Limited-Data CT for Underwater Pipeline Inspection

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Tuesday 24th September, 2019

IDA Matematik 2019, Copenhagen, Denmark.

Industrial collaborator:

FORCE TECHNOLOGY

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Arvid P. L. Böttiger – Exensor Technology

Jürgen Frikel – OTH Regensburg

Todd Quinto – Tufts University

DTU Compute

Department of Applied Mathematics and Computer Science

Subsea CT-Scanner by FORCE Technology, Denmark

Alternative to ultrasound: use X-ray scanning to compute cross-sectional images of oil pipes lying on the seabed, to detect defects, cracks, etc. in the pipe.

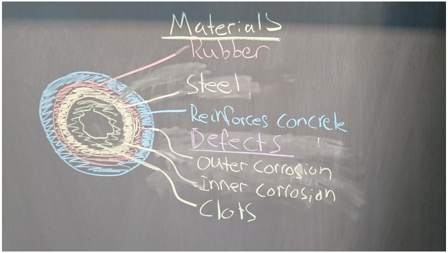
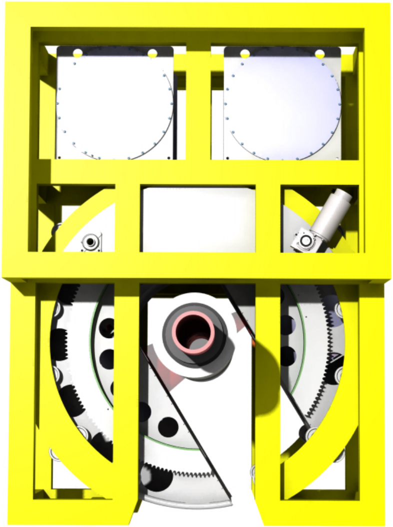


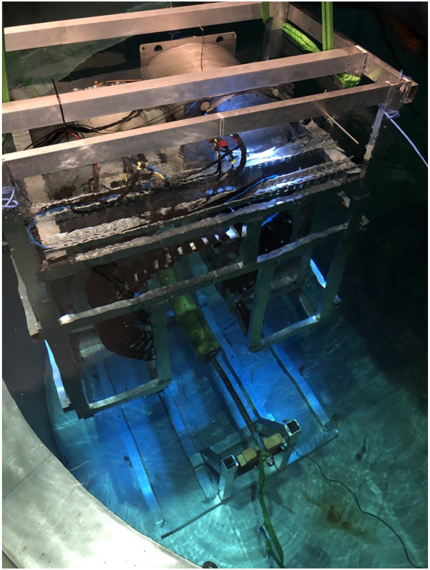
Illustration courtesy of FORCE Technology.

Subsea CT-Scanner by FORCE Technology, Denmark



https://forcetechnology.com/da/innovation/afsluttede-projekter/subsea-inspektion-konstruktion-levetid-reparation-planlaegning

Subsea CT-Scanner by FORCE Technology, Denmark



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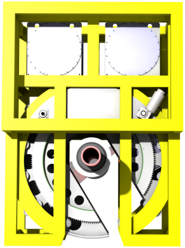
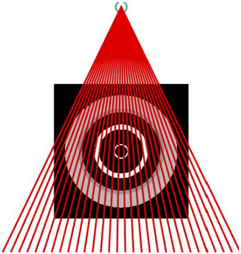
Comparing Geometries

Limitations in the scanner device

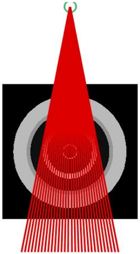
Data: *Narrow* high intensity X-Ray beam → available from a limited view.

Goal: Design set-up to capture as much detail of the pipe as possible.

