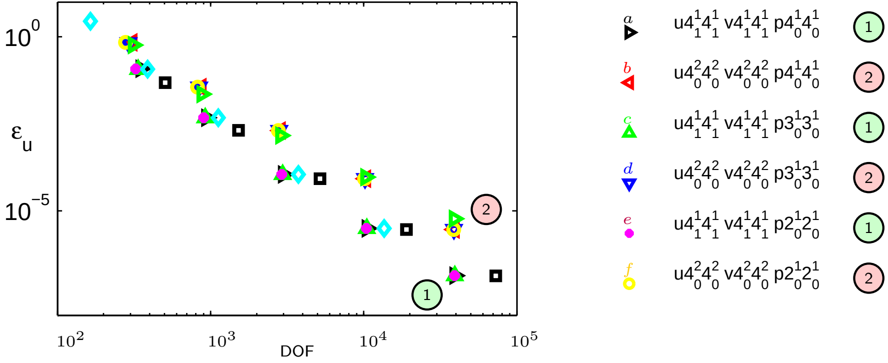


a	$u4^1_14^1_1\ v4^1_14^1_1\ p4^1_04^1_0$	1
b	$u4^2_04^2_0\ v4^2_04^2_0\ p4^1_04^1_0$	2
c	$u4^1_14^1_1\ v4^1_14^1_1\ p3^1_03^1_0$	1
d	$u4^2_04^2_0\ v4^2_04^2_0\ p3^1_03^1_0$	2
e	$u4^1_14^1_1\ v4^1_14^1_1\ p2^1_02^1_0$	1
f	$u4^2_04^2_0\ v4^2_04^2_0\ p2^1_02^1_0$	2

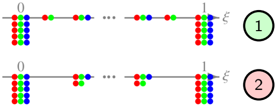
1

2

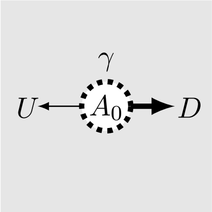
Discretizations with higher regularity perform better



	Smoothness	Mesh Density
Strategy 1	**High**	High
Strategy 2	Low	**Low**



We design a body with minimal drag

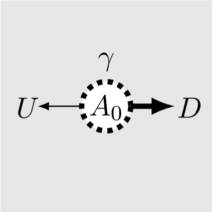


[Pironneau, 1973; 1974; Mohammadi & Pironneau, 2010]

We design a body with minimal drag

Aim

Design boundary γ of body with area A_0 travelling at constant speed U to minimize the drag D



[Pironneau, 1973; 1974; Mohammadi & Pironneau, 2010]

We design a body with minimal drag

Aim

Design boundary γ of body with area A_0 travelling at constant speed U to minimize the drag D

Optimization Problem

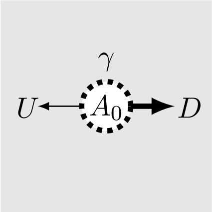
$$\min_{\gamma(\bar{\boldsymbol{x}}_b)} \quad c = D + \epsilon R \quad \text{drag objective}$$

$$\text{s. t.} \quad \text{Area} \geq A_0 \quad \text{area constraint}$$

$$\text{linear design constraints} \quad \boldsymbol{L}_-(\bar{\boldsymbol{x}}_b) \leq \boldsymbol{L}(\bar{\boldsymbol{x}}_b) \leq \boldsymbol{L}_+(\bar{\boldsymbol{x}}_b)$$

$$\text{governing equations} \quad \boldsymbol{MU} = \boldsymbol{F}$$

$$D = \int_{\boldsymbol{\gamma}} \Big(-p\boldsymbol{I} + \mu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T)\Big)\, \boldsymbol{n}\ \mathrm{d}s \cdot \boldsymbol{e}_u$$



