Motion Modeling in CT Imaging

Caleb Rottman[1], Martin Bauer[2], Klas Modin[3], Sarang Joshi[1]

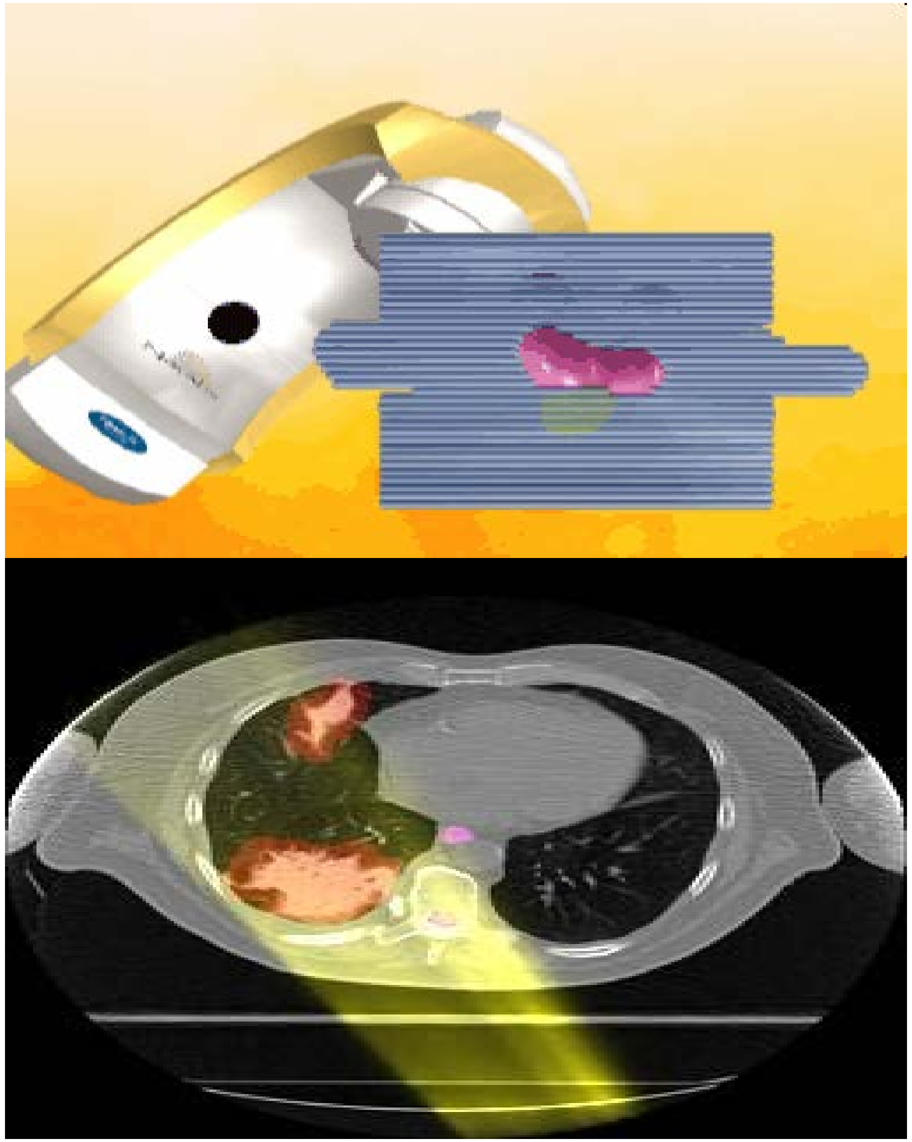
[1]Department of Bioengineering, SCI Institute, University of Utah

[2]Fakultät für Mathematik, Universität Wien

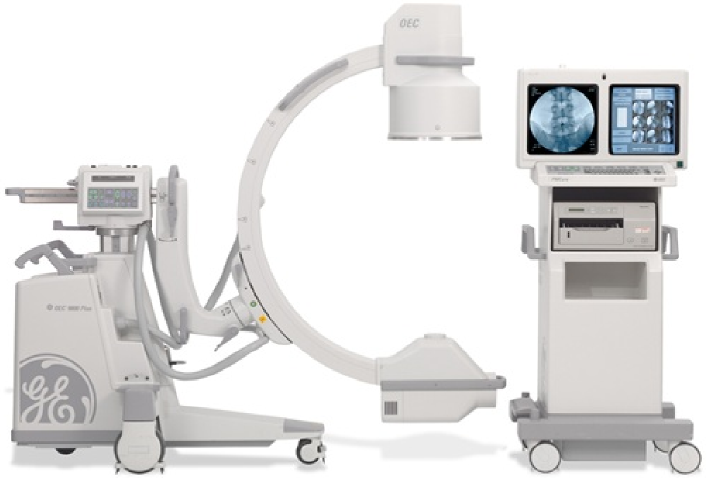
[3]Department of Mathematical Sciences, Chalmers University of Technology and the University of Gothenburg

Motivating Problem: Radiation Oncology

- **Stereotactic Body Radiation Therapy** (SBRT) - computer controlled delivery delivery of **extremely-high-doses** of radiation that conform precisely to the irregular shape of a particular patient's tumor.
- The extreme precision and dose conformity of SBRT makes the technique particularly susceptible to normal, respiratory-induced motion.
- This can result in **under dosing** of the targeted lesion and **overdosing** of surrounding healthy tissue-Resulting in local failure and complications.



Motivation – 3D CT Reconstruction



<u>Fixed-room CT scanner</u>

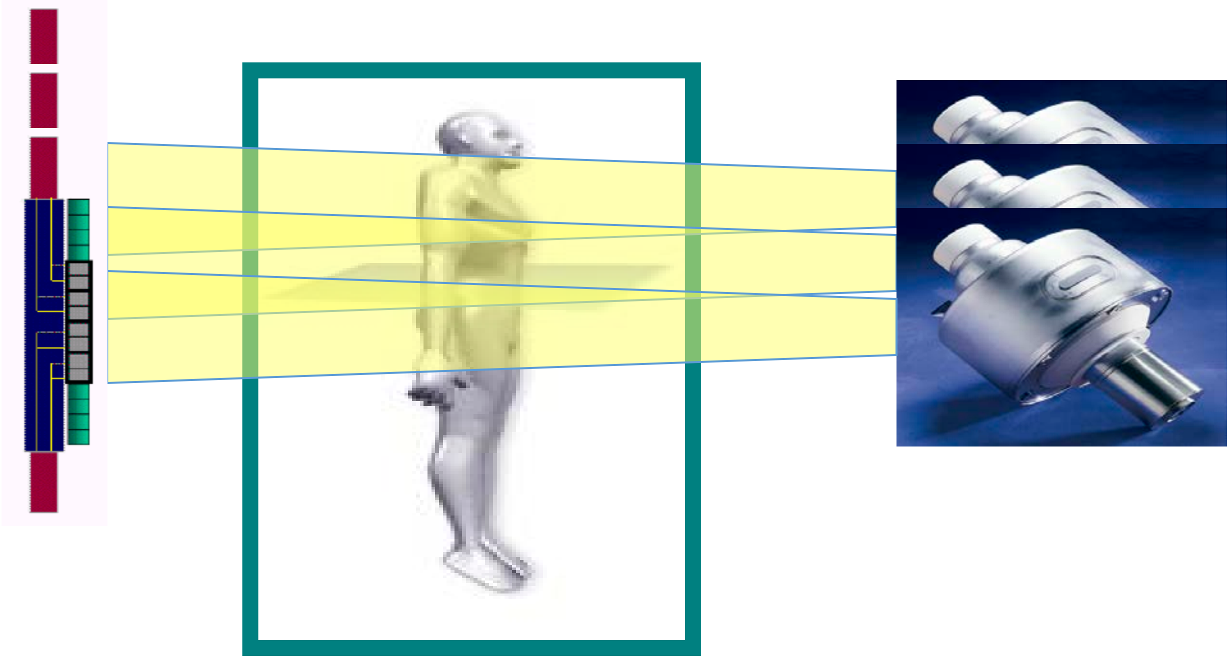
- Designed for 3D imaging
- Fixed/calibrated geometry
- Immobile, expensive

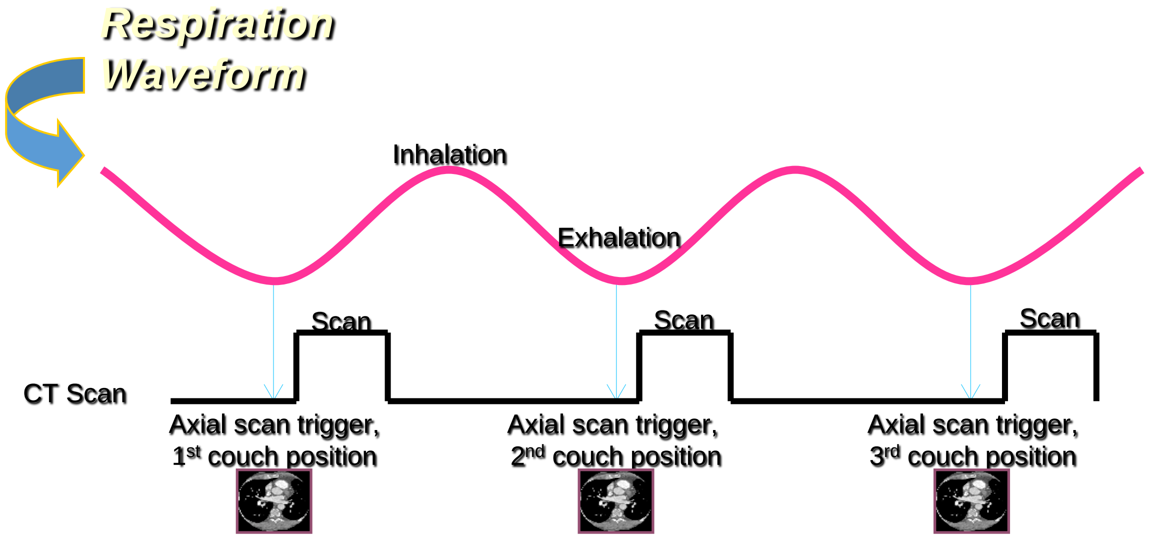
<u>Mobile C-arm</u>

- Designed for 2D imaging
- **Variable/uncalibrated geometry**
- Non-isocentric, limited angle
- Mobile, inexpensive
- 3D reconstruction rare

Motivating Problem: Radiation Oncology

The recent advent of the fast multi slice CT scanners have enabled the development of Respiratory Correlated CT (RCCT) imaging techniques to study organ motion during breathing (i.e. 4D Imaging).





Motivating Problem: Radiation Oncology

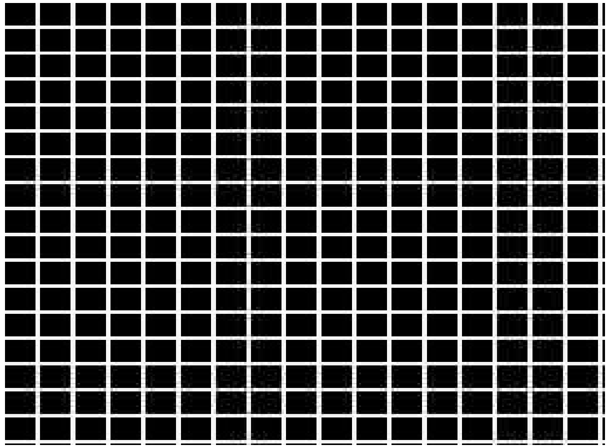
Acquisition of such imaging data allows for visualization of a dynamic CT ‘movie loop’ of the patient’s unique, internal anatomy.

•Problem: How can we effectively use the 4DRCCT data to improve Radiation Treatment Planning and Delivery for SBRT of liver malignancies?

➢Need to first model anatomical motion. Watching a movie of anatomy is not the same as modeling

Introduction to Diffeomorphisms

- Diffeomorphisms: one-to-one onto (invertible) and differential transformations. Preserve topology.


- Space of all Diffeomorphisms forms a group under composition:

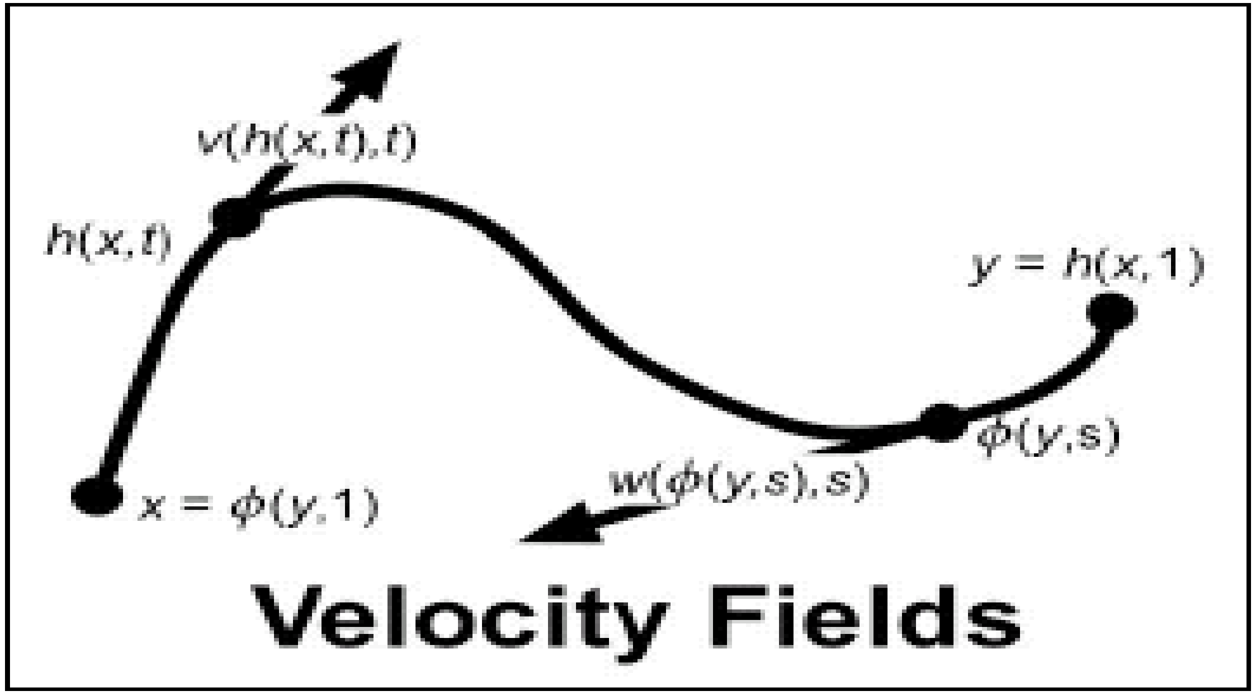
$$h_1, h_2 \in Diff(\Omega) : h = h_1 \circ h_2 \in Diff(\Omega)$$

- Space of diffeomorphisms not a vector space.

$$h_1, h_2 \in Diff(\Omega) : h = h_1 + h_2 \notin Diff(\Omega)$$

Large deformation diffeomorphisms.

