Limited-Data CT for Underwater Pipeline Inspection

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Subsea CT-Scanner by FORCE Technology, Denmark

Alternative to ultrasound: use X-ray scanning to compute cross-sectional images of oil pipes lying on the seabed, to detect defects, cracks, etc. in the pipe.

Illustration courtesy of FORCE Technology.
Subsea CT-Scanner by FORCE Technology, Denmark

Subsea CT-Scanner by FORCE Technology, Denmark

Comparing Geometries

Limitations in the scanner device

Data: Narrow high intensity X-Ray beam → available from a limited view.

Goal: Design set-up to capture as much detail of the pipe as possible.

Full illumination Not possible

Centered Possible set-up

Off-centered Possible set-up

Nicolai A. B. Riis: Limited-Data CT for Underwater Pipeline Inspection
Comparing Geometries

Limitations in the scanner device

Data: Narrow high intensity X-Ray beam → available from a limited view.

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The Computed Tomography Forward Model

Continuous formulation, limited data

The measured projections \( g \) for the object \( f \) are described by

\[
g(\theta, s) = (\mathcal{R}\ell f)(\theta, s) + \text{noise}
\]

where \((\mathcal{R}\ell f)(\theta, s) = (\mathcal{R}f)(\theta, s)\) for those pairs \((\theta, s)\) corresponding to the \(\ell\)imited illumination, and \(\mathcal{R}\) is the Radon transform.

Corresponding algebraic model

The measured data \( b \) for a discretized object \( x \) is described by

\[
b = A\ell x + e,
\]

\( b \in \mathbb{R}^m \), \( x \in \mathbb{R}^n \), \( A\ell \in \mathbb{R}^{m \times n} \),

where \( e \in \mathbb{R}^m \) with \( e_i \sim N(0, \sigma^2) \) and \( A\ell \) is the discretion of \( \mathcal{R}\ell \).
Full and Two Different Limited Illuminations
Characterising What We Can Measure – Centered Beam

**Microlocal analysis:** a singularity at position $\chi$ with direction $\xi$ is visible if and only if data from the line through $\chi$ perpendicular to $\xi$ is present.

Measured data from one view/projection.

Visible from one view/projection.

Visible from all views/projections.
**Characterising What We Can Measure – Off-Center Beam**

**Microlocal analysis:** a singularity at position $\chi$ with direction $\xi$ is visible if and only if data from the line through $\chi$ perpendicular to $\xi$ is present.

Measured data from one view/projection.

Visible from one view/projection.

Visible from all views/projections.
Reconstruction: Incorporating Sparsity

We need a robust algorithm that can utilize all information in the data.

Use the algebraic system

\[ b = A^\ell \, x + e, \quad b \in \mathbb{R}^m, \quad x \in \mathbb{R}^n, \quad A^\ell \in \mathbb{R}^{m \times n}. \]

and compute a sparse representation of the solution \( x \) in a “basis” that is well suited for the problem.

If we can represent \( x \) with few non-zero basis functions, we have fewer unknowns to determine → same data, fewer unknowns, “easier” problem.
Representing pipe in a sparse “basis”

**Frame decomposition** Given an object, \( x \), we decompose it into building blocks

\[ x = \sum_{\mu} c_\mu \varphi_\mu, \]

where \( \varphi_\mu \) are the building blocks and \( c_\mu = \langle x, \varphi_\mu \rangle \). are coefficients.

*If the object is fully represented by few building blocks, we have a sparse representation.*
Representing pipe in a sparse “basis”

Frame decomposition Given an object, \( x \), we decompose it into building blocks

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Example:
Representing pipe in a sparse “basis”

**Frame decomposition** Given an object, $\mathbf{x}$, we decompose it into building blocks

$$\mathbf{x} = \sum_{\mu} c_{\mu} \varphi_{\mu},$$

where $\varphi_{\mu}$ are the building blocks and $c_{\mu} = \langle \mathbf{x}, \varphi_{\mu} \rangle$ are coefficients.

*If the object is fully represented by few building blocks, we have a sparse representation.

Example:

\[
\begin{align*}
\text{circle} & = c_1 + c_2 + c_3 + c_4 + c_5 \\
\end{align*}
\]

Why is this useful?
Why enforce sparsity of frame coefficients?