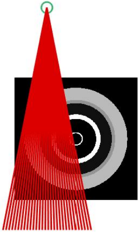




Full illumination
Not possible



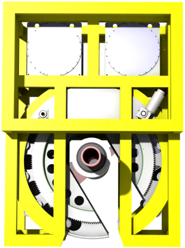
Centered
Possible set-up



Off-centered
Possible set-up

Comparing Geometries

Limitations in the scanner device



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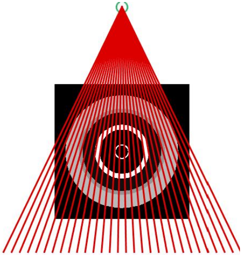

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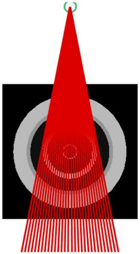

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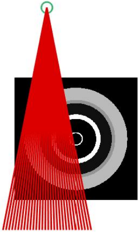


Off-centered
Possible set-up

The Computed Tomograhy Forward Model

Continuous formulation, limited data

The measured projections g for the object f are described by

$$g(\theta, s) = (\mathcal{R}^{\ell} f)(\theta, s) + \text{noise}$$

where $(\mathcal{R}^{\ell} f)(\theta, s) = (\mathcal{R} f)(\theta, s)$ for those pairs (θ, s) corresponding to the ℓimited illumination, and $\mathcal{R}$ is the Radon transform.

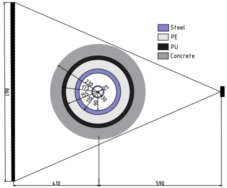
Corresponding algebraic model

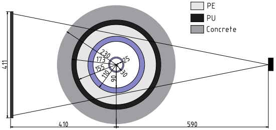
The measured data $\boldsymbol{b}$ for a discretized object $\boldsymbol{x}$ is described by

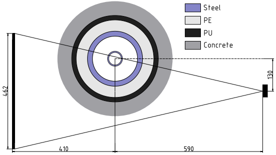
$$\boldsymbol{b} = \boldsymbol{A}^{\ell} \boldsymbol{x} + \boldsymbol{e}, \qquad \boldsymbol{b} \in \mathbb{R}^m, \qquad \boldsymbol{x} \in \mathbb{R}^n, \qquad \boldsymbol{A}^{\ell} \in \mathbb{R}^{m \times n},$$

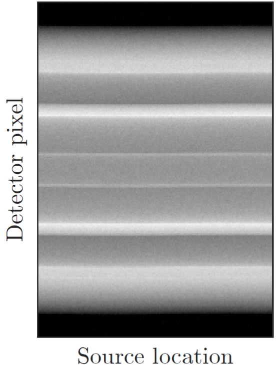
where $\boldsymbol{e} \in \mathbb{R}^m$ with $e_i \sim N(0, \sigma^2)$ and $\boldsymbol{A}^{\ell}$ is the discretion of $\mathcal{R}^{\ell}$.

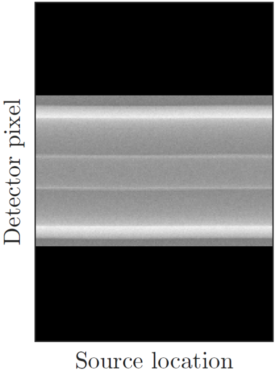
Full and Two Different Limited Illuminations

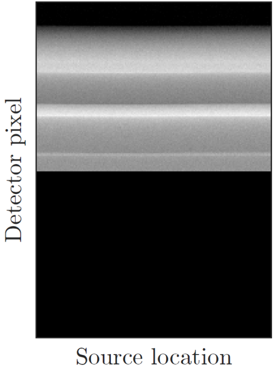






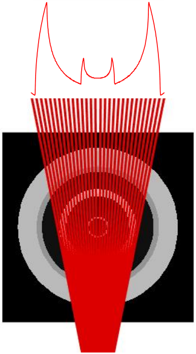




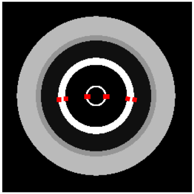


Characterising What We Can Measure – Centered Beam

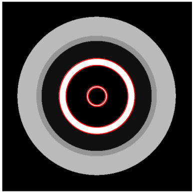
Microlocal analysis: a singularity at position χ with direction ξ is visible if and only if data from the line through χ perpendicular to ξ is present.



Measured data from one view/projection.



Visible from one view/projection.

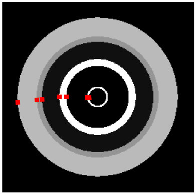


Visible from all views/projections.

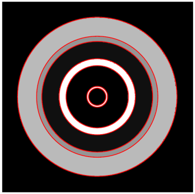
Characterising What We Can Measure – Off-Center Beam

Microlocal analysis: a singularity at position χ with direction ξ is visible if and only if data from the line through χ perpendicular to ξ is present.