- $Diff(\Omega)$ infinite dimensional “Lie Group”.
- Tangent space: The space of smooth vector velocity fields.
- Construct deformations by integrating flows of velocity fields.



$$
\begin{aligned}
y &= h(x,1) = x + \int_0^1 v(h(x,\tau),\tau)\,d\tau \\
x &= \phi(y,1) = y + \int_0^1 w(\phi(y,\tau),\tau)\,d\tau
\end{aligned}
$$

Metric on the Group of Diffeomorphisms:

- Induce a metric via a Sobolev norm on the velocity fields. Distance defined as the length of geodesics under this norm.
- Distance between *e*, the identity and any diffeomorphism is defined via the geodesic equation: (L differential operator in space only)

$h(x)$

$$d^2(e,h) = \min_{v} \int_0^1 \langle Lv(t), v(t) \rangle dt \quad \text{subject to} : h(x) = x + \int_0^1 v(h(x,t),t)dt$$

- Right invariant distance between any two diffeomorphisms is defined as:

$$d(h_1, h_2) = d(e, h_2 \circ h_1^{-1})$$

$$d(h_1, h_2) = d(e, h_2 \circ h_1^{-1})$$

Geodesic Equations

- Minimum energy paths follow geodesic equations.
- Evolution equations usually given in terms of momentum

$$m(t) = Lv(t)$$

- The Euler-Poincare equations for diffeomorphisms: (EPDIFF)

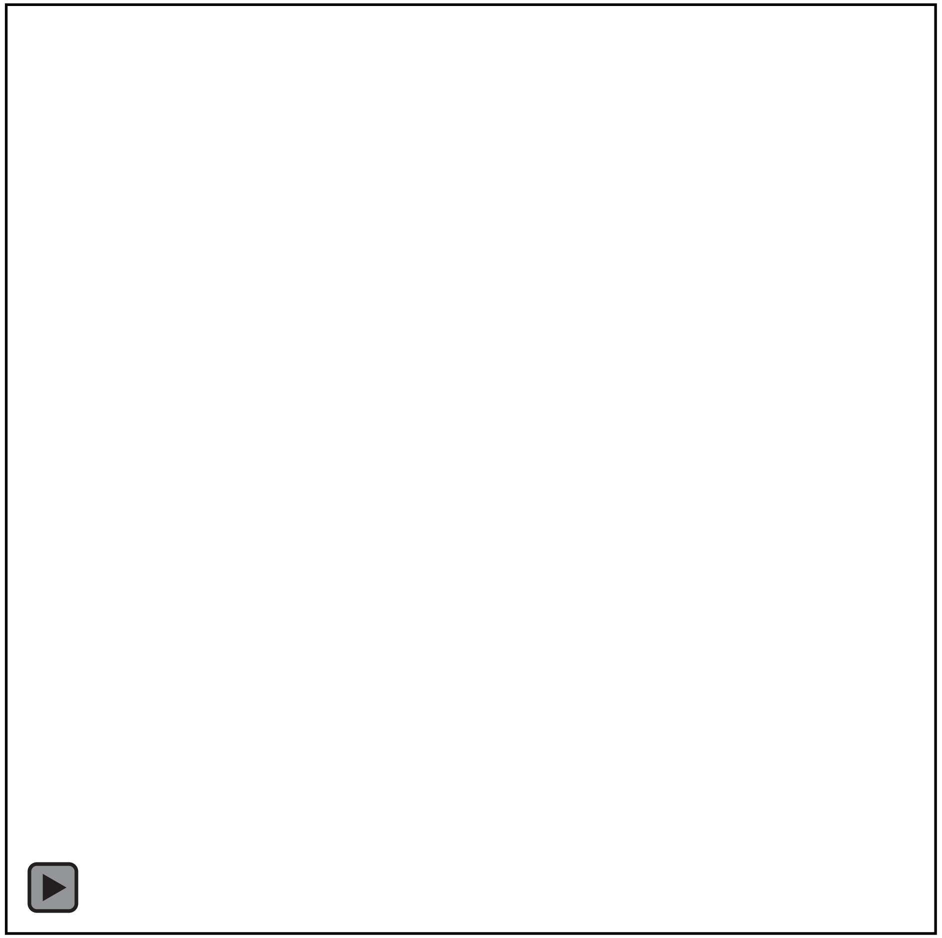
$$\frac{dm(t)}{dt} = -Dm(t)v(t) - m(t)(\nabla \cdot v(t)) - Dm(t)^T v(t)$$

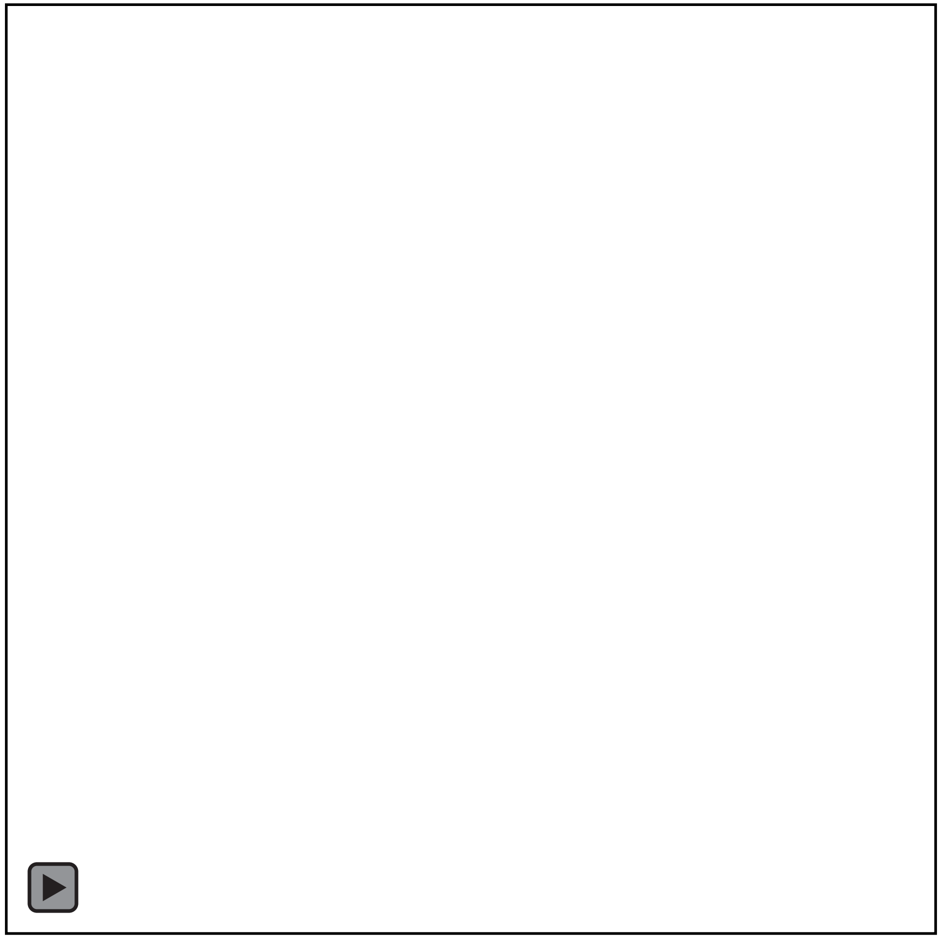
Relationship to Fluid Deformations

- **Newtonian fluid flows generate diffeomorphisms:** John P. Heller "An Unmixing Demonstration," *American Journal of Physics*, **28**, 348-353 (1960).

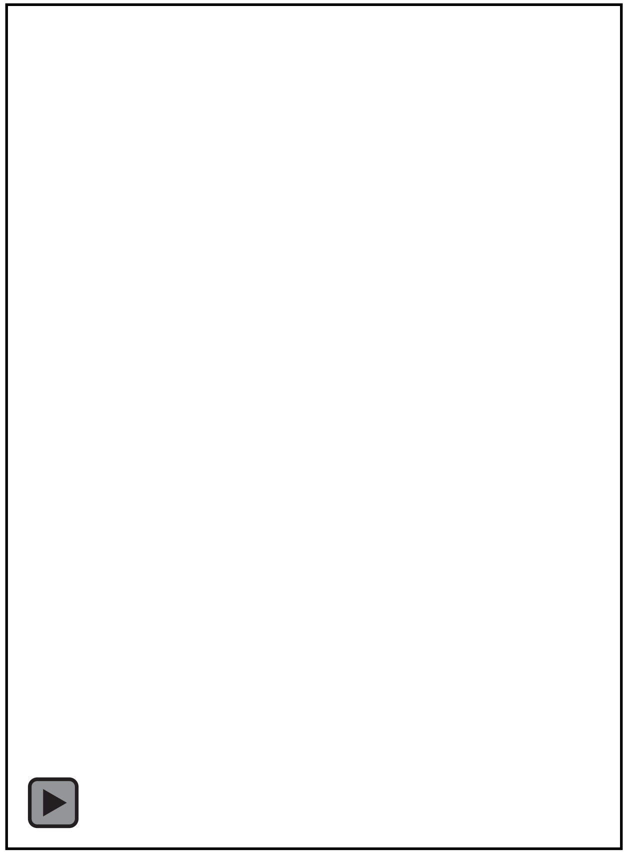
- Euler's equation: Geodesics on Sdiff with L^{2} metric.
  - *Mathematical methods of classical mechanics*, by Vladimir Arnold (Springer)
  - https://terrytao.wordpress.com/2010/06/07/the-euler-arnold-equation/

Back to Lungs





Diffeomorphic registration of lung CT images



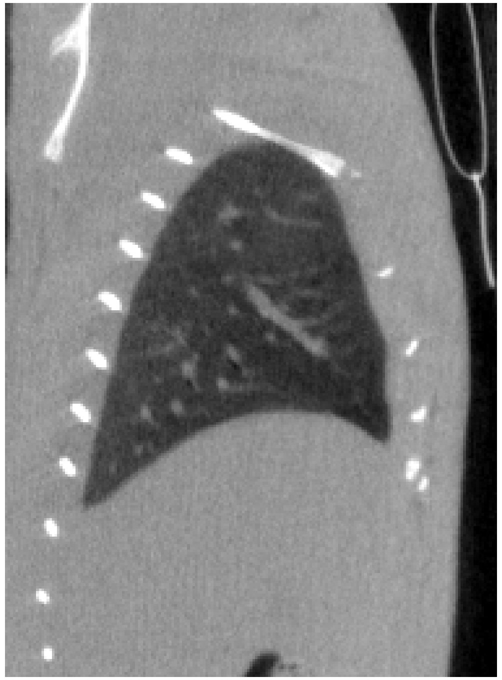
- Rat imaged at 11 time points of breathing cycle using a ventilator
- CBCT reconstruction using FDK

- Goal: find diffeomorphic (bijective and smooth) transformations that accurately model:
  - Physics (conservation of mass[1,2])
  - Physiology (local tissue compressibility)

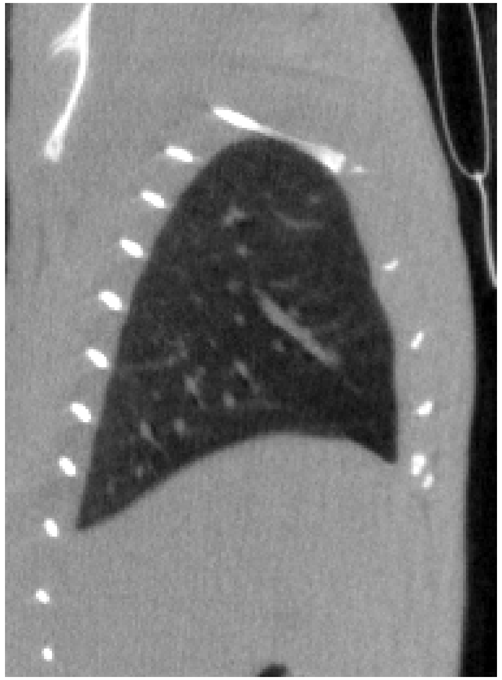
[1]Yin, Hoffman and Lin. Mass preserving non-rigid registration of CT lung images using cubic B-spline. Medical Physics 36(9) 2009.
[2]Gorbunova, Sporring, Lo, Loeve, Tiddens, Nielsen, Dirksen, and de Bruijne, Mass preserving image registration for lung CT, Med. Image Anal., 16(4) 2012.