Noise/ artefact reduction:

Decompose

\[ c_1 + c_2 + c_3 + c_4 + c_5 = \]

\[ \begin{array}{c}
\text{Decompose} \\
\text{Why enforce sparsity of frame coefficients?}
\end{array} \]
Why enforce sparsity of frame coefficients?

Noise/artefact reduction:

分解

$c_1 + c_2 + c_3$

$c_4 + c_5$

$= $
Why enforce sparsity of frame coefficients?

Noise/artefact reduction:

- Decompose
- $c_1 + c_2 + c_3$
- $+ c_4 + c_5 = c_1$

Too strong prior!
Not all images are fully represented by the decomposition.
Shearlets: A tight frame

Tight frames, $\Phi = \{ \varphi_\mu \}$, generalises ONB, i.e., for all images $x$ we have

$$x = \sum_\mu \langle x, \varphi_\mu \rangle \varphi_\mu.$$ 

Examples of 2D Shearlets:

Shearlets yield a sparse representation of defects, contours, etc.
Shearlet-based optimization problem

Recall discrete model of scanning process: \( b = A^\ell x + e. \)

**Reconstruction with a weighted shearlet-based sparsity penalty**

Optimization problem:

\[
\begin{align*}
\arg \min_{x \geq 0} & \quad \frac{1}{2} \| A^\ell x - b \|_2^2 + \alpha \| W c \|_1, \\
\text{s.t.} & \quad c = \Phi x
\end{align*}
\]

with \( \alpha > 0 \) regularization parameter, \( W = \text{diag}(w_i) \in \mathbb{R}^{p \times p} \) with weights \( w_i > 0 \), \( \Phi \in \mathbb{R}^{p \times n} \) is the shearlet analysis transform and \( c_\mu = \langle x, \varphi_\mu \rangle \) are coefficients.

Shearlet transform contains the basis functions: \( \Phi^T = [\varphi_1, \ldots, \varphi_p] \).
Shearlet-based optimization problem

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Shearlet transform contains the basis functions: $\Phi^T = [\varphi_1, \ldots, \varphi_p]$.

Data-fitting
Shearlet-based optimization problem

Recall discrete model of scanning process: \( \mathbf{b} = \mathbf{A}^{\ell} \mathbf{x} + \mathbf{e} \).

Reconstruction with a weighted shearlet-based sparsity penalty

Optimization problem:

\[
\begin{align*}
\arg \min_{\mathbf{x} \geq 0} & \quad \frac{1}{2} \| \mathbf{A}^{\ell} \mathbf{x} - \mathbf{b} \|_2^2 + \alpha \| \mathbf{W} \mathbf{c} \|_1, \\
\text{s.t.} & \quad \mathbf{c} = \mathbf{\Phi} \mathbf{x}
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\]

with \( \alpha > 0 \) regularization parameter, \( \mathbf{W} = \text{diag}(w_i) \in \mathbb{R}^{p \times p} \) with weights \( w_i > 0 \), \( \mathbf{\Phi} \in \mathbb{R}^{p \times n} \) is the shearlet analysis transform and \( c_\mu = \langle \mathbf{x}, \varphi_\mu \rangle \) are coefficients.

Shearlet transform contains the basis functions: \( \mathbf{\Phi}^T = [\varphi_1, \ldots, \varphi_p] \).

Weighted sparsity penalty on shearlet coefficients
Final algorithm

Recall optimization problem:

\[
\begin{align*}
\arg \min_{x \geq 0} & \frac{1}{2} \| A^\ell x - b \|_2^2 + \alpha \| W c \|_1, \\
\text{s.t. } & c = \Phi x
\end{align*}
\]

ADMM-based algorithm

We solve it using the ADMM method [Boyd et. al. 2011]. Auxiliary variable \( c = \Phi x \). Iterative updates:

\[
\begin{align*}
x^{k+1} & := \min_{x \geq 0} \frac{1}{2} \| A^\ell x - b \|_2^2 + \frac{\rho}{2} \| \Phi x - c^k + u^k \|_2^2, \\
c^{k+1} & := \min_{c} \alpha \| W c \|_1 + \frac{\rho}{2} \| \Phi x^{k+1} - c + u^k \|_2^2, \\
u^{k+1} & := u^k + \Phi x^{k+1} - c^{k+1},
\end{align*}
\]

where \( u \) are the scaled Lagrange multipliers and \( \rho > 0 \) the penalty parameter.