Measured data from one view/projection.



Visible from one view/projection.



Visible from all views/projections.

Reconstruction: Incorporating Sparsity

We need a robust algorithm that can utilize all information in the data.

Use the algebraic system

$$\boldsymbol{b} = \boldsymbol{A}^{\ell}\,\boldsymbol{x} + \mathbf{e}, \qquad \boldsymbol{b} \in \mathbb{R}^m, \qquad \boldsymbol{x} \in \mathbb{R}^n, \qquad \boldsymbol{A}^{\ell} \in \mathbb{R}^{m\times n}.$$

and compute a sparse representation of the solution $\boldsymbol{x}$ in a “basis” that is well suited for the problem.
If we can represent $\boldsymbol{x}$ with few non-zero basis functions, we have fewer unknowns to determine $\rightarrow$ same data, fewer unknowns, “easier” problem.

Representing pipe in a sparse “basis”

Frame decomposition Given an object, x, we decompose it into building blocks

$$\boldsymbol{x} = \sum_{\mu} c_\mu \boldsymbol{\varphi}_\mu,$$

where $\boldsymbol{\varphi}_\mu$ are the building blocks and $c_\mu = \langle \boldsymbol{x}, \boldsymbol{\varphi}_\mu \rangle$. are coefficients.
**If the object is fully represented by few building blocks, we have a sparse representation.*

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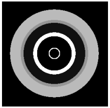
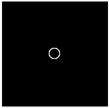
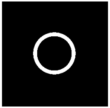
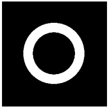
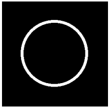
Example:

 $= c_1$  $+ c_2$  $+ c_3$ 

$+c_4$  $+ c_5$ 

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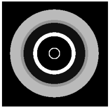
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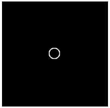
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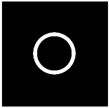
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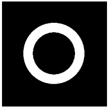
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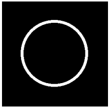
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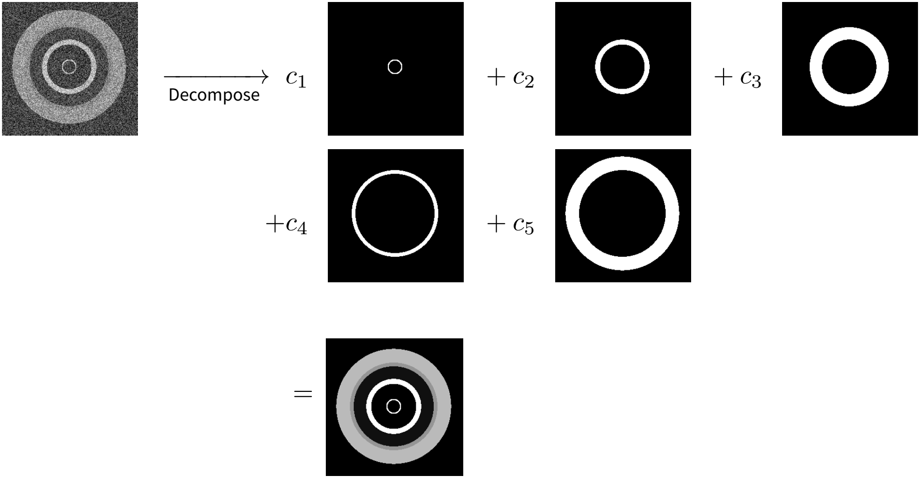
Why is this useful?

Why enforce sparsity of frame coefficients?

Noise/artefact reduction:



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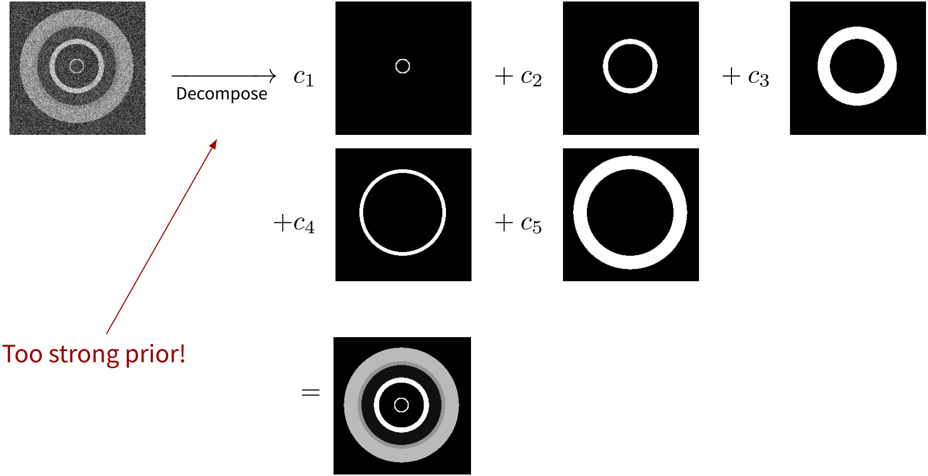

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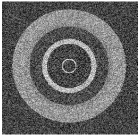
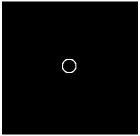


$\xrightarrow[\text{Decompose}]{}$ c_1



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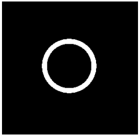

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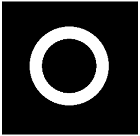

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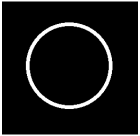

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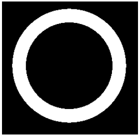

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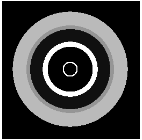


Too strong prior!

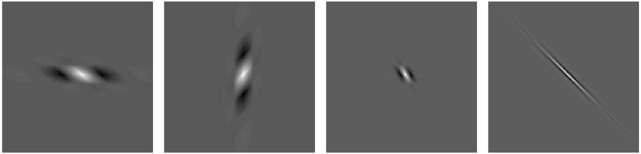
Not all images are fully represented by the decompositon.

Shearlets: A tight frame

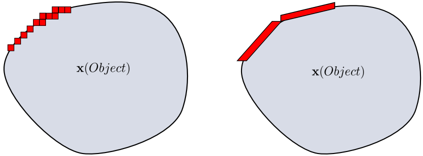
Tight frames, $\mathbf{\Phi} = \{\varphi_\mu\}$, generalises ONB, i.e., for all images x we have

$$x = \sum_\mu \langle x, \varphi_\mu \rangle \varphi_\mu .$$

Examples of 2D Shearlets:



Shearlets yield a sparse representation of defects, contours, etc.



Shearlet-based optimization problem

Recall discrete model of scanning process: $\boldsymbol{b} = \boldsymbol{A}^{\ell} \boldsymbol{x} + \mathbf{e}$.

Reconstruction with a weighted shearlet-based sparsity penalty

Optimization problem:

$$\begin{aligned} &\arg\min_{\mathbf{x}\geq\mathbf{0}} \tfrac{1}{2}\|\boldsymbol{A}^{\ell}\mathbf{x} - \mathbf{b}\|_2^2 + \alpha\|\mathbf{W}\,\mathbf{c}\|_1, \\ &\text{s.t. } \mathbf{c} = \boldsymbol{\Phi}\boldsymbol{x} \end{aligned}$$

with $\alpha > 0$ regularization parameter, $\mathbf{W} = \text{diag}(w_i) \in \mathbb{R}^{p\times p}$ with weights $w_i > 0$, $\boldsymbol{\Phi} \in \mathbb{R}^{p\times n}$ is the shearlet analysis transform and $c_\mu = \langle \boldsymbol{x}, \boldsymbol{\varphi}_\mu \rangle$ are coefficients.

Shearlet transform contains the basis functions: $\boldsymbol{\Phi}^T = \begin{bmatrix} \boldsymbol{\varphi}_1, & \dots, & \boldsymbol{\varphi}_p \end{bmatrix}$.