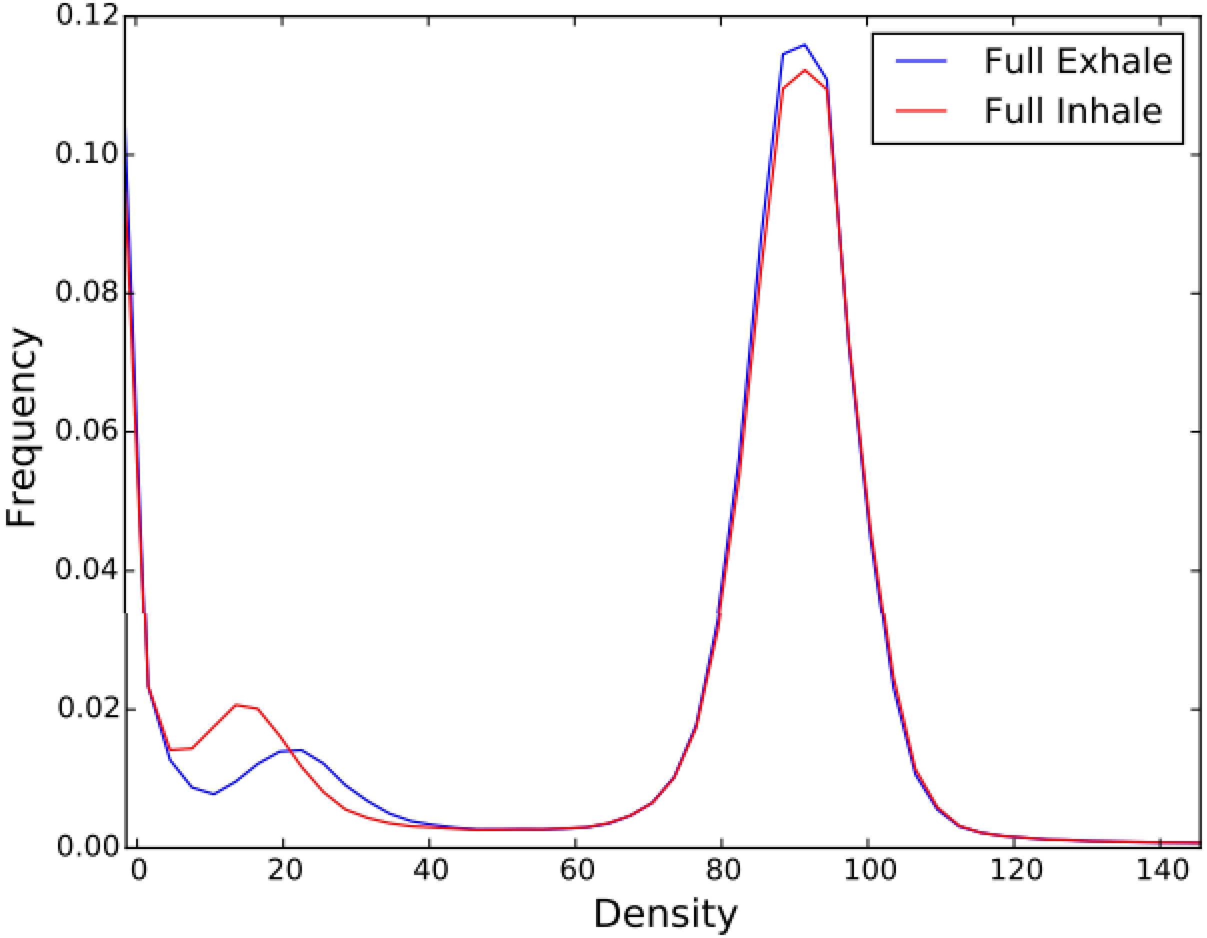
Diffeomorphic registration of lung CT images

Full Exhale



Full Inhale





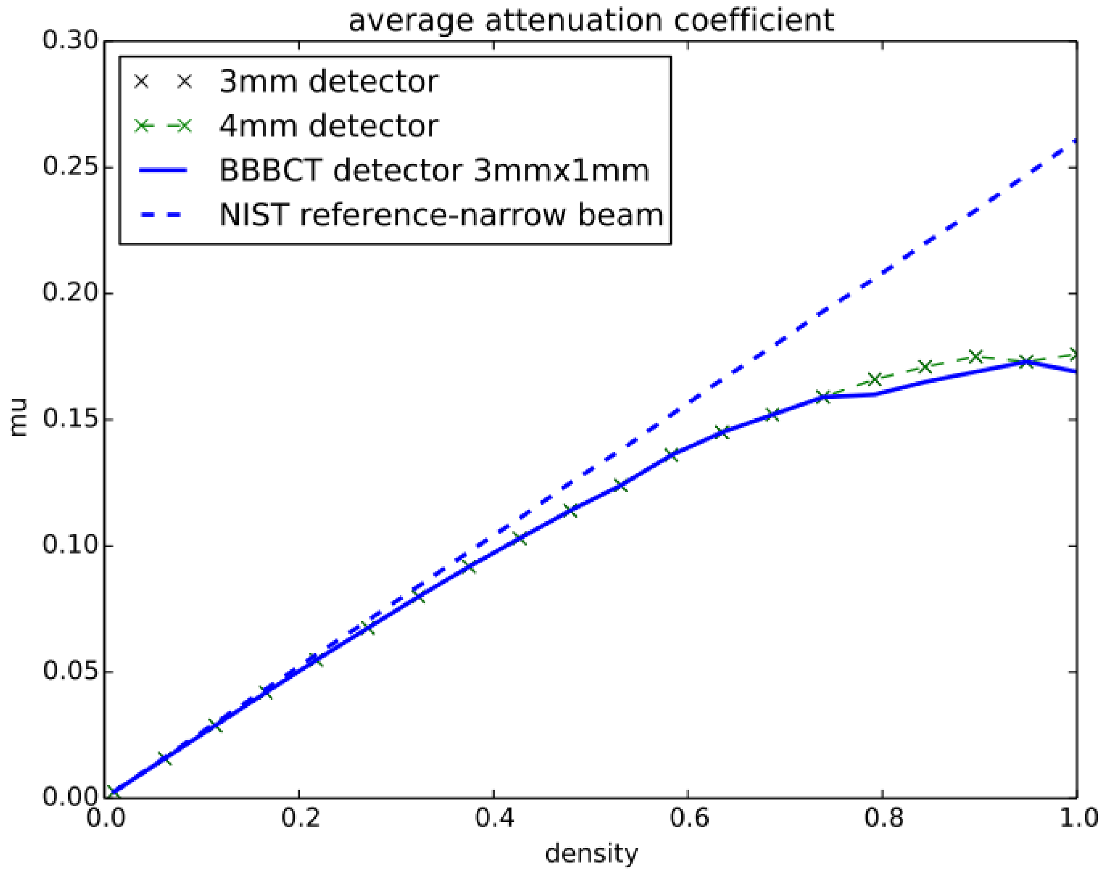
Linear attenuation coefficient (μ)

- CT reconstruction estimates $\mu(x)$, linear attenuation coefficient

  $$\mu = \alpha_m \rho_m$$

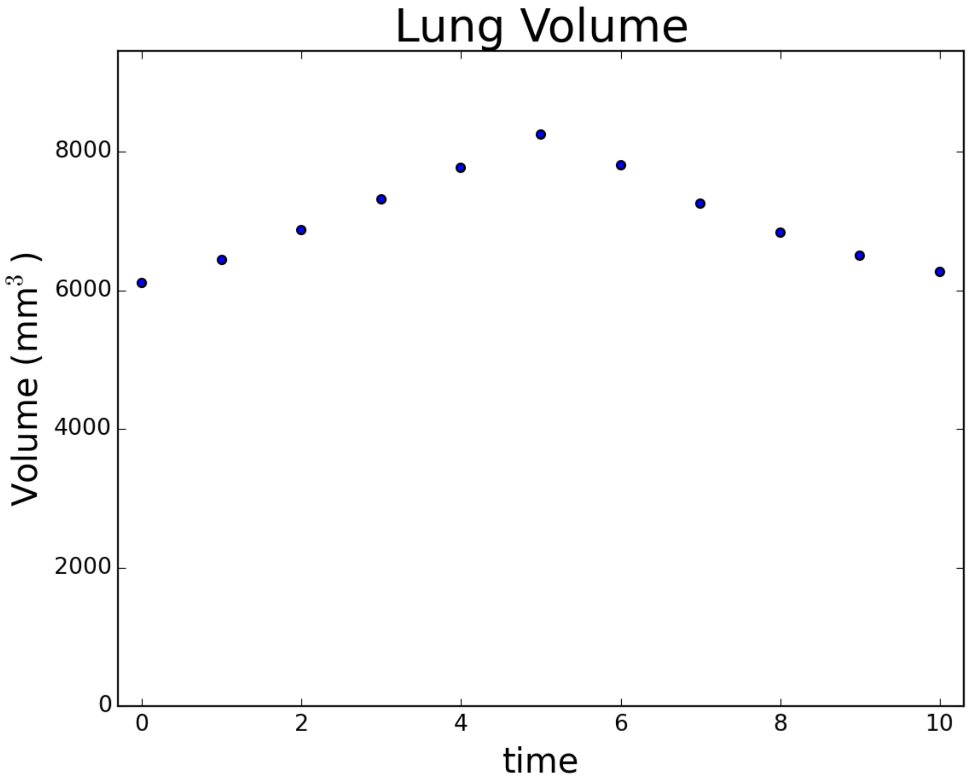
  α_m = mass attenuation coefficient (of a material), ρ_m = mass density
- The linear attenuation coefficient of a material is proportional to its mass density
- Conservation of mass -> conservation of linear attenuation coefficient
- Due to the simplified projection model, reconstruction algorithms don't estimate $\mu(x)$ directly (X-ray scatter, secondary photons, beam hardening, etc.)