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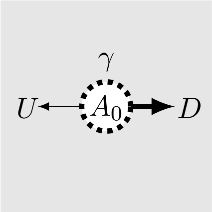
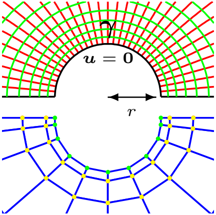
$$\text{governing equations} \quad \boldsymbol{M}\boldsymbol{U} = \boldsymbol{F}$$

$$D = \int_\gamma \Big(-p\boldsymbol{I} + \mu(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)\Big)\,\boldsymbol{n}\ \mathrm{d}s \cdot \boldsymbol{e}_u$$





[Pironneau, 1973; 1974; Mohammadi & Pironneau, 2010]

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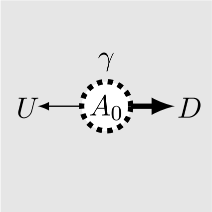
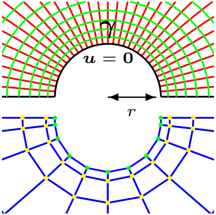
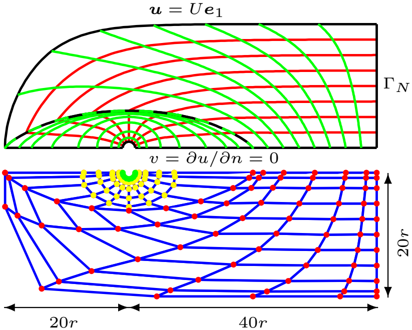
$$\text{governing equations} \quad \boldsymbol{MU} = \boldsymbol{F}$$

$$D = \int_{\boldsymbol{\gamma}} \Big(-p\boldsymbol{I} + \mu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T) \Big) \boldsymbol{n} \, \mathrm{d}s \cdot \boldsymbol{e}_u$$

[Pironneau, 1973; 1974; Mohammadi & Pironneau, 2010]

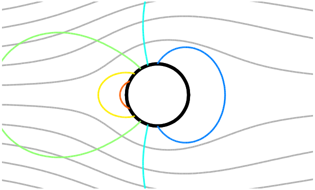






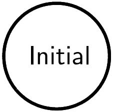
The optimal shape is longer and more slender for higher speeds

Flow before optimization (U=1)

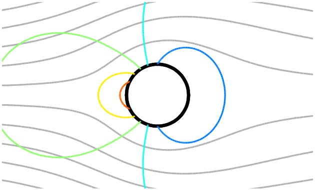


The optimal shape is longer and more slender for higher speeds

Optimal shapes for different speeds



Flow before optimization (U=1)



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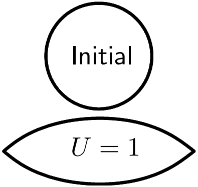
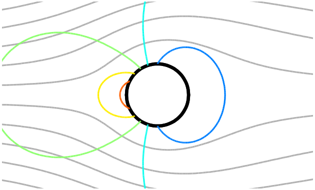
Optimal shapes for different speeds



Flow before optimization (U=1)



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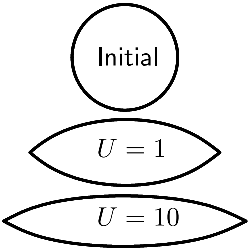
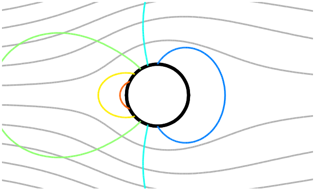
Optimal shapes for different speeds

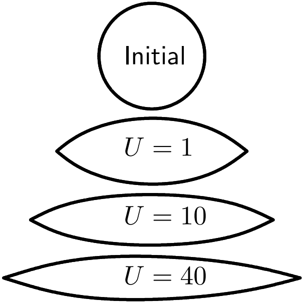


Flow before optimization (U=1)

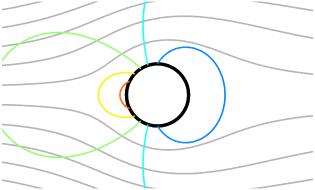


The optimal shape is longer and more slender for higher speeds

Optimal shapes for different speeds

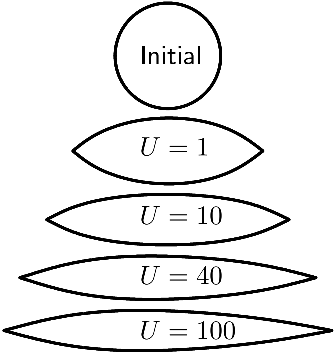


Flow before optimization (U=1)

