



3rd place algorithm

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Core Imaging Library (CIL) - Team



Open challenge on limited-angle reconstruction



https://www.fips.fi/HTC2022.php

CIL team entry finished 3rd – beaten by two teams using machine learning



Test data provided and scan setup

- Four 360 degree measured sinogram data sets to generate limited-angle data from
- Reference reconstructions and segmentations



Table 1: Limited-angle tomography difficulty groups			
Group Angular range Angular increment Number of projections			
1	90°	0.5°	181
2	<mark>80°</mark>	0.5°	161
3	70°	0.5°	141
4	<mark>60°</mark>	0.5°	121
5	50°	0.5°	101
6	40°	0.5°	81
7	30°	0.5°	61



Assessment of (segmented) reconstructions

The reconstructions will be assessed quantitatively, comparing the reconstructed binary image I_r with the ground truth binary image I_t , assigning a numeric score. I_r is assumed to have a dimension of 512 x 512 pixels, otherwise a score 0 will be given to the reconstruction I_r .

The score is based on the confusion matrix of the classification of the pixels between empty (0) or material (1). The confusion matrix is composed by

$$TP = \sum_{i,j} (I_t \cap I_r)_{ij}$$
$$FP = \sum_{i,j} (\bar{I}_t \cap I_r)_{ij}$$
$$FN = \sum_{i,j} (I_t \cap \bar{I}_r)_{ij}$$
$$TN = \sum_{i,j} (\bar{I}_t \cap \bar{I}_r)_{ij}$$
$$\mathbf{M} = \begin{bmatrix} TP & FN\\ FP & TN \end{bmatrix}$$

The score of the reconstruction is given by the Matthews correlation coefficient (MCC)

$$S = \frac{TP \times TN - FP \times FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}$$

where $S \in [-1, 1]$. A score of +1 (best) represents a perfect reconstruction, 0 no better than random reconstruction, and -1 (worst) indicates total disagreement between reconstruction and ground truth. A python code that implements the scoring will be provided to the competitors. The same code will be used to assess the algorithms.

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Motivation



- See how far "conventional" CT pre-processing plus variational methods could go
- Use existing general purpose CIL tools as much as possible – limited time for new dev
- 5 submissions, variations of Preprocessing Reconstruction Segmentation

Segmentation

- 'blind' segmentation of the test data
- Not our expertise!
- Otsu triple-threshold worked consistently for the test data at 30 degrees

- Otsu thresholded segmentation
 - Identifies signal peak

- Otsu triple-thresholded segmentation
 - Strong signal
 - Messy signal
 - Messy background
 - Strong background



Renormalization of sinogram

- Background attenuation should have a mean at zero
- Test data had an offset i.e. the normalization image was brighter than the data
 - Convert data back to I/IO, renormalize for a peak at 1, convert back to absorption



Beam hardening correction

- Lower energy rays are preferentially absorbed leading to a non-linear measurement
- Single material scan can be linearised to an effective monochromatic energy
- Correction to the linear attenuation of acrylic at 24.7 KeV, mu = 0.0409 mm⁻¹





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Zero-padding

- The reconstruction window extends outside the field of view
- Causes a non-zero background outside the radius of the detector
- Zero-Padding the acquisition data corrects this





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Reconstruction: Exploit prior knowledge

Construct optimization problem to express what we know:

- Single homogeneous material
- Sharp edges
- Object is approximately disk shaped
- Zero attenuation outside the object
- Constant value of 0.0409 mm⁻¹ inside the object
- Edges perpendicular to projection angles are the most difficult (micro-local analysis)

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Prior knowledge: homogeneous material with sharp edges





LS + isoTV Mask: none 50 deg. Score: 0.713



Prior knowledge: approximately disk shaped



I.D. Coope Circle fitting by linear and nonlinear least squares in 2D https://link.springer.com/article/10.1007/BF00939613

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Disk shape with known attenuation as constraints

$$\min_{\mathbf{u}} \quad a_1 \|\mathbf{A}\mathbf{u} - \mathbf{b}\|_2^2 + a_2 \mathrm{ITV}(\mathbf{u})$$

s.t. $\mathbf{0} \le \mathbf{u} \le v\mathbf{m}$

LS + isoTV Mask: fitted 50 deg. Score: 0.919



Anisotropic TV

- Blurred edges along central projection direction
- Rotate to align with coordinate axis
- Apply single-directional TV to encourage edges in blurred direction
- Remember to check convergence!
- Rotate back