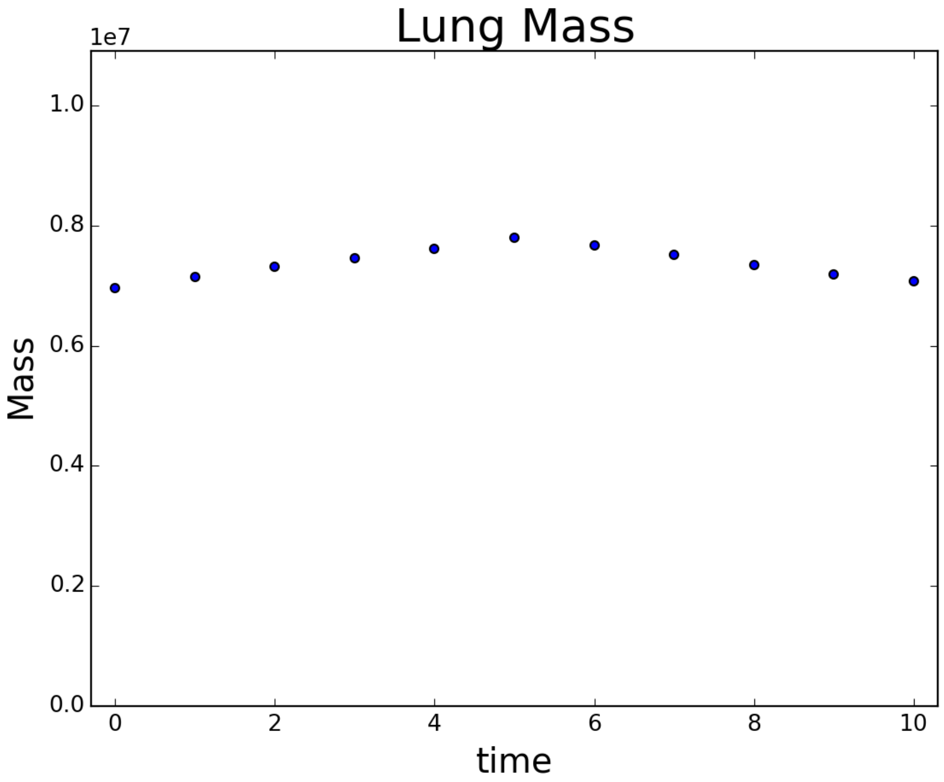
Monte-Carlo Simulations



Conservation of mass

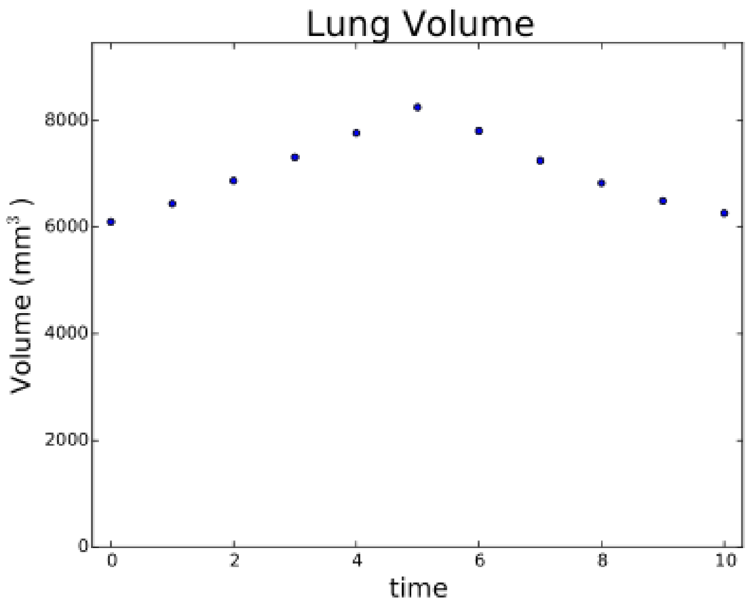
- Using nominal densities from CBCT

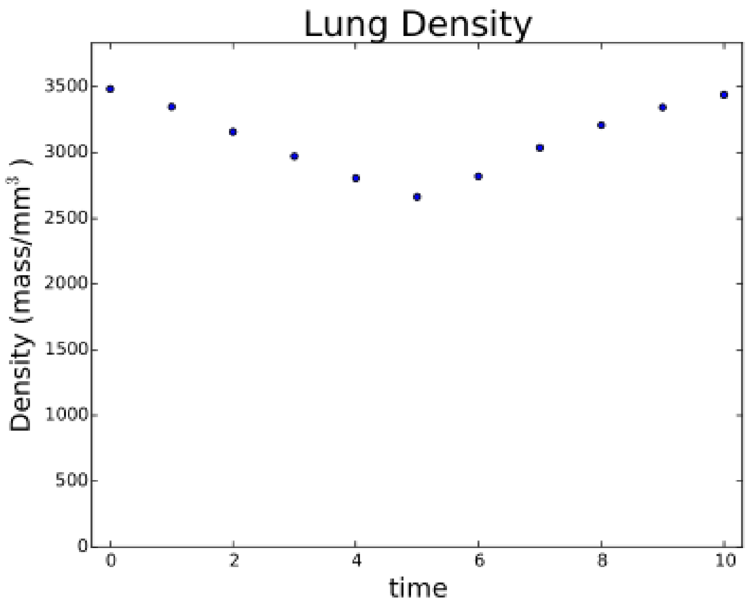


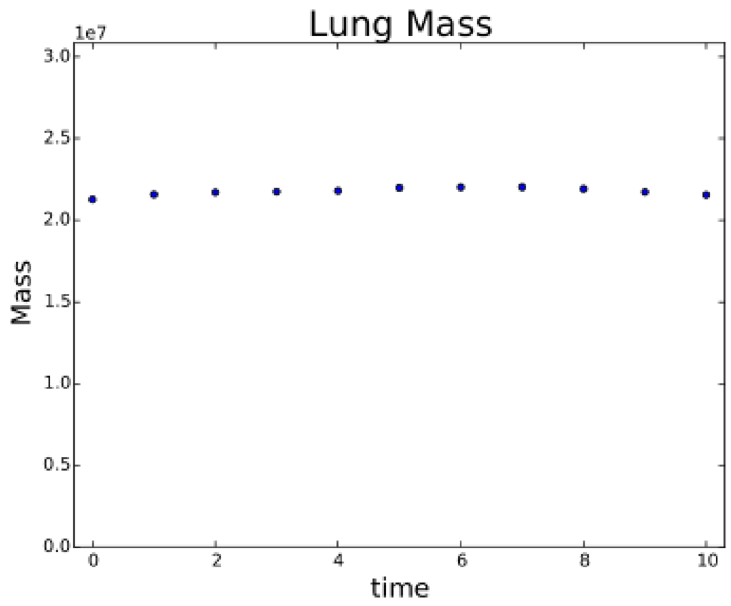


Volume, Density, Mass

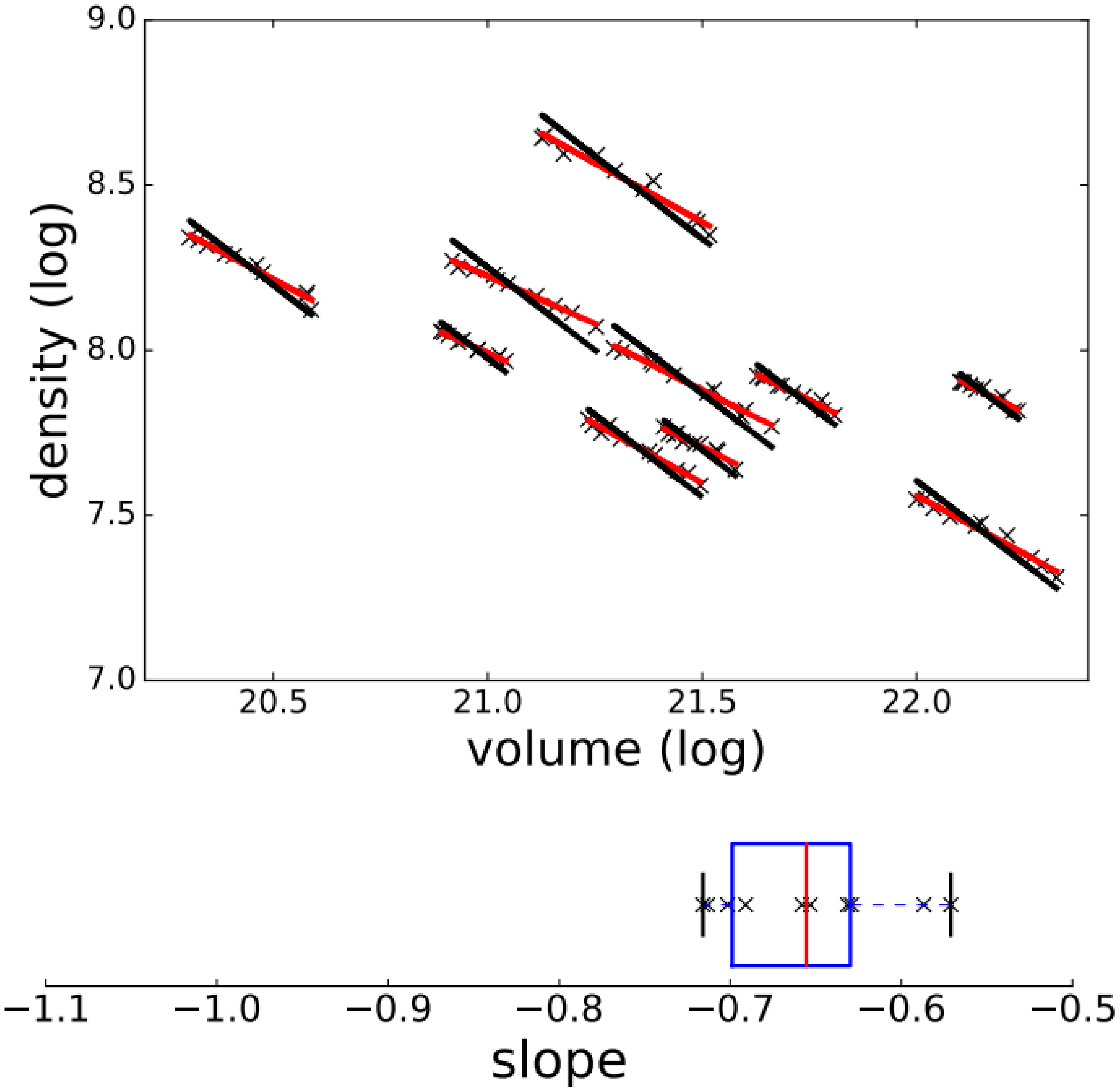
- Using $\mu = \hat{\mu}^2$







Analysis of 10 Human 4D-RCCT Data Sets



- Imperially estimate the power transformation from the data.
- Optimal power transformation is 1.64

Our Method

- View problem as density matching instead of image matching[1]
- Place physiological constraint on lung mechanics
  - Lungs/air is compressible
    - Mixture of soft tissue and air
  - Rest of the body is incompressible (essentially divergence free)
- Use a Left (Right) invariant metric on Diff and a Left (Right) action on Densities.

- Fundamentally different than LDDMM: Right invariant metric and Left action.

[1]Bauer, M., Joshi, S., Modin, K.: Diffeomorphic density matching by optimal information transport. SIAM Journal on Imaging Sciences 8 (3), 1718-1751

Density

- A density μ is a volume form on Ω
  - Non-negative function with a volume element

$$\mu = I\,dx$$

$$dx = dx^1 \wedge dx^2 \wedge dx^3$$

$$\mathrm{vol}(\Omega) = \int_\Omega dx^1 \wedge dx^2 \wedge dx^3$$

- The space of densities, Dens(Ω), is an infinite-dimensional Fréchet manifold

Diffeomorphism group action

L^2 image action

Right action: composition

$$(\varphi, I) \mapsto \varphi^*(I) = I \circ \varphi$$

Left action: composition

$$(\varphi, I) \mapsto \varphi_*(I) = I \circ \varphi^{-1}$$

Density action

Right action: pushforward on volume forms

$$(\varphi, \mu) \mapsto \varphi^*(\mu) = |D\varphi| \mu \circ \varphi = \big(|D\varphi| I \circ \varphi\big)\, dx$$

Left action: pullback on volume forms

$$(\varphi, \mu) \mapsto \varphi_*(\mu) = |D\varphi^{-1}| \mu \circ \varphi^{-1} = \big(|D\varphi^{-1}| I \circ \varphi^{-1}\big)\, dx$$

Conservation of mass

$$\int_\Omega \mu = \int_\Omega I(x)\,dx$$
$$\int_\Omega \varphi_*(\mu) = \int_\Omega |D\varphi^{-1}(x)| I \circ \varphi^{-1}(x)\,dx$$

Change of variables $\quad x \mapsto \varphi(y),\ \ dx \mapsto |D\varphi| dy,\ \ |D\varphi^{-1}(x)| \mapsto \dfrac{1}{|D\varphi(y)|}$

$$= \int_\Omega \frac{1}{|D\varphi(y)|} I \circ \varphi^{-1} \circ \varphi(y)\,|D\varphi(y)| dy$$
$$= \int_\Omega I(y)\,dy$$

Fisher-Rao metric on densities

$$G^F_\mu(\alpha,\beta) = \frac{1}{4}\int_\Omega \frac{\alpha}{\mu}\frac{\beta}{\mu}\mu$$

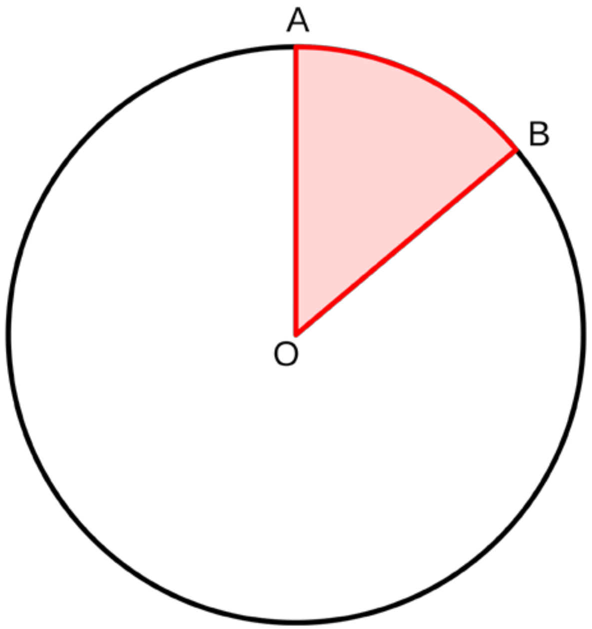
$$\alpha,\beta \in T_\mu \mathrm{Dens}(\Omega)$$

- Using the W-map
  - isometry between Dens(Ω) and $S^\infty(\Omega)$

$$W : \mu \mapsto \sqrt{\frac{\mu}{dx}}$$

- Since distances and geodesics are explicit on the sphere, the Fisher-Rao distance is the distance on the sphere

Fisher-Rao distance



- Fisher-Rao distance
  - Geodesic distance on S^∞

$$d_F(\mu_0, \mu_1) = \theta = \arccos\left(\langle \sqrt{\mu_0}, \sqrt{\mu_1} \rangle_{L^2}\right)$$

- Geodesics on S^∞ are explicit:

$$[0,1] \ni t \mapsto \left(\frac{\sin((1-t)\theta)}{\sin\theta} \sqrt{\mu_0} + \frac{\sin(t\theta)}{\sin\theta} \sqrt{\mu_1} \right)^2$$

- For infinite volume, the geodesic distance becomes the chord distance, or Hellinger distance

$$d_F^2(I_0 dx, I_1 dx) = \int_\Omega (\sqrt{I_0} - \sqrt{I_1})^2 dx$$

Fisher-Rao distance

- The Fisher-Rao metric is the unique[1] Riemannian metric on the space of densities that is invariant under the action of a diffeomorphism

$$
\begin{aligned}
d_F^2(I_0 dx, I_1 dx) &= d_F^2(\varphi_*(I_0 dx), \varphi_*(I_1 dx)) \qquad\qquad \forall \varphi \in \mathrm{Diff}(\Omega) \\
&= \int_\Omega (\sqrt{|D\varphi^{-1}(x)| I_0 \circ \varphi^{-1}} - \sqrt{|D\varphi^{-1}(x)| I_1 \circ \varphi^{-1}})^2 dx
\end{aligned}
$$

$$
x \mapsto \varphi(y), \;\; dx \mapsto |D\varphi| dy, \;\; |D\varphi^{-1}(x)| \mapsto \frac{1}{|D\varphi(y)|}
$$

$$
\begin{aligned}
&= \int_\Omega \Big(\sqrt{\frac{1}{|D\varphi(y)|} I_0} - \sqrt{\frac{1}{|D\varphi(y)|} I_1}\Big)^2 |D\varphi(y)| dy \\
&= \int_\Omega (\sqrt{I_0} - \sqrt{I_1})^2 dy
\end{aligned}
$$

[1]Bauer, M., Bruveris, M., Michor, P.W.: Uniqueness of the Fisher–Rao metric on the space of smooth densities. http://arxiv.org/abs/1411.5577, submitted (2015)

Descending Metric on Diff

The infinitesimal action of a vector field on a density (Lie derivative) is

$$\mathcal{L}_u \mu = \mathrm{div}_\mu(u)\mu$$

The information metric on Diff descends to the Fisher-Rao metric on Dens

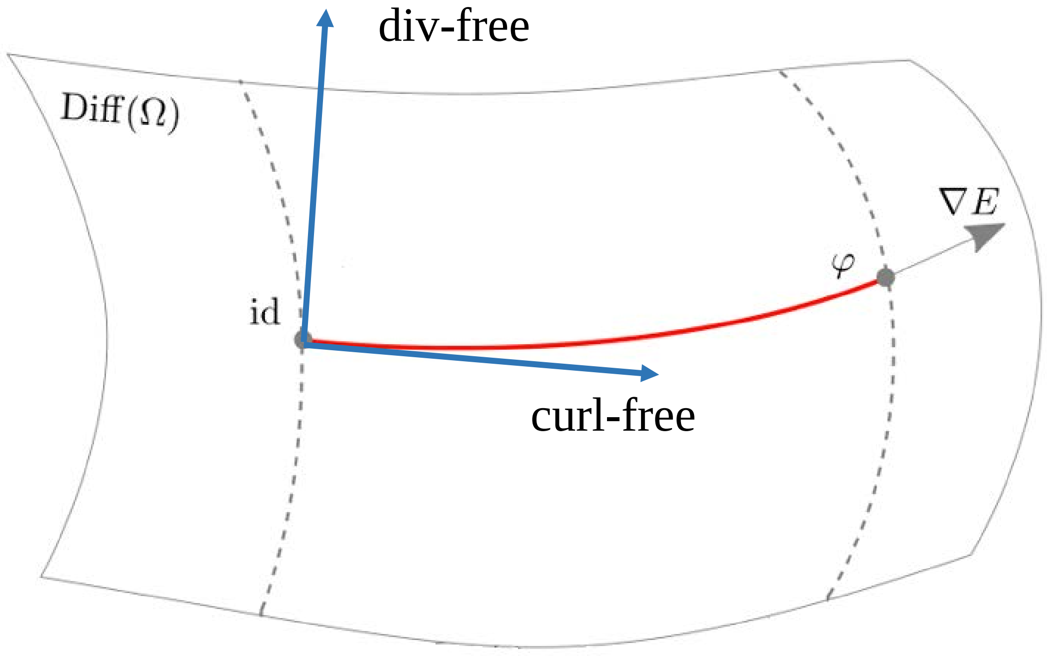
$$G^I = \int_\Omega \langle \Delta u, v \rangle d\mu(x)$$

$$\Delta u = \nabla \mathrm{div}(u) - \nabla \times \nabla u = -(\delta \mathrm{d} u^\flat + \mathrm{d}\delta u^\flat)^\sharp$$

$$\alpha = \mathcal{L}_u \mu, \;\; \beta = \mathcal{L}_v \mu$$

$$G^I(u,v) = \int_\Omega \frac{\alpha}{\mu}\frac{\beta}{\mu} d\mu(x) = G^{FR}(\alpha, \beta)$$

$$\mathrm{Dens}(\Omega) = \mathrm{Diff}(\Omega)/\mathrm{SDiff}(\Omega)$$



By Helmholtz-Hodge the vertical (div-free) and horizontal (curl-free) vector fields are orthogonal