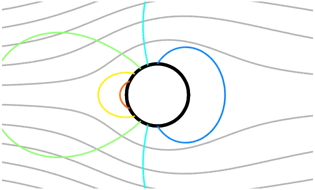


The optimal shape is longer and more slender for higher speeds

Optimal shapes for different speeds



Flow before optimization (U=1)



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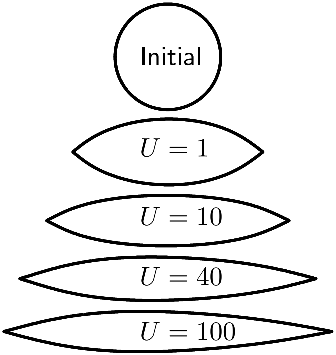
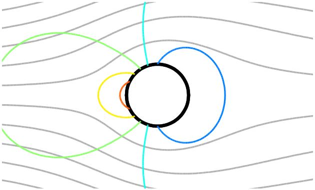
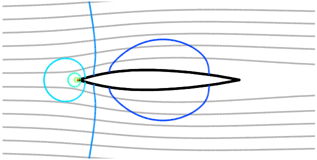
Optimal shapes for different speeds



Flow before optimization (U=1)



Flow after optimization (U=100)

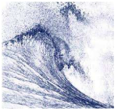


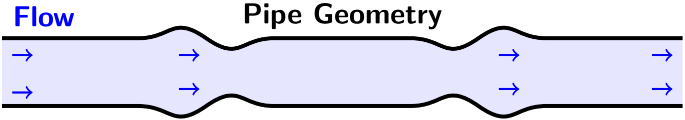
Note: low Reynolds numbers

We model geometric effects on sound propagation through flow in 2D ducts

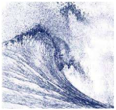
Pipe Geometry

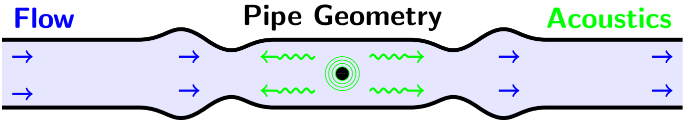
We model geometric effects on sound propagation through flow in 2D ducts

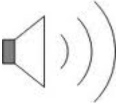




We model geometric effects on sound propagation through flow in 2D ducts







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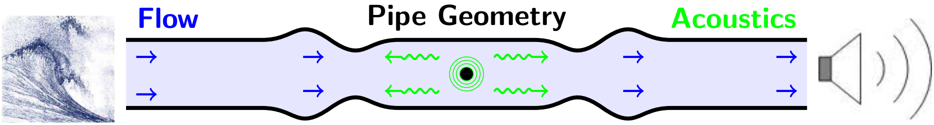
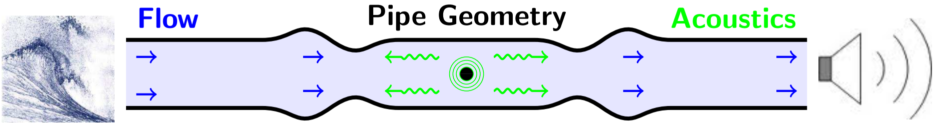


p	pressure
$\boldsymbol{u}$	velocity
ρ	density

We model geometric effects on sound propagation through flow in 2D ducts

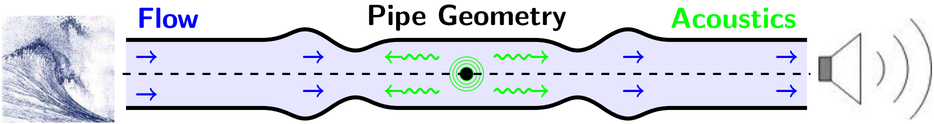


p	pressure
$\boldsymbol{u}$	velocity
ρ	density

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla p - \mu\nabla^2\boldsymbol{u} = 0$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}) = 0$$

