Limited-Data CT for Underwater Pipeline Inspection

Per Christian Hansen

Joint work with:

Jacob Frøsig, Nicolai A. B. Riis, Yiqiu Dong, Rasmus D. Kongskov – Technical University of Denmark

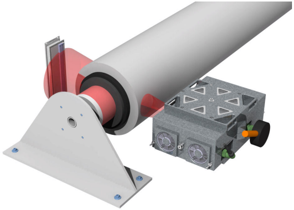
Arvid P. L. Böttiger, Torben Klit Pedersen – FORCE Technology

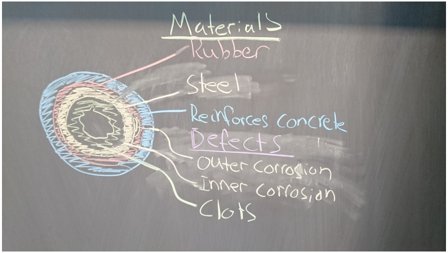
Jürgen Frikel – OTH Regensburg

Todd Quinto – Tufts University

Subsea CT-Scanner by FORCE Technology, Denmark

Alt. to ultrasound: use X-ray scanning to compute cross-sectional images of oil pipes lying on the seabed, to detect defects, cracks, etc. in the pipe.



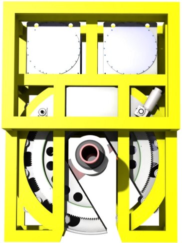


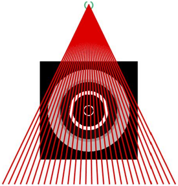
Illustrations courtesy of FORCE Technology.

The Geometry of the Problem

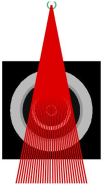
Limitations in the scanner device ⇒ only a part of the pipe can be illuminated by the fan-beam.

Design a set-up + algorithm that allows us to reconstruct as much of the pipe as possible from the limited data.

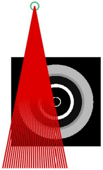




Full illumination, centered.
Not possible.



Partial illumination, centered.
Possible set-up.



Partial illumination, off-center.
Also possible.

The CT Forward Model

Continuous formulation, limited data

The measured projections g for the object f are described by

$$g(\theta, s) = (\mathcal{R}^\ell f)(\theta, s) + \text{noise}$$

where $(\mathcal{R}^\ell f)(\theta, s) = (\mathcal{R} f)(\theta, s)$ for those pairs (θ, s) corresponding to the ℓimited illumination, and $\mathcal{R}$ is the Radon transform.

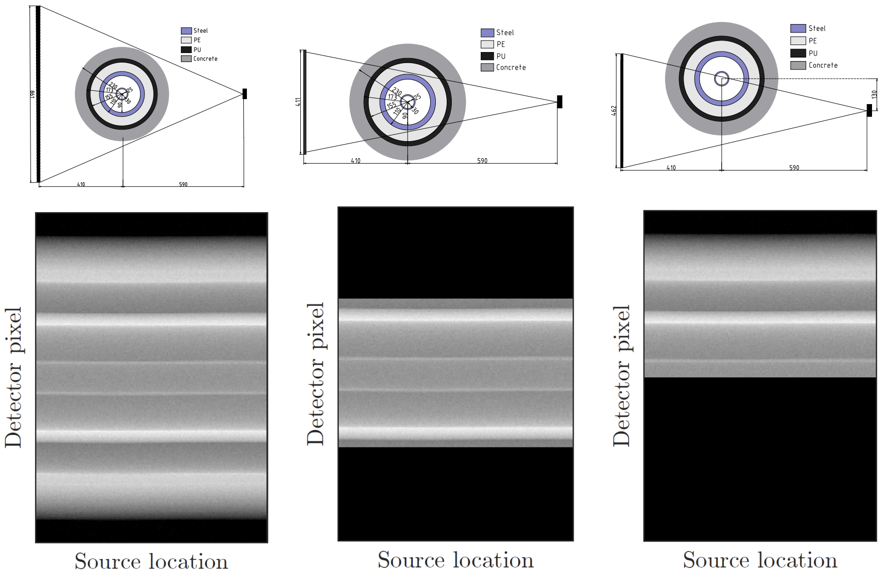
Corresponding algebraic model

The measured data $\boldsymbol{b}$ for a discretized object $\boldsymbol{x}$ is described by

$$\boldsymbol{b} = \boldsymbol{A}^\ell \boldsymbol{x} + \boldsymbol{e}, \qquad \boldsymbol{b} \in \mathbb{R}^m, \qquad \boldsymbol{x} \in \mathbb{R}^n, \qquad \boldsymbol{A}^\ell \in \mathbb{R}^{m \times n},$$

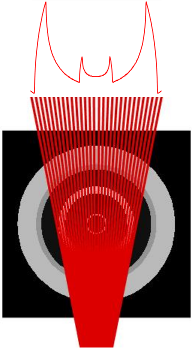
where $\boldsymbol{e} \in \mathbb{R}^m$ with $e_i \sim N(0, \sigma^2)$ and $\boldsymbol{A}^\ell$ is the discretion of $\mathcal{R}^\ell$.

Full and Two Different Limited Illuminations

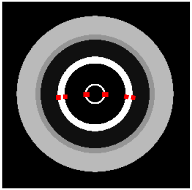


Characterising What We Can Measure – Centered Beam

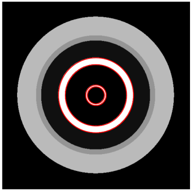
Microlocal analysis: a singularity at position χ with direction ξ is visible if and only if data from the line through χ perpendicular to ξ is present.