Riemannian submersion

Lemma 2.4 (see [27]). *Under the identification* $\mathrm{Dens}(M) \simeq \mathrm{Diff}_{\mathrm{vol}}(M)\backslash\mathrm{Diff}(M)$, *the information metric* G^I, *given by* (4), *descends to the Fisher–Rao metric* $\bar{G}^F$, *given by* (3), *i.e.,* $\pi\colon (\mathrm{Diff}(M), G^I) \to (\mathrm{Dens}(M), \bar{G}^F)$ *is a Riemannian submersion. The horizontal distribution is right invariant, given by*

$$\mathcal{H}_\varphi = \{U \in T_\varphi \mathrm{Diff}(M); U \circ \varphi^{-1} = \mathrm{grad}(f), f \in C^\infty(M)\}.$$

- Horizontal geodesics on Diff descend to Fisher-Rao geodesics on Dens
- There is a unique horizontal lift of curves in Dens to curves in Diff

Left invariant penalty using Fisher-Rao

- Distance between L^2 image action and density action for positive functions

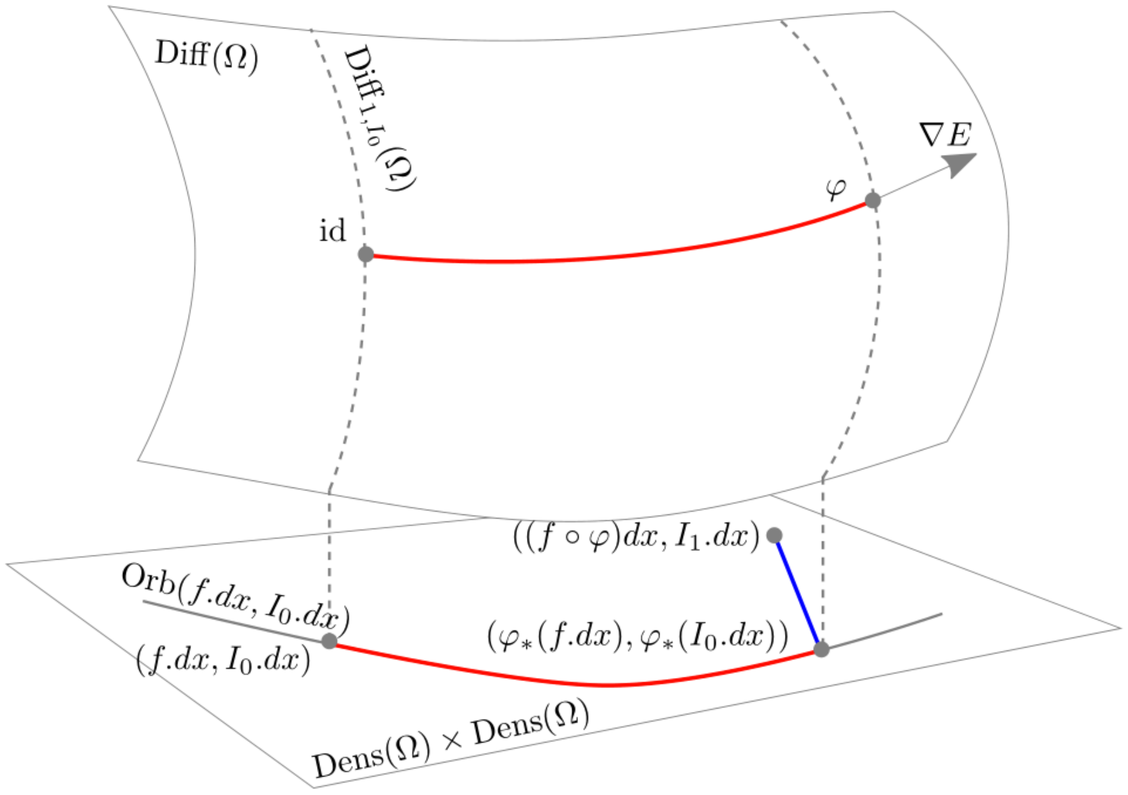
$$
\begin{aligned}
E(\varphi) &= d_F^2(\varphi_*(f\,dx), f\circ\varphi^{-1}\,dx) \\
&= \int_\Omega \left(\sqrt{|D\varphi^{-1}|}\sqrt{f\circ\varphi^{-1}} - \sqrt{f\circ\varphi^{-1}}\right)^2 dx \\
&= \int_\Omega \left(\sqrt{|D\varphi^{-1}|} - 1\right)^2 f\circ\varphi^{-1} dx
\end{aligned}
$$

- Since the information metric descends to the Fisher-Rao, we define the metric on Dens and then horizontally lift to Diff

Energy functional

- Minimization problem on the product space Dens(Ω) x Dens(Ω) using the product distance

$$E(\varphi) = d_F^2(\varphi_*(f\,dx), f \circ \varphi^{-1}\,dx) + d_F^2((\varphi_*(I_0\,dx), I_1\,dx)$$



Energy functional

$$E(\varphi) = \underbrace{\int_\Omega (\sqrt{|D\varphi^{-1}|} - 1)^2 \, f \circ \varphi^{-1} \, dx}_{E_1(\varphi)} + \underbrace{\int_\Omega \left(\sqrt{|D\varphi^{-1}| I_0 \circ \varphi^{-1}} - \sqrt{I_1} \right)^2 dx}_{E_2(\varphi)}$$

- E_1 is the regularity measure
  - Weighted by f
- E_2 is the matching term

Energy functional

- Minimizers of E_1 are not unique
  - The functional is invariant under volume preserving diffeomorphisms
- Strategy:
  - The fact that the metric is descending with respect to the metric on Diff can be used to ensure that the gradient flow is *infinitesimally optimal*, i.e., always orthogonal to the null-space (horizontal lift of the density flow to diff)

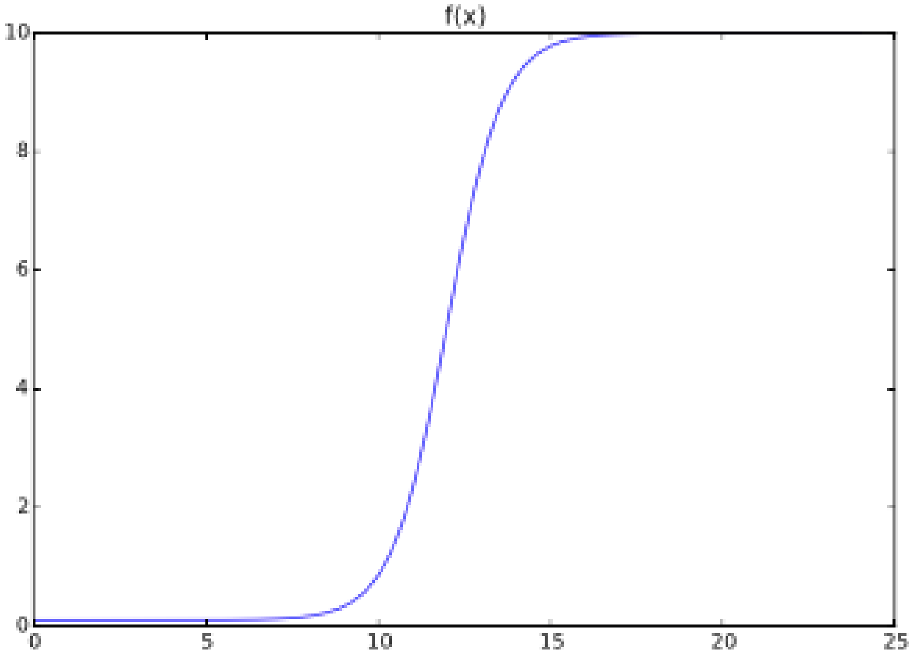
Final Algorithm

Choose $\epsilon > 0$
Set $\varphi^{-1} = \mathrm{id}$
Set $|D\varphi^{-1}| = 1$
for $iter = 1...\mathrm{NumIters}$ **do**
  Compute $\varphi_* I_0 = I_0 \circ \varphi^{-1} |D\varphi^{-1}|$
  Compute $u = -\nabla\big(f \circ \varphi^{-1}(1 - \sqrt{|D\varphi^{-1}|})\big) - \sqrt{\varphi_* I_0}\nabla\sqrt{I_1} + \nabla(\sqrt{\varphi_* I_0})\sqrt{I_1}$
  Compute $v = -\Delta^{-1}(u)$
  Update $\varphi^{-1} \mapsto \varphi^{-1}(y + \epsilon v)$
  Update $|D\varphi^{-1}| \mapsto |D\varphi^{-1}| \circ \varphi^{-1} \mathrm{e}^{-\epsilon \mathrm{div}(v)}$
end for

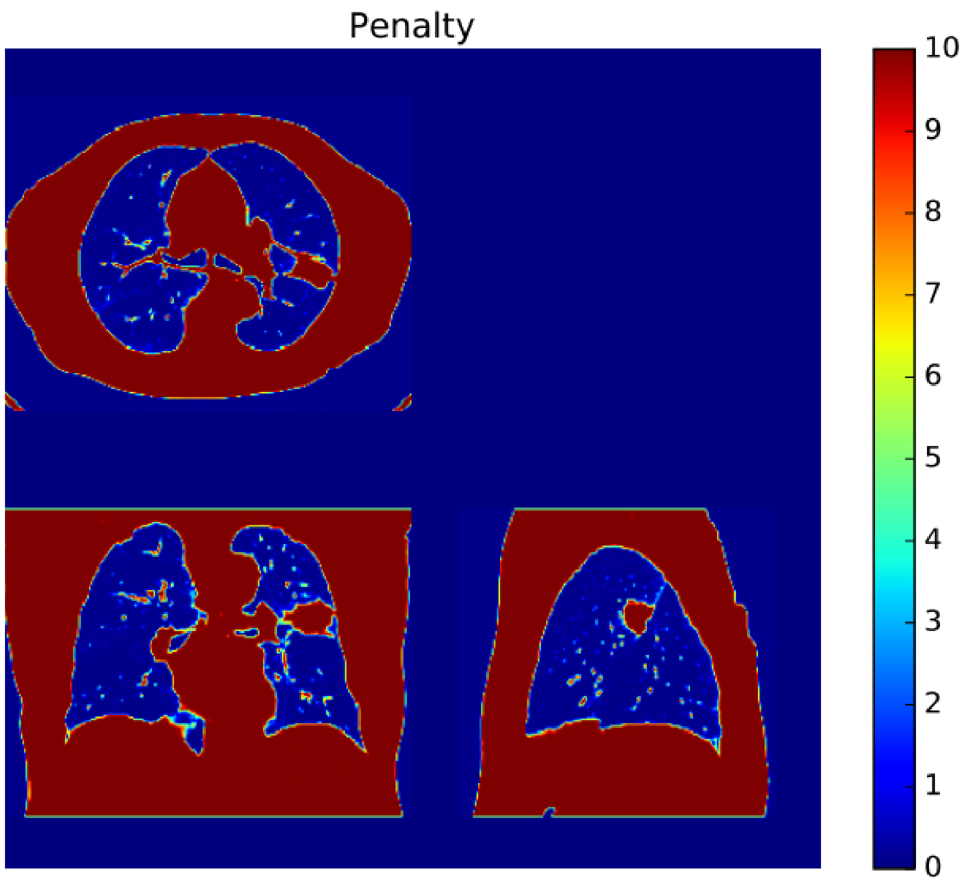
Weighting density

- Use a soft threshold on image values

$$f(I_0(x)) = \mathrm{sig}(I_0(x)) \in [0.1, 10]$$

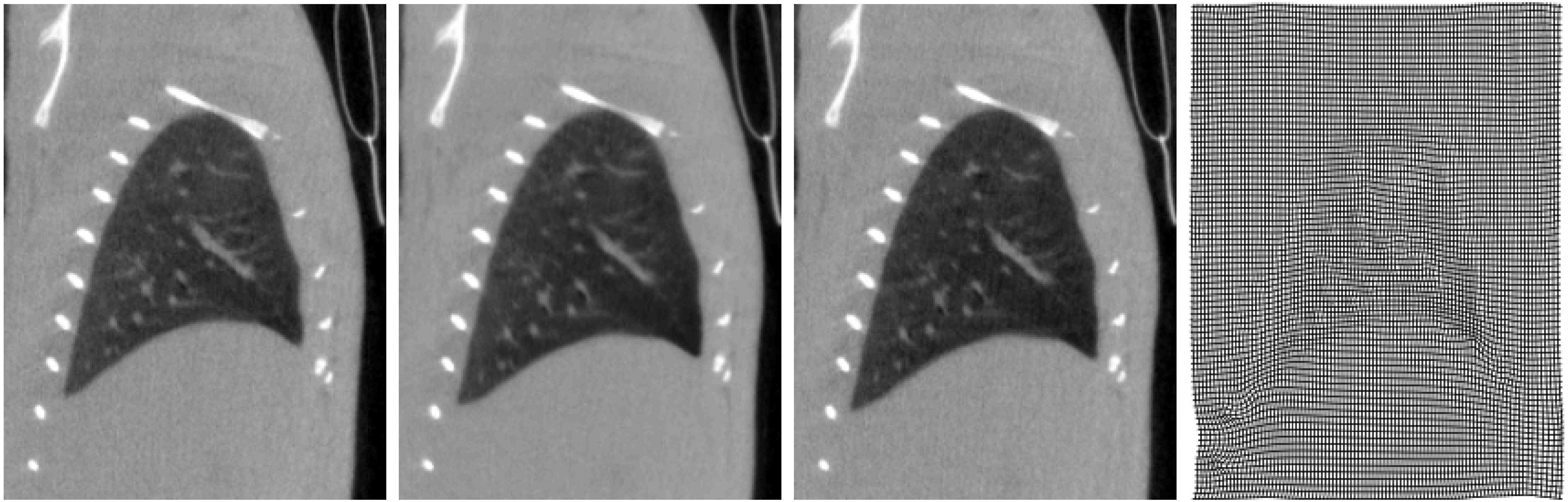


$\mathrm{sig}(x)$



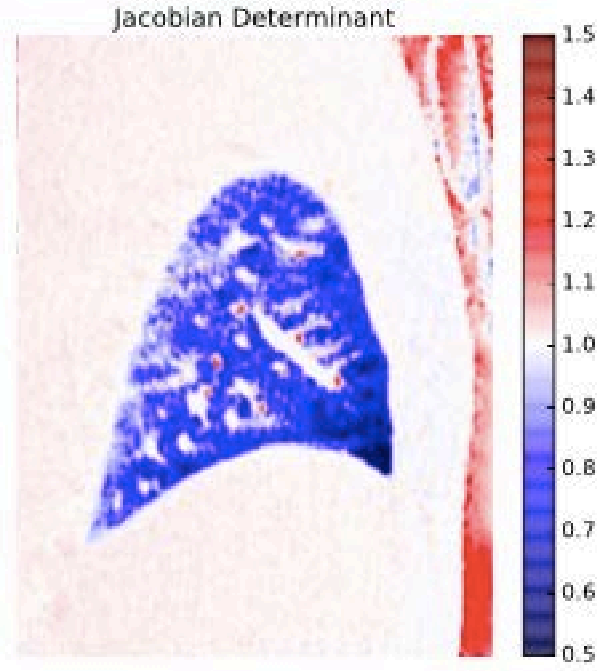
$\mathrm{sig}(I_{ex})$

Results (Rat)