We model geometric effects on sound propagation through flow in 2D ducts



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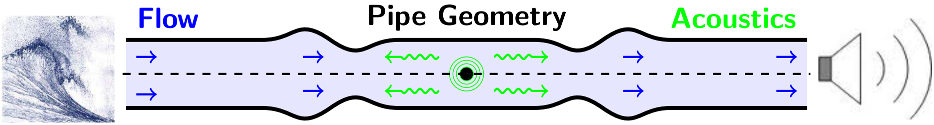
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\begin{aligned} p &= p_0 + p' \\ \boldsymbol{u} &= \boldsymbol{u}_0 + \boldsymbol{u}' \\ \rho &= \rho_0 + \rho' \end{aligned}$$

Background Flow: p_0, $\boldsymbol{u}_0$, ρ_0

Acoustic Disturbance: p', $\boldsymbol{u}'$, ρ'

We model geometric effects on sound propagation through flow in 2D ducts



p pressure
$\boldsymbol{u}$ velocity
ρ density

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla p - \mu\nabla^2\boldsymbol{u} = 0$$
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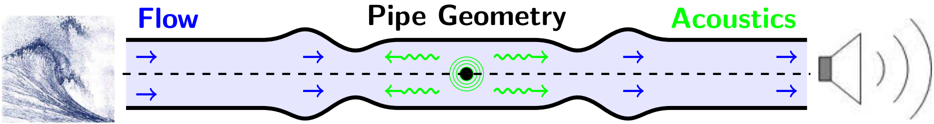
1. Background Flow: p_0, $\boldsymbol{u}_0$, ρ_0

$\Downarrow$

2. Acoustic Disturbance: p', $\boldsymbol{u}'$, ρ'

We model geometric effects on sound propagation through flow in 2D ducts



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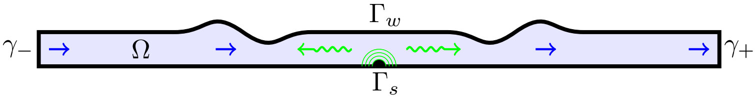
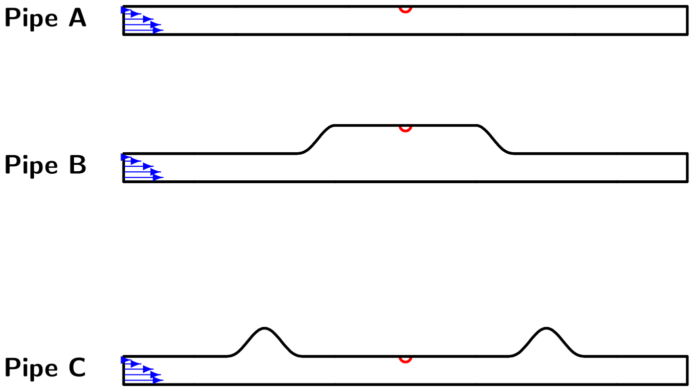
$\Downarrow$