The updates are calculated using:
\( x^{k+1} \): CGLS + non-negativity projection.
\( c^{k+1} \): Element-wise soft thresholding.
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Synthetic and measured data from both geometries

Synthetic centered
Projection angle
Detector pixel

Measured centered
Projection angle
Detector pixel

Synthetic off-centered
Projection angle
Detector pixel

Measured off-centered
Projection angle
Reconstructions from Real Data – Both Geometries

**Centered beam:** there are many artifacts.

Prev. alg.

![Image](prev_alg.png)

Our alg.

![Image](our_alg.png)

**Off-center beam:** singularities are easy to detect; artifacts are reduced.

Prev. alg.

![Image](prev_alg_off_center.png)

Our alg.

![Image](our_alg_off_center.png)
Reconstructions from Real Data – Zoom

**Off-center beam:** singularities are easy to detect; artifacts are reduced.

Prev. alg.  

Our alg.
Reconstructions from Real Data – Zoom

**Off-center beam:** singularities are easy to detect; artifacts are reduced.

$$\arg \min_{x \geq 0} \|Ax - b\|_2^2 + \lambda \|\nabla x\|_{2,1}$$

Our alg.
## Centered Versus Off-Center Beam

<table>
<thead>
<tr>
<th>Pros</th>
<th>Centered beam</th>
<th>Off-center beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good reconstruction in the center domain.</td>
<td>Captures singularities outside the center domain.</td>
<td></td>
</tr>
<tr>
<td>Terrible reconstruction outside the center domain.</td>
<td>Less good reconstruction in the center domain.</td>
<td></td>
</tr>
<tr>
<td>Requires less projections because the center domain is well covered by rays.</td>
<td>Requires more projections to give good reconstruction everywhere. <strong>Better suited for this application.</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Table: Centered Versus Off-Center Beam**
Conclusions

• For technical reasons the X-ray beam cannot cover the whole pipe.
• An off-centered beam can give a satisfactory reconstruction.
• A weighted shearlets-based sparsity penalty gives better reconstructions – especially with few projections.
• It is important to include weights in the sparsity penalty.

Future work:
• Optimize the algorithm for performance and robustness.
• Design heuristics for choosing the weights and the reg. parameter.
• Derive more theory for the continuous model with limited data.
• Quantify the uncertainties in the model and the solution.
Conclusions

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• Quantify the uncertainties in the model and the solution.
**Uncertain view angle CT**

\[ b = A(\theta) x + e, \quad \theta \sim \pi_\theta(\cdot), \ e \sim \pi_e(\cdot). \]  

(1)

**Measured / fixed:**
- \( b \in \mathbb{R}^m \): measured noisy sinogram.
- \( A \in \mathbb{R}^{m \times n} \): discretized Radon transform.

**Known but with uncertainty:**
- \( \theta \in \mathbb{R}^q \): view angles.
- \( e \in \mathbb{R}^m \): measurement noise.

**Unknown:**
- \( x \in \mathbb{R}^n \): attenuation coefficients.
**Formulation of problem**

**Uncertain view angle CT**

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b = A(\theta) \, x + e, \quad \theta \sim \pi_\theta(\cdot), \ e \sim \pi_e(\cdot). \quad (1)
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Actual data comes from \( b := A(\bar{\theta}) \, x + e^{\text{noise}} \).
Introduction

Formulation of problem

Uncertain view angle CT

\[ b = A(\theta) x + e, \quad \theta \sim \pi_\theta(\cdot), \; e \sim \pi_e(\cdot). \] (1)

Goal:
Reconstruct \( x \) from \( b \) with uncertainty in \( \theta \) and \( e \! \)!

Applications:
Inaccuracies in rotation, patient motion etc.
Introduction

Early results

\[ \text{TV} (\lambda = 10^{-4}) \]

\[ \text{STV} (\lambda = 10^{-4}) \]

\[ \text{TV}(\lambda = 3 \cdot 10^{-4}) \]

\[ \text{STV} (\lambda = 3 \cdot 10^{-4}) \]
Introduction

Early results 2

TV ($\lambda = 1.33 \cdot 10^{-4}$)  

STV ($\lambda = 1.33 \cdot 10^{-4}$)

TV ($\lambda = 3.16 \cdot 10^{-4}$)  

STV ($\lambda = 3.16 \cdot 10^{-4}$)
Ongoing Research - Uncertain View Angles

- Large scale problems (2D → 3D)
- Apply to pipe scanner (with shearlet regularizer)
Thank you!

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