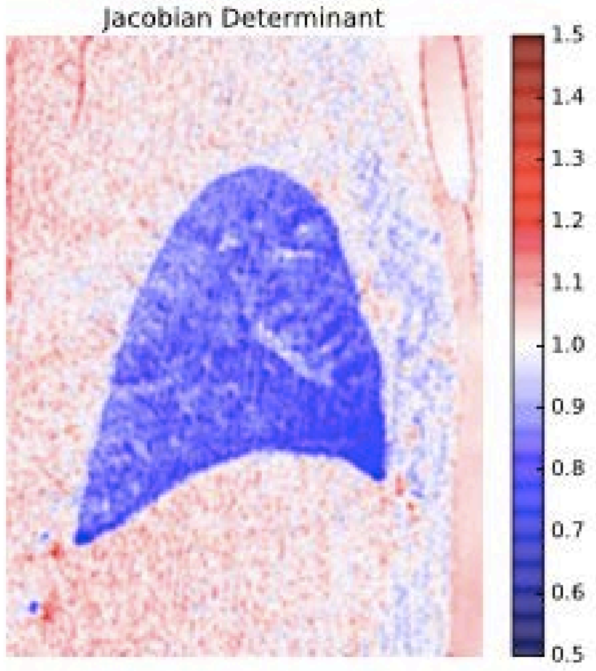


I_{ex} $\varphi_*(I_{ex}dx)$ I_{in} φ^{-1}

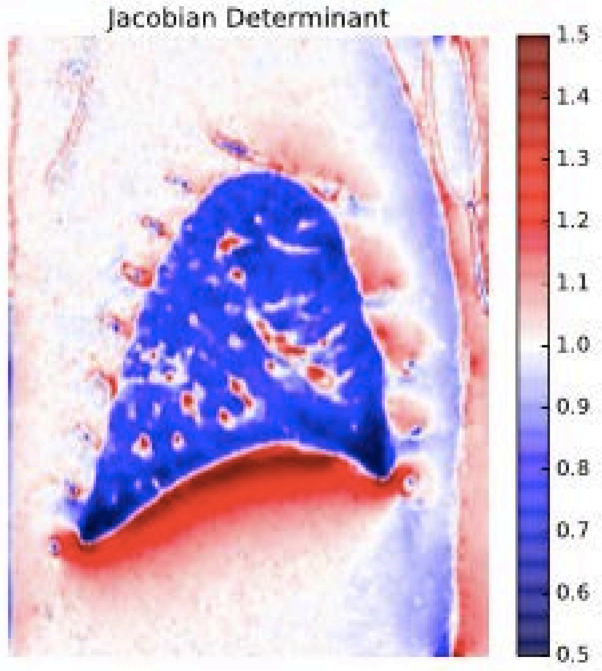
Results (Rat)



$f(x) = \mathrm{sig}(I_0(x))$

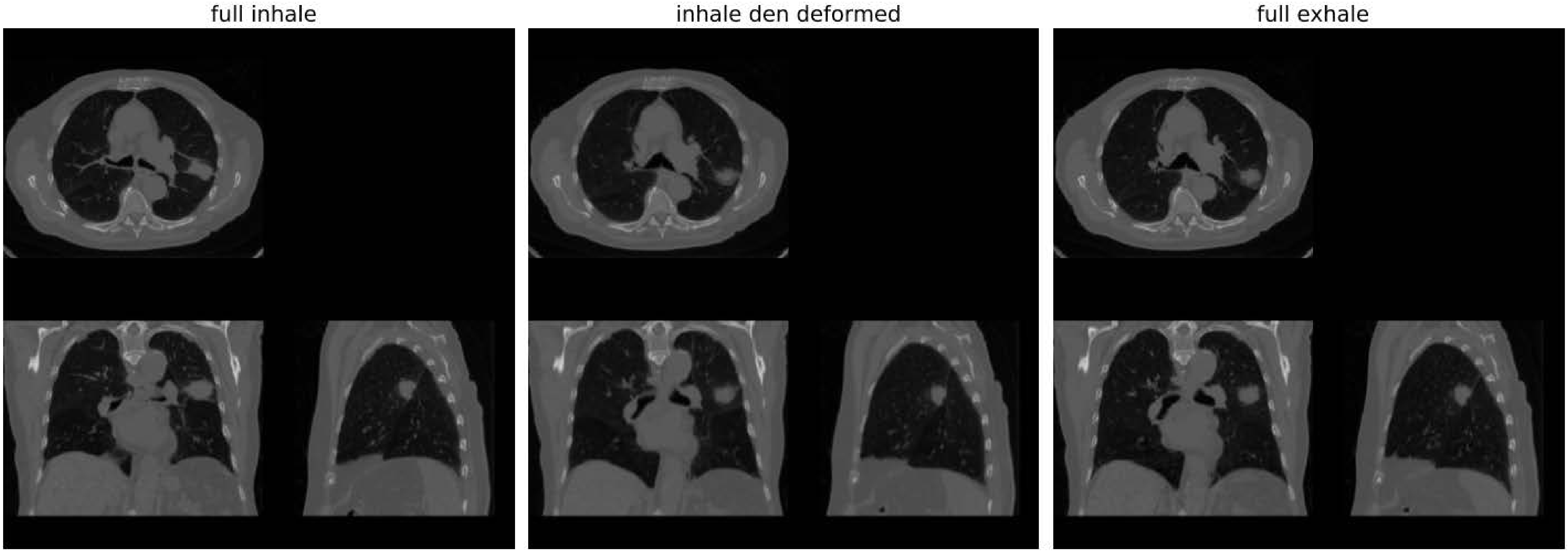


$f(x) = 1$



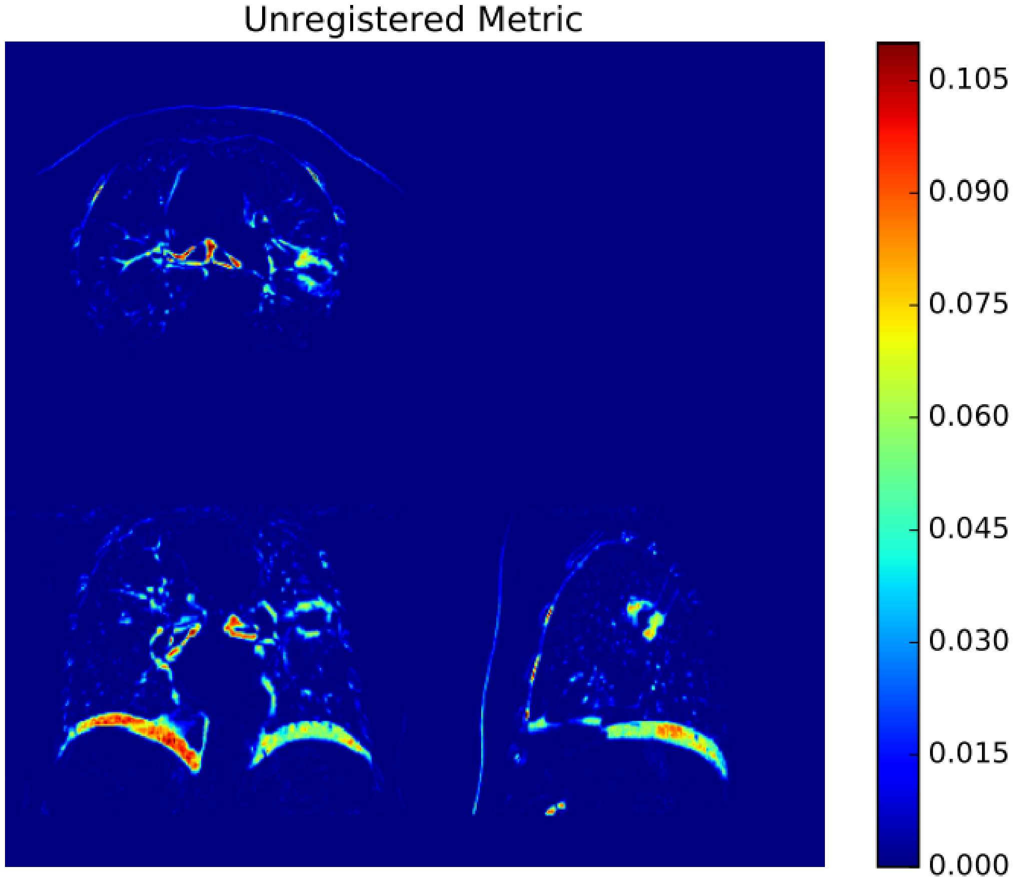
LDDMM

Results (Human)