2. Acoustic Disturbance: p', $\boldsymbol{u}'$, ρ'

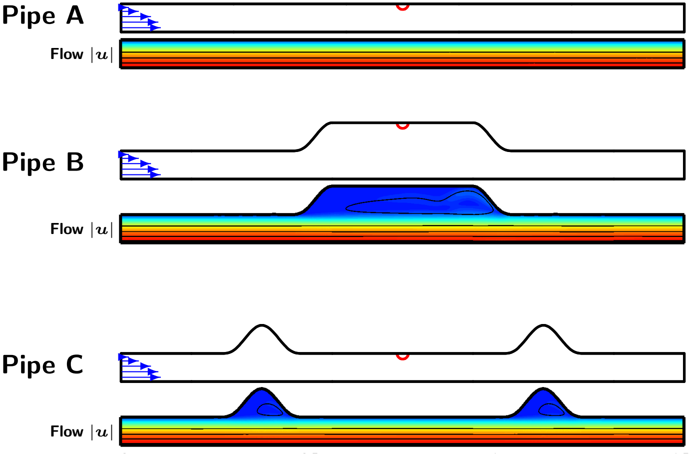


We examine how the flow affects the sound in different ducts



Fluid	Air
Radius	1 cm
Speed	1 m/s
Frequency	∼25 kHz

We examine how the flow affects the sound in different ducts



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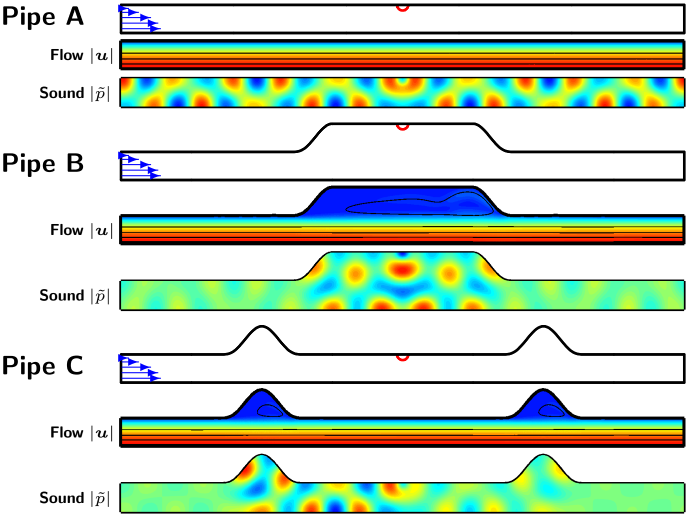
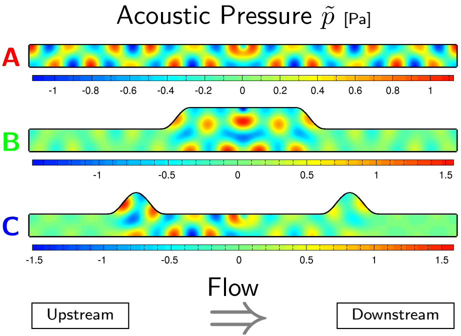


Fluid	Air
Radius	1 cm
Speed	1 m/s
Frequency	∼25 kHz

The corrugated duct exhibits stronger flow-acoustic coupling



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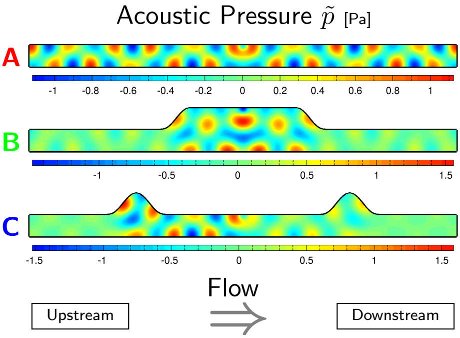
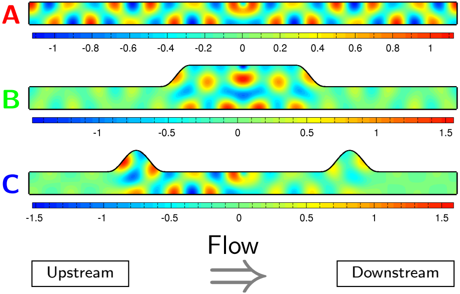


Measure of Flow-Acoustic Coupling

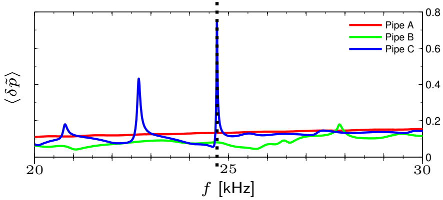
$$\langle \delta \tilde{p} \rangle = \frac{\iint_\Omega |\tilde{p}(\boldsymbol{x}) - \tilde{p}(-\boldsymbol{x})| \, \mathrm{d}A}{\iint_\Omega |\tilde{p}(\boldsymbol{x})| \, \mathrm{d}A}$$

The corrugated duct exhibits stronger flow-acoustic coupling

Acoustic Pressure $\tilde{p}$ [Pa]



