Micro Project 3: Variational Methods

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1 Enter the Real World

On the course website you will find a Zip file with some MATLAB MAT-files, and each of these contains a sinogram that corresponds to the center slice of a set of cone beam projections. The Zip file also includes a MATLAB script that demonstrates how to compute a basic reconstruction using a subset of the projections. You may use this example or the data that you measured in the first week as a starting point for the micro project. Of course, you may also simulate measurements if you decide to investigate something that is difficult to evaluate when working with real data. In addition, you may use MATLAB or Python/CIL for the micro project.

2 The Micro Project

The goal of the micro project is to investigate the use of optimization and variational methods for X-ray CT. The project is open, and you decide what to investigate. Here are some ideas:

- How many projections do we need? Investigate how many projections you need to obtain a reasonably "good" reconstruction based on the TV-regularized reconstruction model.
- What can we learn from the residual? Using a regularized reconstruction model (*e.g.*, Tikhonov), investigate what $Ax^\star(\gamma) - b$ looks like (*i.e.*, visualized as a sinogram) for different values of the regularization parameter $\gamma > 0$. Can the residual image be used to select a suitable regularization parameter?
- Pick a regularized reconstruction model (e.g., Tikhonov or TV) and implement a method that "roughly" traces the associated trade-off curve. For example, you could use "warm-starting" to speed up the computation of reconstructions $x^\star(\gamma_i)$ for a sequence of regularization parameters $\gamma_1, \ldots, \gamma_p$.
- Compare the "performance" of an algebraic and a variational reconstruction method on a real data set. Examples of performance measures include subjective reconstruction quality, CPU time, residual image, residual norm, and/or severity of artifacts.
- Investigate what role the smoothing parameter plays when a smooth approximation of the total variation penalty is used for TV-regularized reconstruction. How does the smoothing parameter affect the reconstruction quality, the number of iterations, etc.?
- Extend the study from the first two weeks, and try e.g. TV regularization for the reconstruction problem with incomplete data. Here you can use synthetic and/or real data in MATLAB or CIL.

3 Practical information and assessment

You will be working in groups of 2–3 students. At the end of Friday, starting at 2pm, each group will present their work to the lecturers and the other groups, and each group has 10–15 minutes for their presentation. All group members are expected to contribute. There is no written report – assessment is based solely on the oral presentation. As part of your

presentation, please explain what you have chosen to investigate, which theory/tools you have used, show your results (plots, reconstructions, etc.), and state your conclusions.