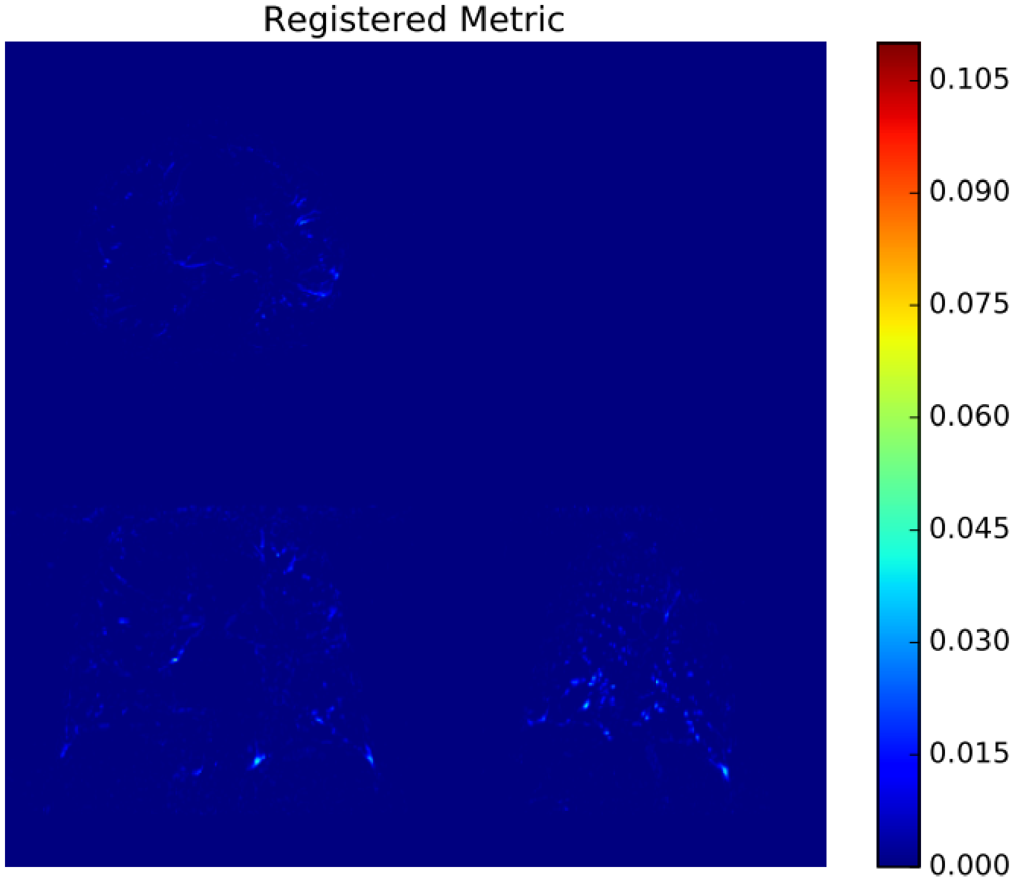


I_{ex} $\varphi_*(I_{ex}\,dx)$ I_{ex}

Results

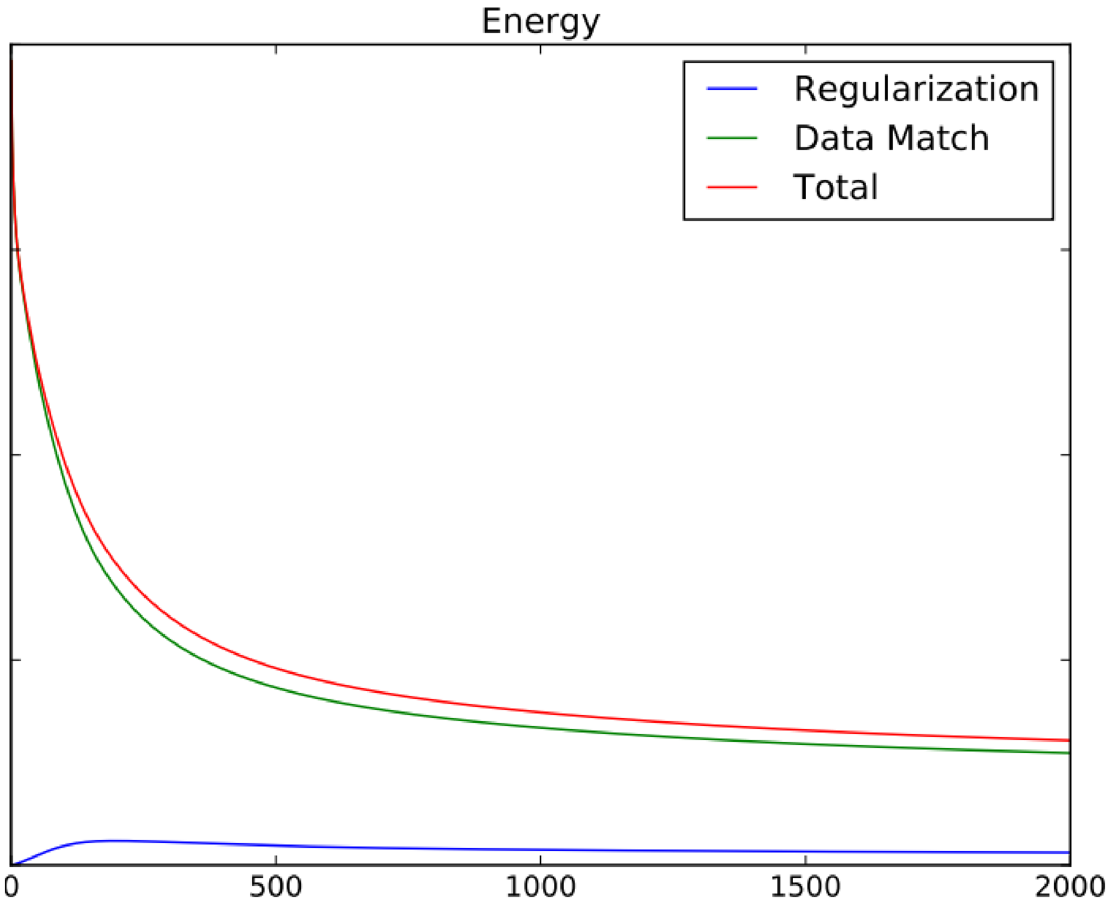


$d_{FR}(I_{in}, I_{ex})$

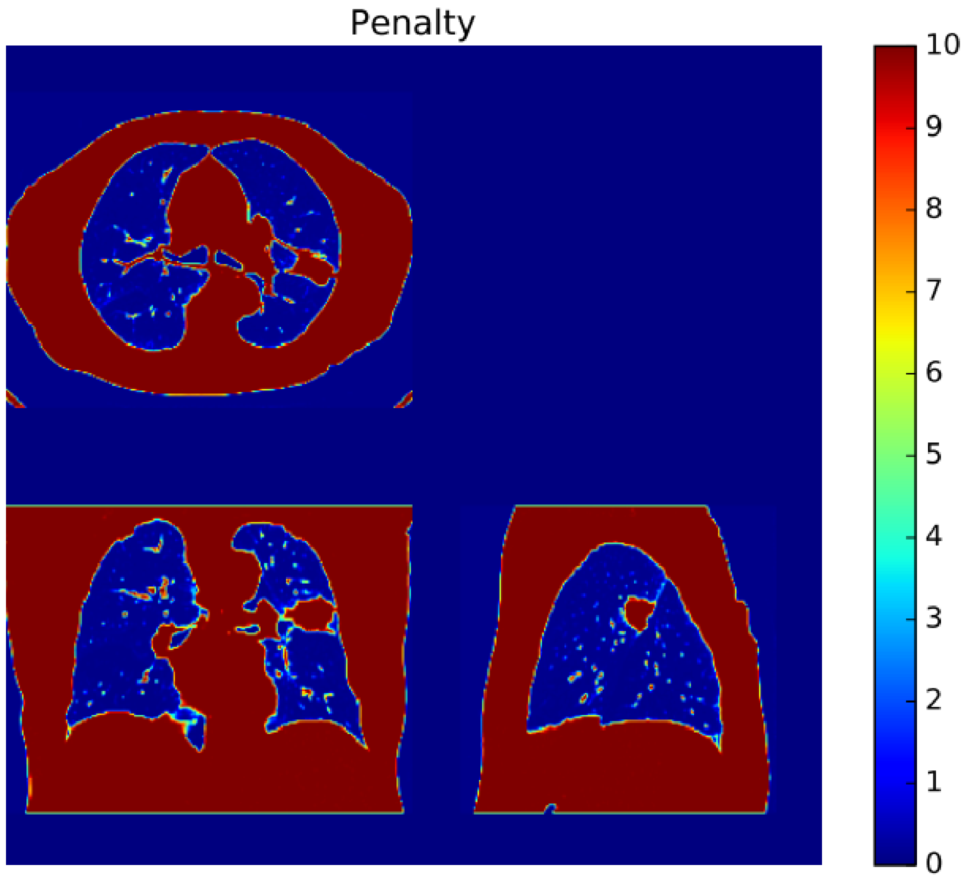


$d_{FR}(I_{in}, \varphi_*(I_{ex}dx))$

Results

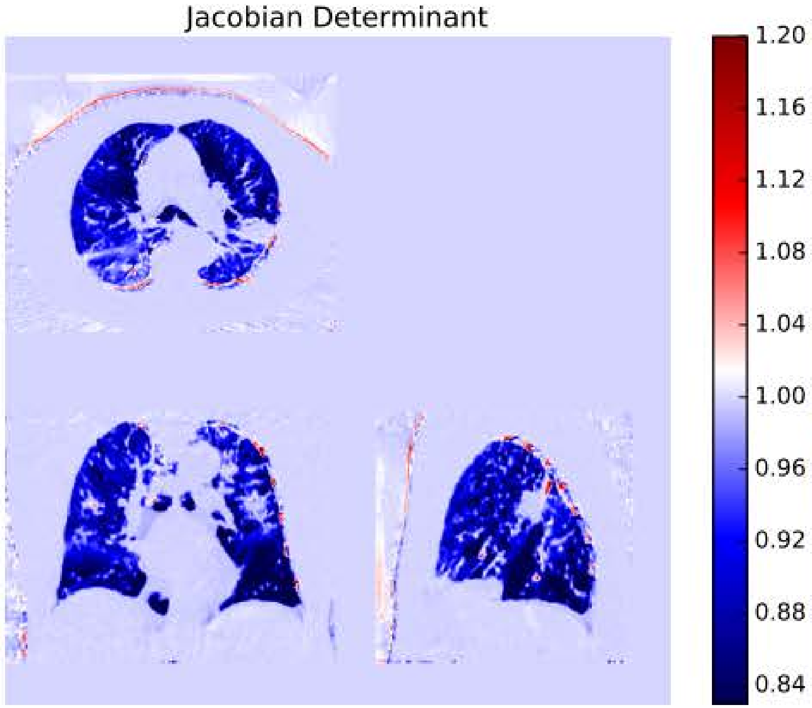


Energy

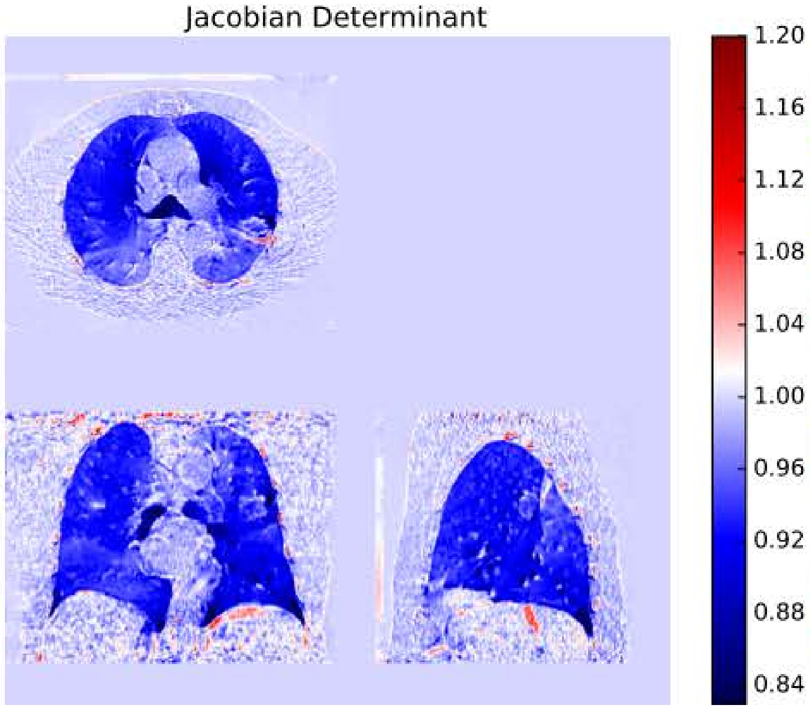


$\mathrm{sig}(I_{ex})$

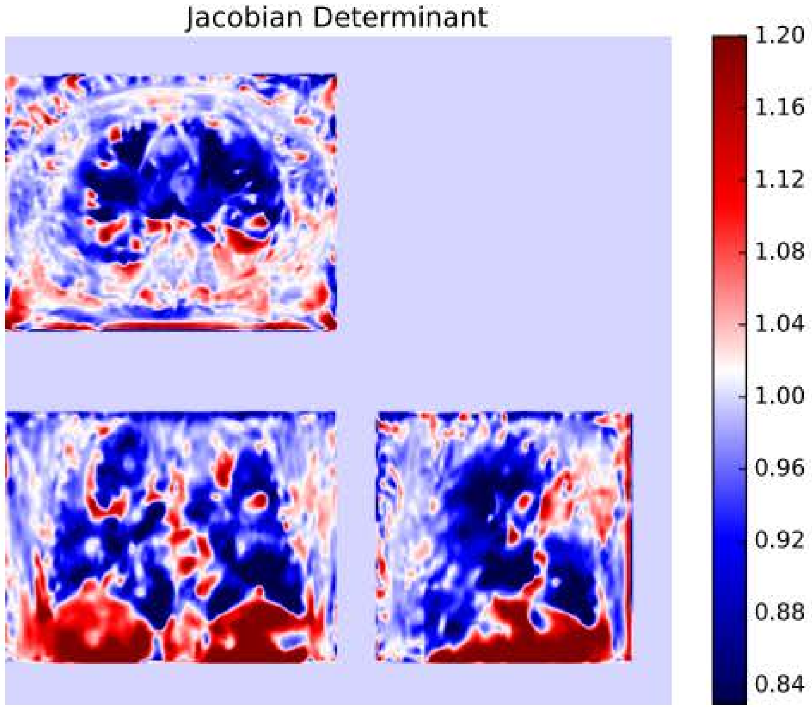
Jacobian Determinants



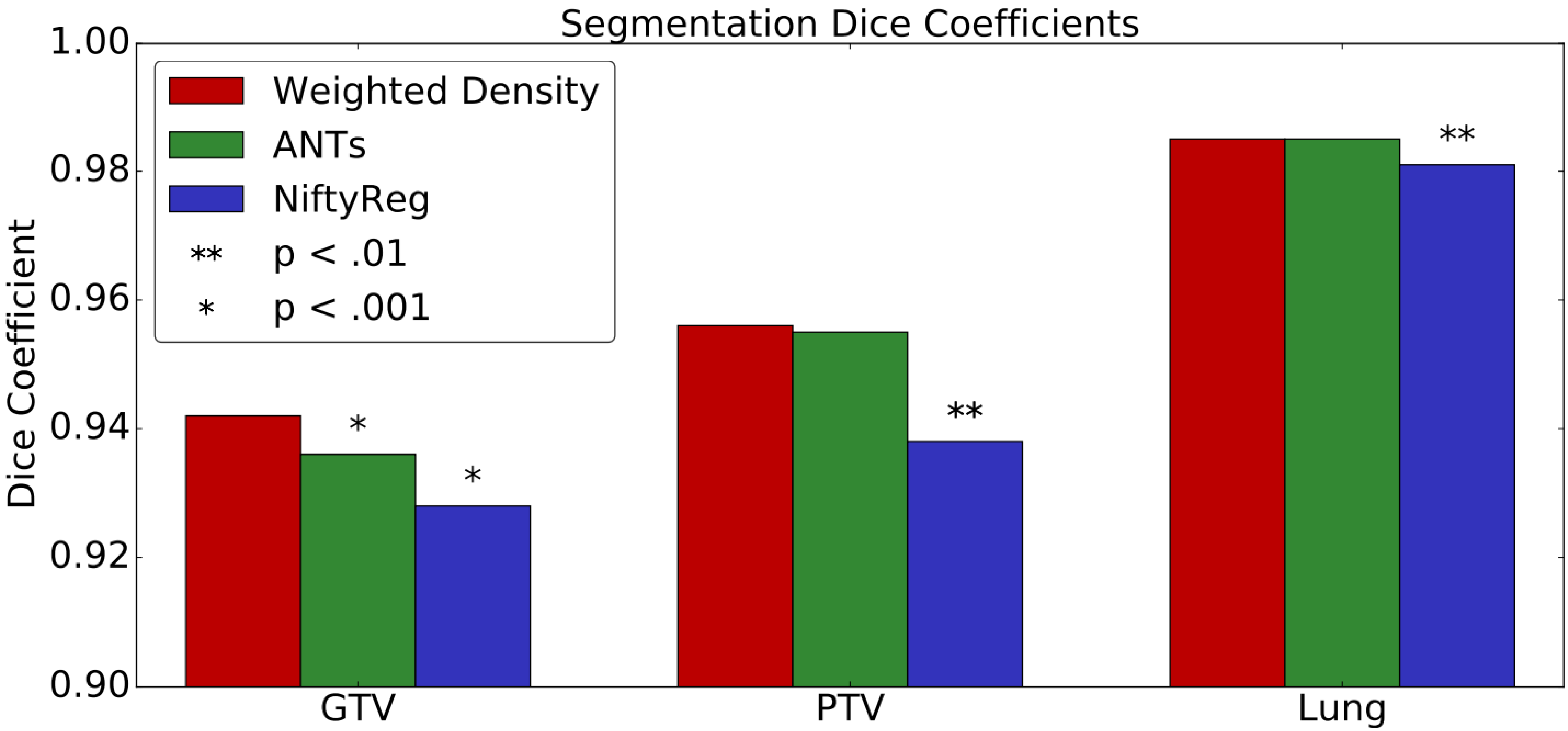
$f(x) = \mathrm{sig}(I_0(x))$



$f(x) = \sigma$



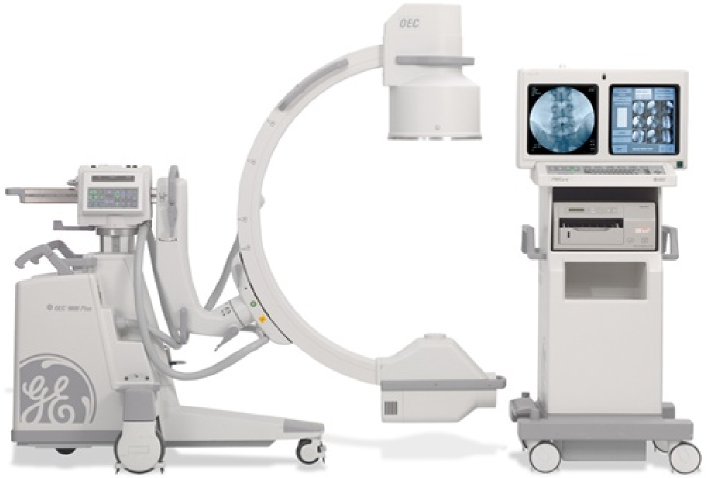
ANTs


Segmentation Dice Coefficients
Dice Coefficient
1.00
0.98
0.96
0.94
0.92
0.90
Weighted Density
ANTs
NiftyReg
** p < .01
* p < .001
*
*
**
**
GTV
PTV
Lung