Sound Frequency Sensitivity



Measure of Flow-Acoustic Coupling

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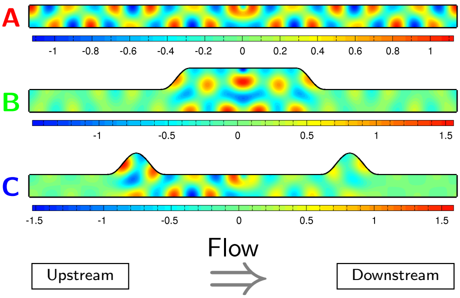

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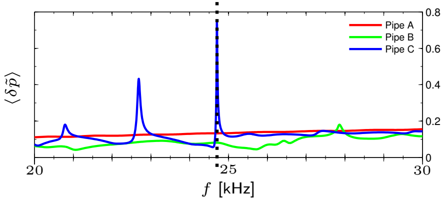
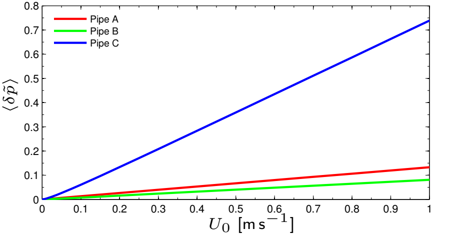


Flow Speed Sensitivity

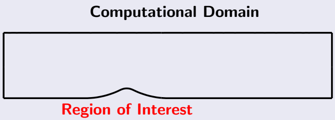


Ongoing work

Flow Discretizations using Locally Refinable B-splines

Ongoing work

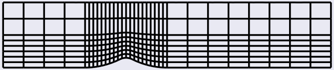
Flow Discretizations using Locally Refinable B-splines



Ongoing work

Flow Discretizations using Locally Refinable B-splines

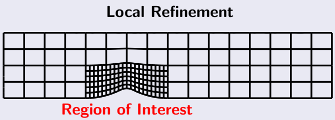
Global Refinement



Region of Interest

Ongoing work

Flow Discretizations using Locally Refinable B-splines



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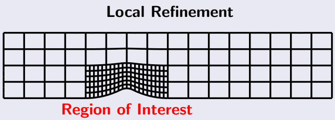

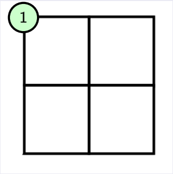

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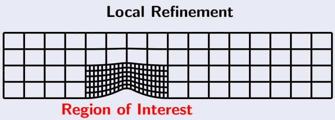

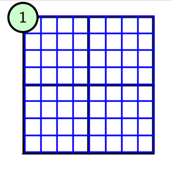

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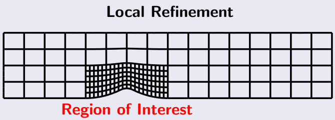

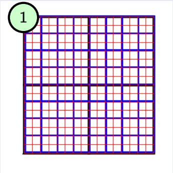

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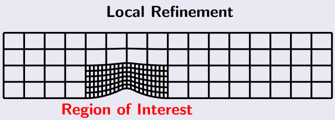

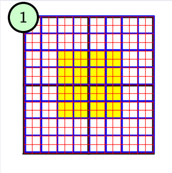

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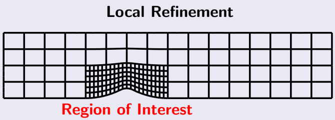

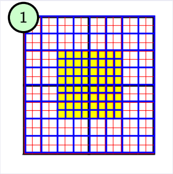

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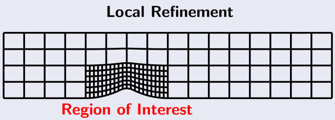

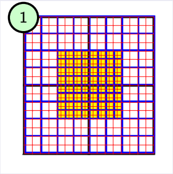

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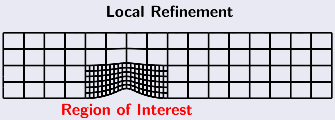