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Data-fitting

Shearlet-based optimization problem

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Weighted sparsity penalty on shearlet coefficients

Final algorithm

Recall optimization problem:

$$\arg\min_{\mathbf{x}\geq\mathbf{0}} \tfrac{1}{2}\|\boldsymbol{A}^{\ell}\mathbf{x}-\mathbf{b}\|_2^2+\alpha\|\mathbf{W}\,\mathbf{c}\|_1,$$
$$\text{s.t. } \mathbf{c}=\boldsymbol{\Phi}\boldsymbol{x}$$

ADMM-based algorithm

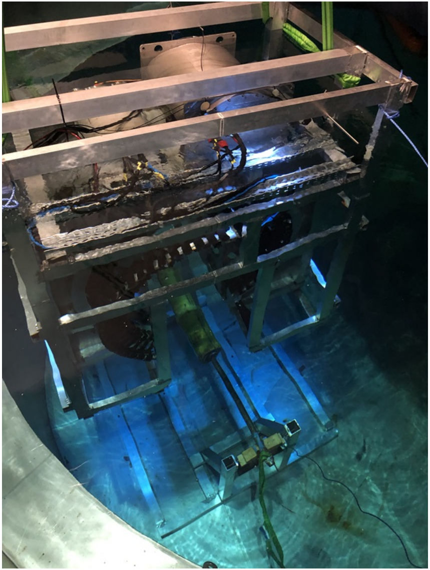
We solve it using the ADMM method [Boyd et. al. 2011]. Auxiliary variable $\mathbf{c}=\boldsymbol{\Phi}\mathbf{x}$. Iterative updates:

$$\begin{aligned}
\mathbf{x}^{k+1} &:= \min_{\boldsymbol{x}\geq\mathbf{0}} \quad \tfrac{1}{2}\|\boldsymbol{A}^{\ell}\,\boldsymbol{x}-\boldsymbol{b}\|_2^2+\tfrac{\rho}{2}\|\boldsymbol{\Phi}\,\boldsymbol{x}-\mathbf{c}^{k}+\mathbf{u}^{k}\|_2^2,\\
\mathbf{c}^{k+1} &:= \min_{\mathbf{c}} \quad \alpha\|\mathbf{W}\,\mathbf{c}\|_1+\tfrac{\rho}{2}\|\boldsymbol{\Phi}\,\boldsymbol{x}^{k+1}-\mathbf{c}+\mathbf{u}^{k}\|_2^2,\\
\mathbf{u}^{k+1} &:= \mathbf{u}^{k}+\boldsymbol{\Phi}\,\boldsymbol{x}^{k+1}-\mathbf{c}^{k+1},
\end{aligned}$$

where $\mathbf{u}$ are the scaled Lagrange multipliers and $\rho>0$ the penalty parameter.

The updates are calculated using:
$\mathbf{x}^{k+1}$: CGLS + non-negativity projection.
$\mathbf{c}^{k+1}$: Element-wise soft thresholding.

Subsea CT-Scanner by FORCE Technology, Denmark



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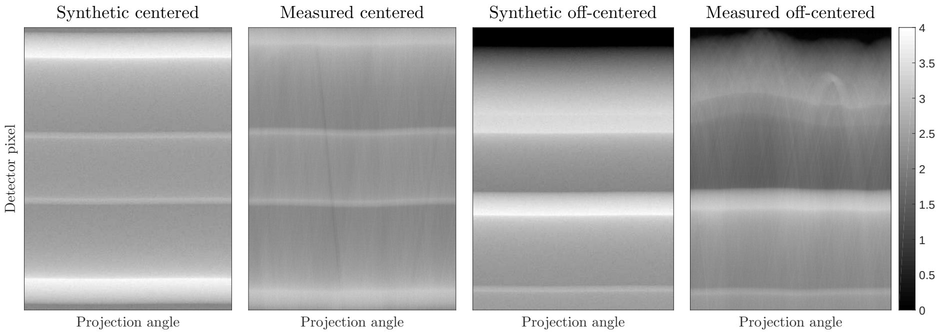
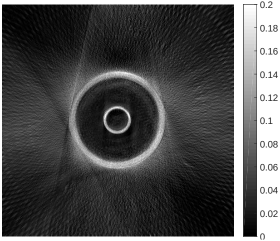
Synthetic and measured data from both geometries

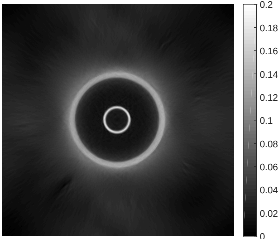


Reconstructions from Real Data – Both Geometries

Centered beam: there are many artifacts.

Prev. alg.