Measured data from one view/projection.

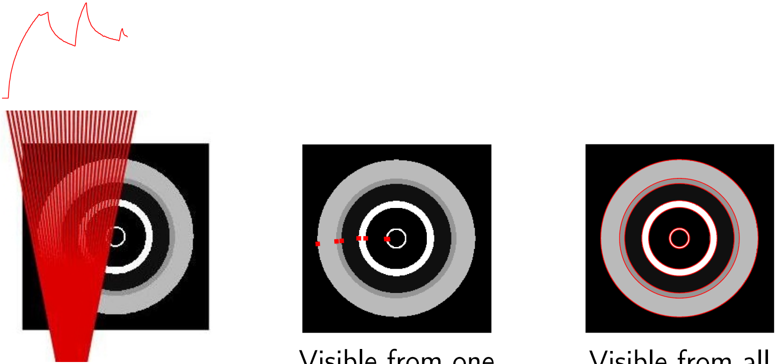


Visible from one view/projection.



Visible from all views/projections.

Characterising What We Can Measure – Off-Center Beam



Measured data from one view/projection.

Visible from one view/projection.

Visible from all views/projections.

Reconstruction – Variational Formulation

Reconstruction with a weighted frame-based sparsity penalty

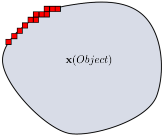
Solve the problem

$$\min_{\boldsymbol{x}} \left\{ \| \boldsymbol{A}^{\ell}\, \boldsymbol{x} - \boldsymbol{b} \|_2^2 + \alpha \| \boldsymbol{W}\, \boldsymbol{c} \|_1 \right\}, \qquad \alpha = \text{reg. parameter}$$

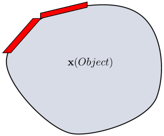
with weights $\boldsymbol{W} = \mathrm{diag}(w_i)$ and tight-frame coefficients $c_i = \langle \phi_i, \boldsymbol{x} \rangle$.

To solve this problem we use the optimization algorithm FISTA – the Fast Iterative Shrinkage-Thresholding Algorithm [Beck & Teboulle 2009].

Shearlets give a good, sparse representation of defects, contours, etc.



Frame with no direction.



Frame with direction.

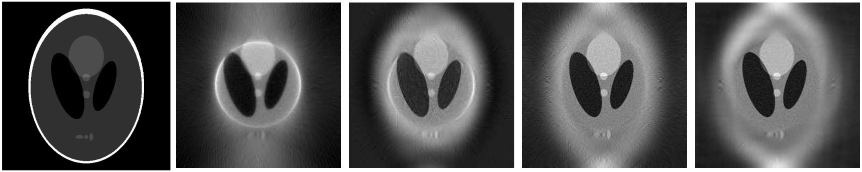
Definition of Weights + Example

Scale weights: w_i^{s} depend solely on the scale, or level, of the frame ϕ_i (smaller "footprint" of $\phi_i \to$ larger weight).

Ray-density weights: w_i^{r} depend solely on the number of rays that intersect the "footprint" of ϕ_i,

$$w_i^{\mathrm{r}} \sim \|\boldsymbol{M}\,\phi_i\|_2 / \|\phi_i\|_2, \qquad \boldsymbol{M} = \mathrm{diag}(\|A(:\,,j)\|_2).$$

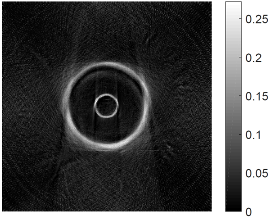
Left to right: phantom, Landweber, and 3 $\times$ our algorithm.

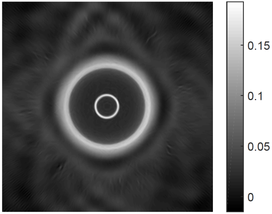


$w_i = 1$ $\qquad$ $w_i = w_i^{\mathrm{s}}$ $\qquad$ $w_i = w_i^{\mathrm{s}} \cdot w_i^{\mathrm{r}}$

Reconstructions from Real Data – Both Geometries

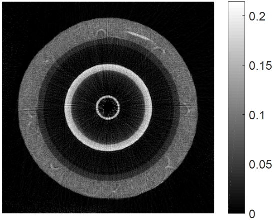
Centered beam: there are many artifacts.

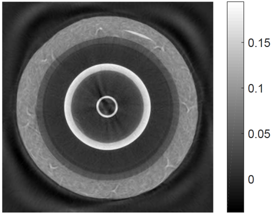
ART




Our alg.

Off-center beam: singularities are easy to detect; artifacts are reduced.

ART




Our alg.

Centered Versus Off-Center Beam

	Centered beam	**Off-center beam**
Pros	Good reconstruction in the center domain.	Captures singularities outside the center domain.
Cons	Terrible reconstruction outside the center domain.	Less good reconstruction in the center domain.
Comments	Requires less projections because the center domain is well covered by rays.	Requires more projections to give good reconstruction everywhere. Better suited for this application.

Conclusions

- For technical reasons the X-ray beam cannot cover the whole pipe.
- An off-centered beam can give a satisfactory reconstruction.
- A weighted shearlets-based sparsity penalty gives better reconstructions than FDK and ART – especially with few projections.
- It is important to include weights in the sparsity penalty.

Future work:

- Optimize the algorithm for performance and robustness.
- Design heuristics for choosing the weights and the reg. parameter.
- Derive more theory for the continuous model with limited data.
- Quantify the uncertainties in the model and the solution.