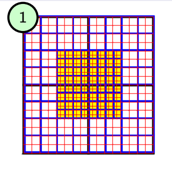

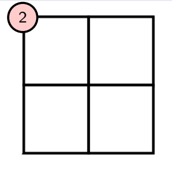

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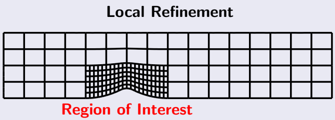

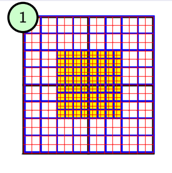

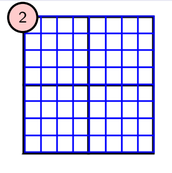

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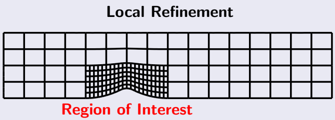

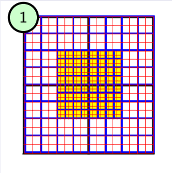

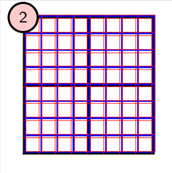

Ongoing work

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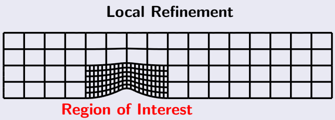

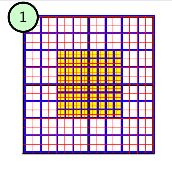

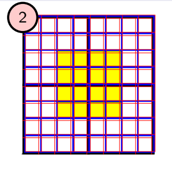

Ongoing work

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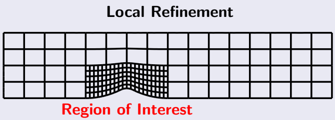

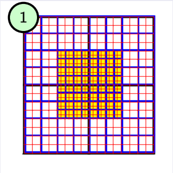

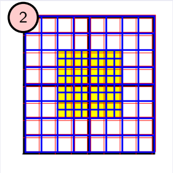

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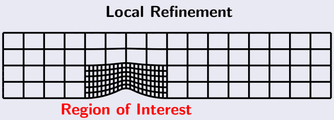

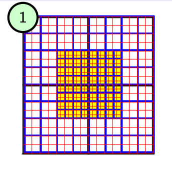

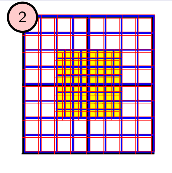

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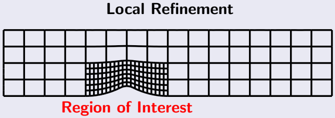
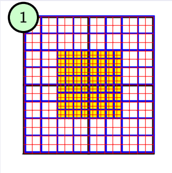
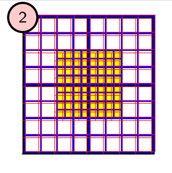
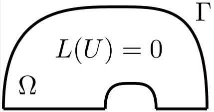






Optimization of Parametrizations for Isogeometric Analysis



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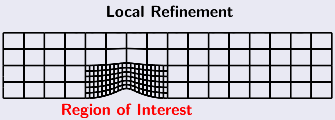

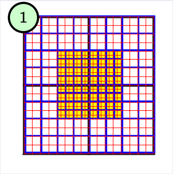

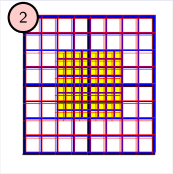

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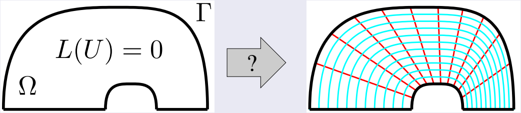

Ongoing work

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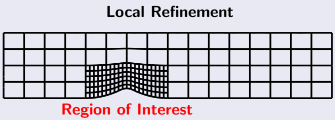

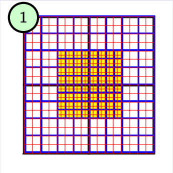

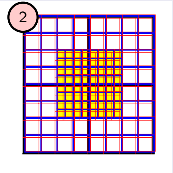

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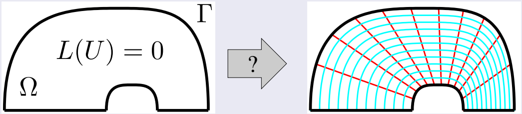