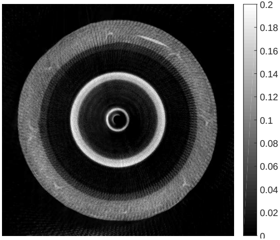


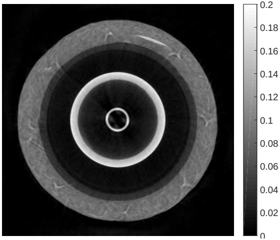


Our alg.

Off-center beam: singularities are easy to detect; artifacts are reduced.

Prev. alg.





Our alg.

Reconstructions from Real Data – Zoom

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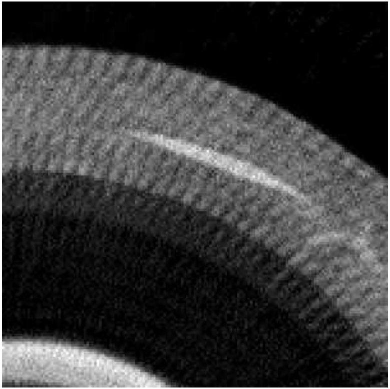

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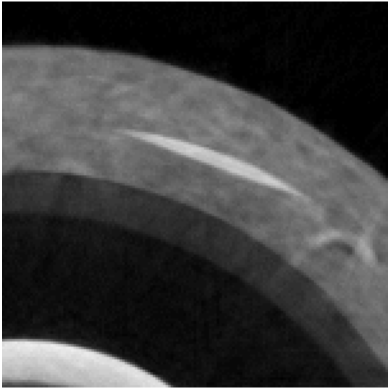
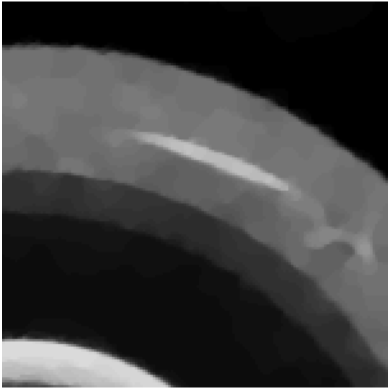


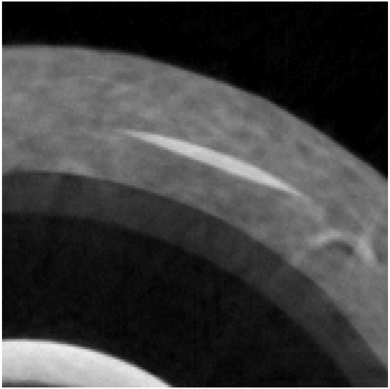
Reconstructions from Real Data – Zoom

Off-center beam: singularities are easy to detect; artifacts are reduced.

$$\arg\min_{\mathbf{x}\geq\mathbf{0}} \|\boldsymbol{A}\boldsymbol{x}-\boldsymbol{b}\|_2^2+\lambda\|\nabla\boldsymbol{x}\|_{2,1}$$

Our alg.





Centered Versus Off-Center Beam

	Centered beam	**Off-center beam**
Pros	Good reconstruction in the center domain.	Captures singularities outside the center domain.
Cons	Terrible reconstruction outside the center domain.	Less good reconstruction in the center domain.
Comments	Requires less projections because the center domain is well covered by rays.	Requires more projections to give good reconstruction everywhere. Better suited for this application.

Conclusions

- For technical reasons the X-ray beam cannot cover the whole pipe.
- An off-centered beam can give a satisfactory reconstruction.
- A weighted shearlets-based sparsity penalty gives better reconstructions – especially with few projections.
- It is important to include weights in the sparsity penalty.

Future work:

- Optimize the algorithm for performance and robustness.
- Design heuristics for choosing the weights and the reg. parameter.
- Derive more theory for the continuous model with limited data.
- Quantify the uncertainties in the model and the solution.

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Formulation of problem

Uncertain view angle CT

$$\boldsymbol{b} = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{x} + \mathbf{e}, \quad \boldsymbol{\theta} \sim \pi_{\boldsymbol{\theta}}(\cdot),\, \mathbf{e} \sim \pi_{\mathbf{e}}(\cdot). \tag{1}$$

Measured / fixed:

$\boldsymbol{b} \in \mathbb{R}^m$: measured noisy sinogram.