LS + isoTV + disk + xTV

$$\min_{\mathbf{u}} \quad a_1 \| \mathbf{A}\mathbf{u} - \mathbf{b} \|_2^2 + a_2 \mathrm{ITV}(\mathbf{u}) + a_3 \mathrm{ATV}_x(\mathbf{u})$$

s.t. $\mathbf{0} \le \mathbf{u} \le v \mathbf{m}$

LS + isoTV + xTV Mask: fitted 50 deg. Score: 0.934



LS + isoTV + disk + xTV (converged)

$$\min_{\mathbf{u}} \quad a_1 \| \mathbf{A}\mathbf{u} - \mathbf{b} \|_2^2 + a_2 \mathrm{ITV}(\mathbf{u}) + a_3 \mathrm{ATV}_x(\mathbf{u})$$

s.t. $\mathbf{0} \le \mathbf{u} \le v \mathbf{m}$

Converged LS + isoTV + xTV Mask: fitted 50 deg. Score: 0.973



Primal Dual Hybrid Gradient (PDHG) method in CIL

CIL offers a range of optimization algorithms, incl GD, FISTA, ADMM and PDHG:

$$\min_{\mathbf{u}} \quad f(\mathbf{K}\mathbf{u}) + g(\mathbf{u})$$

where
$$f(\mathbf{K}\mathbf{u}) = \sum_{i} f_i(\mathbf{K}_i\mathbf{u})$$

Rewrite our optimization problem for PDHG:

$$\min_{\mathbf{u}} \quad a_1 \|\mathbf{A}\mathbf{u} - \mathbf{b}\|_2^2 + a_2 \|\mathbf{D}\mathbf{u}\|_{2,1} + a_3 \|\mathbf{D}_{\mathbf{x}}\mathbf{u}\|_1 + \chi_{[\mathbf{0},v\mathbf{m}]}(\mathbf{u})$$

$$f = \begin{pmatrix} a_1 \| \cdot - \mathbf{b} \|_2^2 \\ a_2 \| \cdot \|_{2,1} \\ a_3 \| \cdot \|_1 \end{pmatrix} \quad \mathbf{K} = \begin{pmatrix} \mathbf{A} \\ \mathbf{D} \\ \mathbf{D}_{\mathbf{x}} \end{pmatrix} \qquad g = \chi_{[\mathbf{0}, v\mathbf{m}]}$$

Solving with CIL – "near-math" syntax

$$f = \begin{pmatrix} a_1 \| \cdot - \mathbf{b} \|_2^2 \\ a_2 \| \cdot \|_{2,1} \\ a_3 \| \cdot \|_1 \end{pmatrix}$$

$$F = BlockFunction(a1*L2NormSquared(data), a2*MixedL21Norm(), a3*L1Norm())$$

$$K = \begin{pmatrix} \mathbf{A} \\ \mathbf{D} \\ \mathbf{D}_{\mathbf{x}} \end{pmatrix}$$

$$K = BlockOperator(ProjectionOperator(ig, ag), GradientOperator(ig), FiniteDifferenceOperator(ig, 'horizontal_x'))$$

$$g = \chi_{[\mathbf{0}, v\mathbf{m}]}$$

$$G = IndicatorBoxPixelwise(lower=0.0, upper=v*m)$$

$$algo = PDHG(initial=ig.allocate(0.0), f=F, g=G, operator=K, max_iteration=2000)$$

$$algo.run()$$

$$g = dordo.pasca@str.acuk @Green (here) = here = here)$$

Mesh type sample – orientation matters!

60° "easy" 40° "very hard" 60° "hard" **Ground truth** Result

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Conclusions

Thanks to organizers – hope to see more challenges!



- HTC submission: github.com/TomographicImaging/CIL-HTC2022-Algo2
- Main site: <u>ccpi.ac.uk/cil</u> ٠
- Discord community: <u>discord.gg/9NTWu9MEGq</u> •
- Job opening(s) at



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- CIL Training course & Hackathon March 20-24, Cambridge, UK
 - "Bring your data",
 - nonlinear problems,
 - deep learning