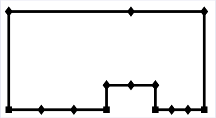

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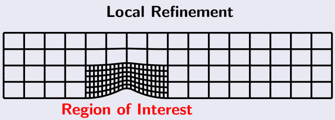

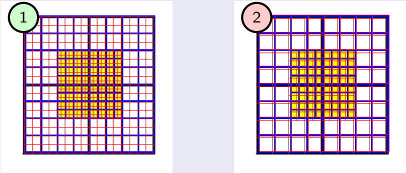

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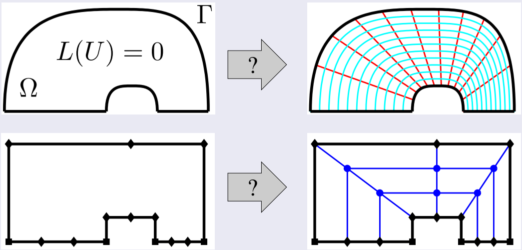

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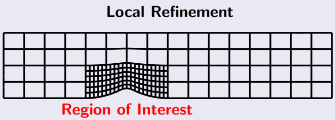

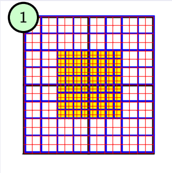

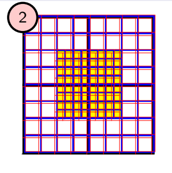

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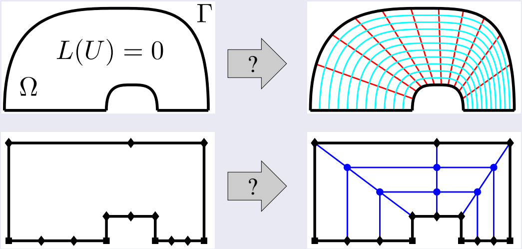


$$\begin{array}{ll} \text{minimize} & c(\Omega, U) \\ \text{such that} & \det(\boldsymbol{J}) > 0 \\ \text{where} & \partial\Omega = \Gamma \\ & L(U) = 0 \end{array}$$

Summary

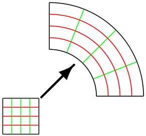
Isogeometric Method

Isogeometric Analysis of Flows

Isogeometric Shape Optimization of Flows

Isogeometric Analysis of Flow Acoustics

Summary



Isogeometric Method

- IGA = FEM (high regularity) + CAD (exact geometry)

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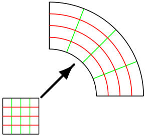

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Isogeometric Analysis of Flows

- Facilitates High-regularity discretizations of flow variables

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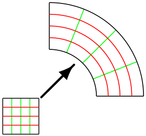

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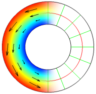

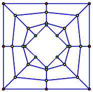

Isogeometric Shape Optimization of Flows

- Unites design and analysis models through B-splines

Isogeometric Analysis of Flow Acoustics

Summary

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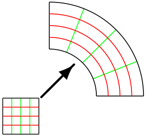

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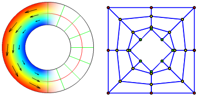
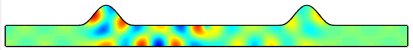
Isogeometric Shape Optimization of Flows

- Unites design and analysis models through B-splines

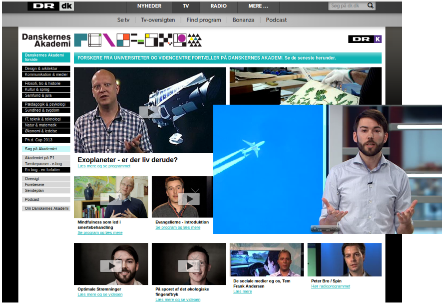


Isogeometric Analysis of Flow Acoustics

- Identifies geometric enhancement of flow-acoustic coupling



The 3 minute version: `http://www.dr.dk/da`



Thank You For Your Attention!