$\mathbf{A} \in \mathbb{R}^{m \times n}$: discretized Radon transform.

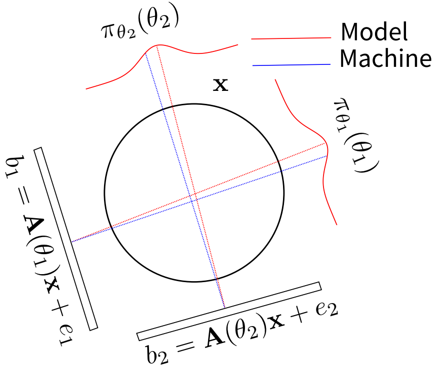
Known but with uncertainty:

$\boldsymbol{\theta} \in \mathbb{R}^q$: view angles.

$\mathbf{e} \in \mathbb{R}^m$: measurement noise.

Unknown:

$\mathbf{x} \in \mathbb{R}^n$: attenuation coefficients.



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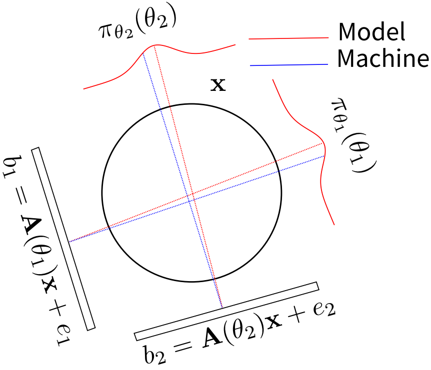
Actual data comes from $\boldsymbol{b} := \mathbf{A}(\bar{\boldsymbol{\theta}})\,\mathbf{x} + \mathbf{e}^{\text{noise}}$.



Formulation of problem

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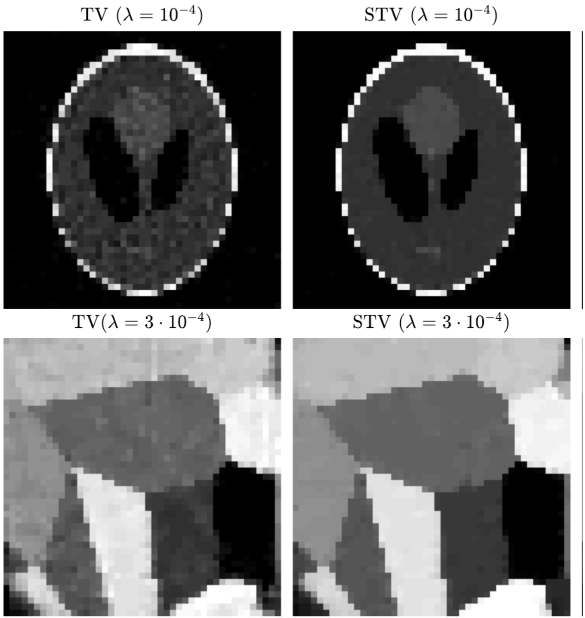
Goal:

Reconstruct $\mathbf{x}$ from $\boldsymbol{b}$ with uncertainty in $\boldsymbol{\theta}$ and $\mathbf{e}$!

Applications:

Inaccuracies in rotation, patient motion etc.

Early results

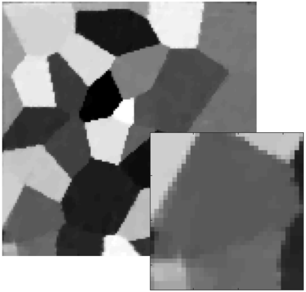


Early results 2

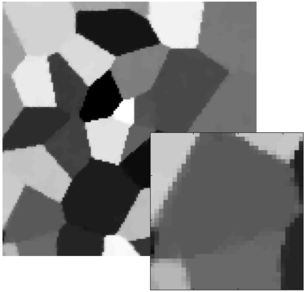
TV $(\lambda = 1.33 \cdot 10^{-4})$

STV $(\lambda = 1.33 \cdot 10^{-4})$

TV $(\lambda = 3.16 \cdot 10^{-4})$



STV $(\lambda = 3.16 \cdot 10^{-4})$



Ongoing Research - Uncertain View Angles

- Large scale problems (2D $\rightarrow$ 3D)
- Apply to pipe scanner (with shearlet regularizer)

Thank you!

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