Ground Truth

Estimated

Motivation – 3D CT Reconstruction



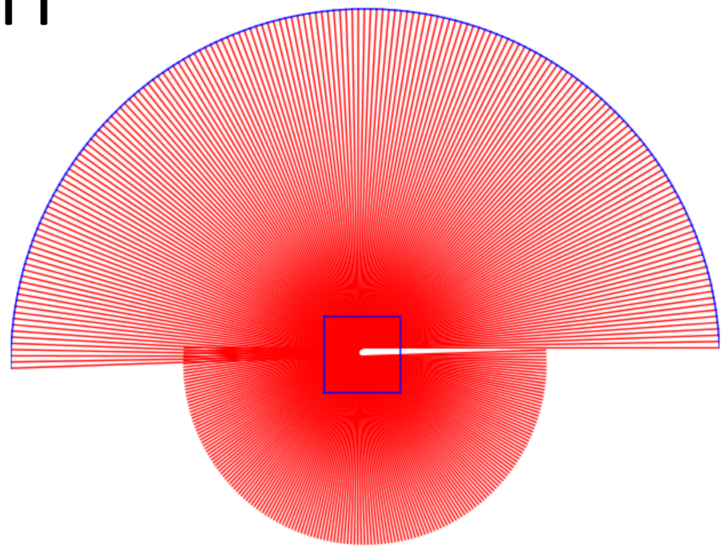
<u>Fixed-room CT scanner</u>

- Designed for 3D imaging
- Fixed/calibrated geometry
- Immobile, expensive

<u>Mobile C-arm</u>

- Designed for 2D imaging
- **Variable/uncalibrated geometry**
- Non-isocentric, limited angle
- Mobile, inexpensive
- 3D reconstruction rare

Mobile C-arm Reconstruction



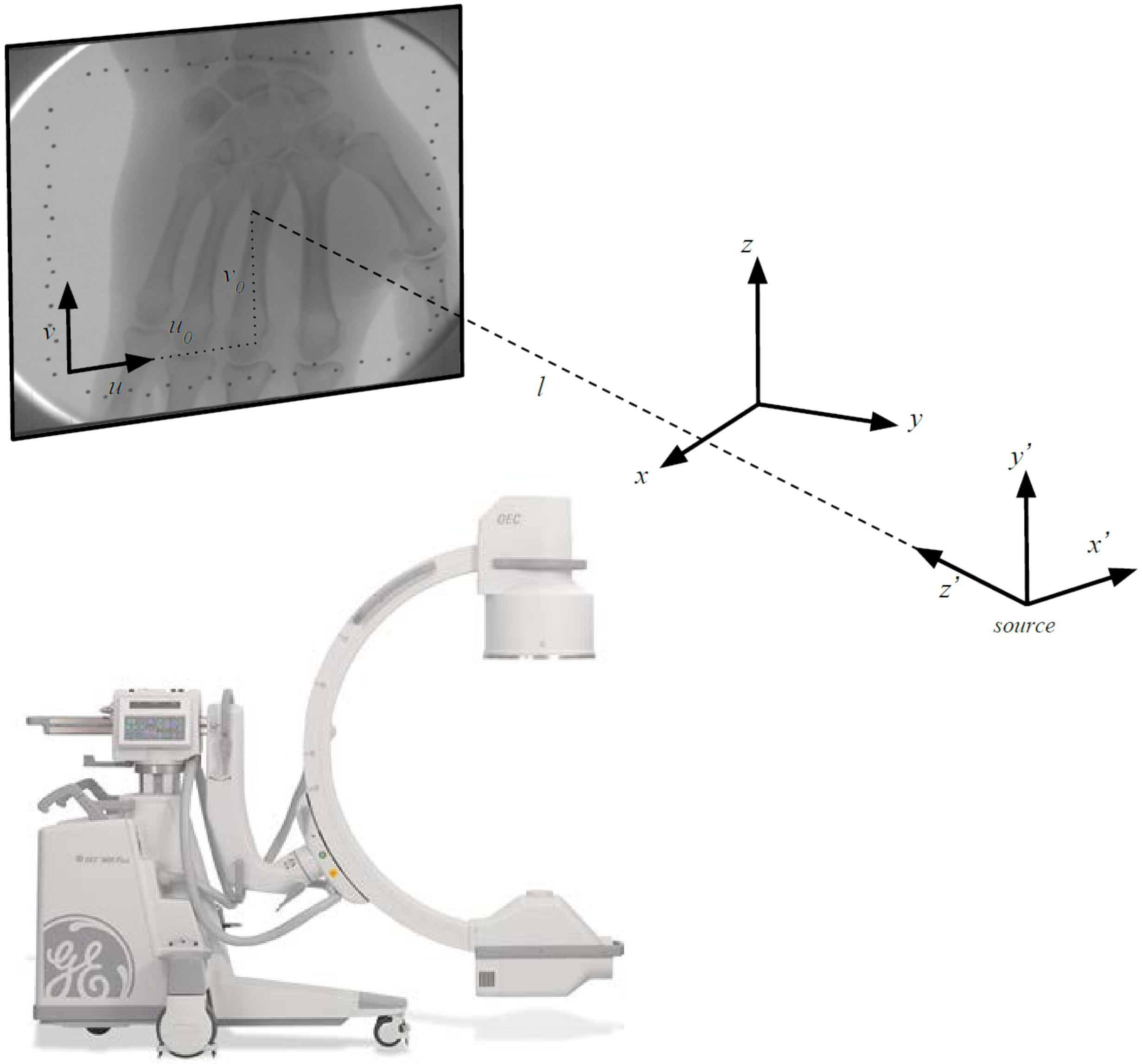
Background: X-ray Computed Tomography (CT)

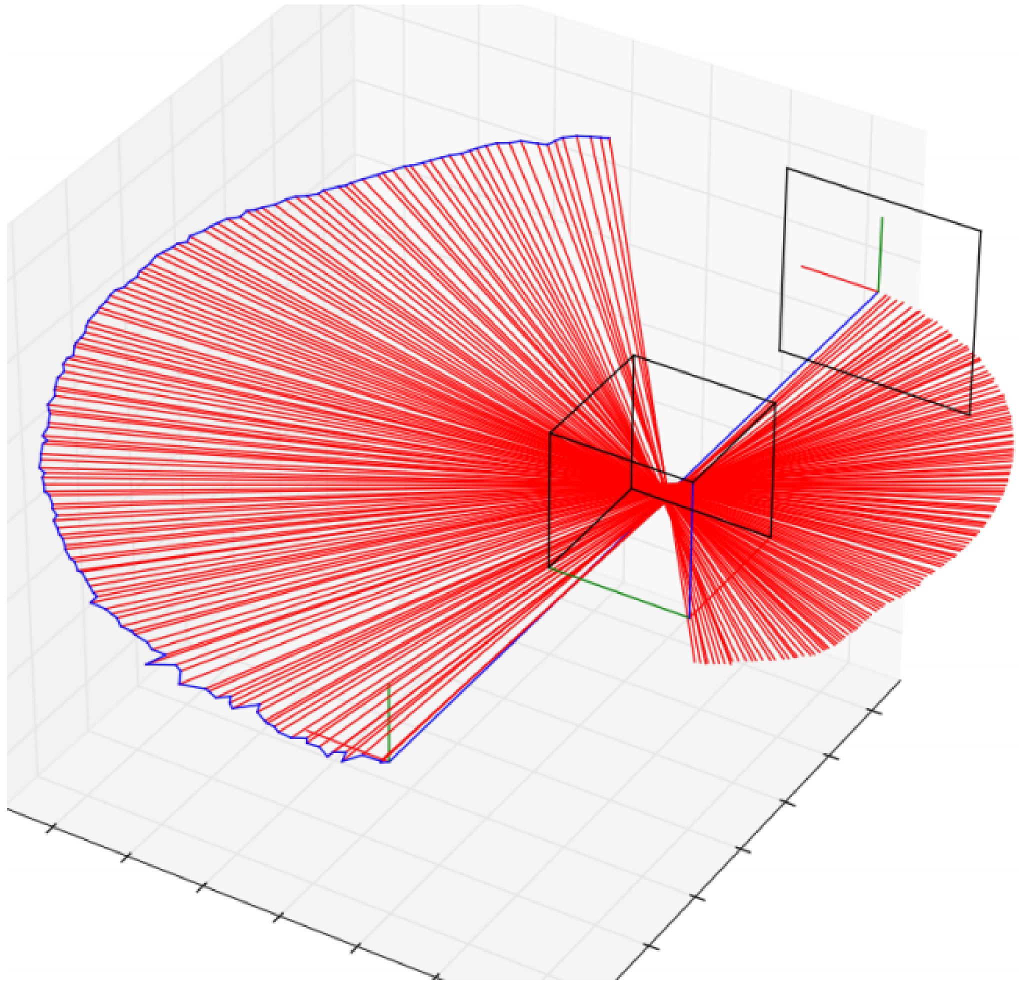
- Problem statement: given multiple X-ray projection images, solve for the 3D volume of linear attenuation coefficients
- “Forward problem” is easy
  - Attenuation due to Compton scattering and photoelectric effect
  - Beer-Lambert law:

$$i_d = i_0 \exp\left(- \int_l \mu(x') dx' \right)$$

- “Inverse problem” is hard

Cone-beam System





Background

- Fixed-room systems
  - GE Innova CT, Siemens Artis Zeego DynaCT
- Mobile C-arms
  - Select few have 3D imaging
    - All isocentric with specialized hardware
  - Most are restricted to 2D fluoroscopic imaging
    - Geometric parameters change from scan to scan

Uncertain Geometry

- Possible solution
  - Improve the hardware (high precision components, optical/RF tracking)
- Proposed solution
  - Design reconstruction framework that is robust to variable geometry
- In practice
  - Somewhere in between?

Proposed Solution

- Traditional CT problem: estimate 3D image given projection data
- Mobile C-arm CT problem: estimate 3D image *and* geometric parameters
- Iterative update: alternate between image updates and parameter updates

Image Update

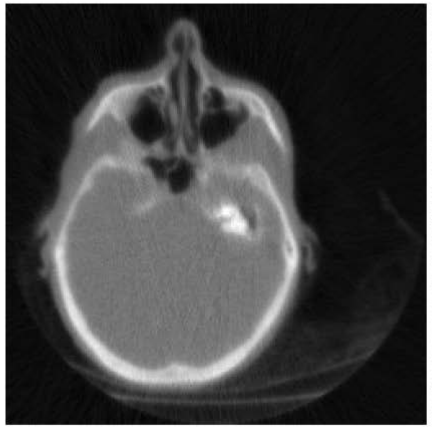
- Image update
  - Ordered subset expectation maximization (OSEM) with total variation (TV) regularization

$$U(I) = \int_{\Omega} |\nabla I(\boldsymbol{x})| \, d\boldsymbol{x}$$

$$I(\boldsymbol{x}) \mapsto \frac{I(\boldsymbol{x})}{\sum_{j \in S} P_j^{\dagger}\{\mathbf{1}(\mathbf{u})\}(\boldsymbol{x}) + \lambda \frac{\partial U(I)}{\partial I(\boldsymbol{x})}} \sum_{j \in S} P_j^{\dagger} \left\{ \frac{f_j^*}{P_j\{I\}} \right\}$$

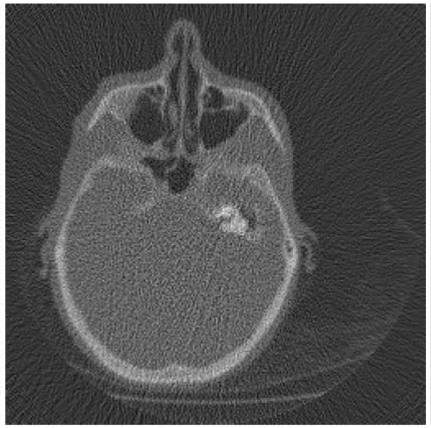
  - Maximizes the log posterior of the 3D image given the projection data

Iterative Reconstruction Methods

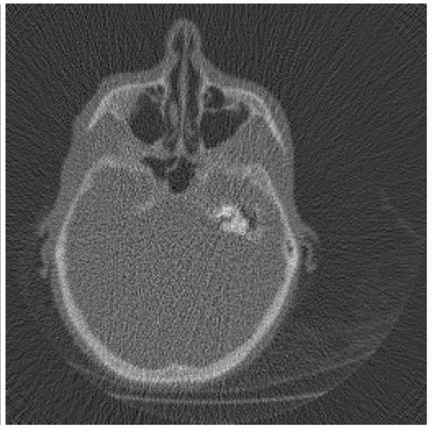


SIRT

SART

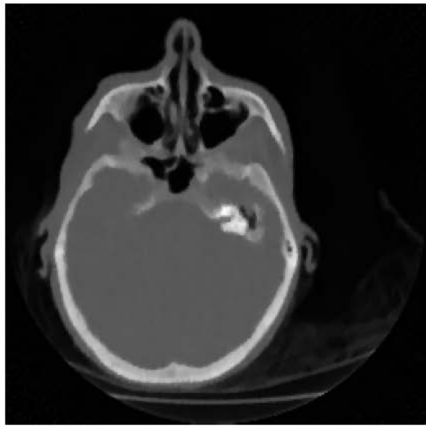


SART-relax

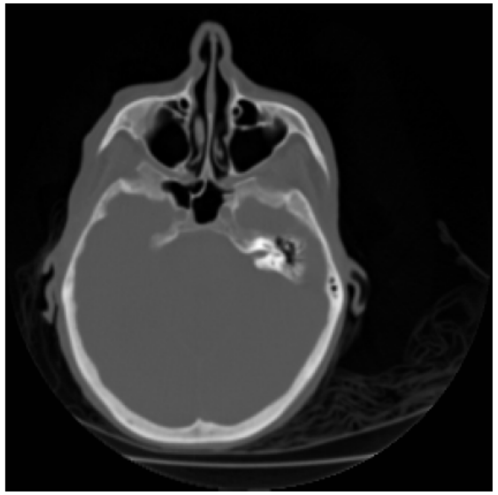


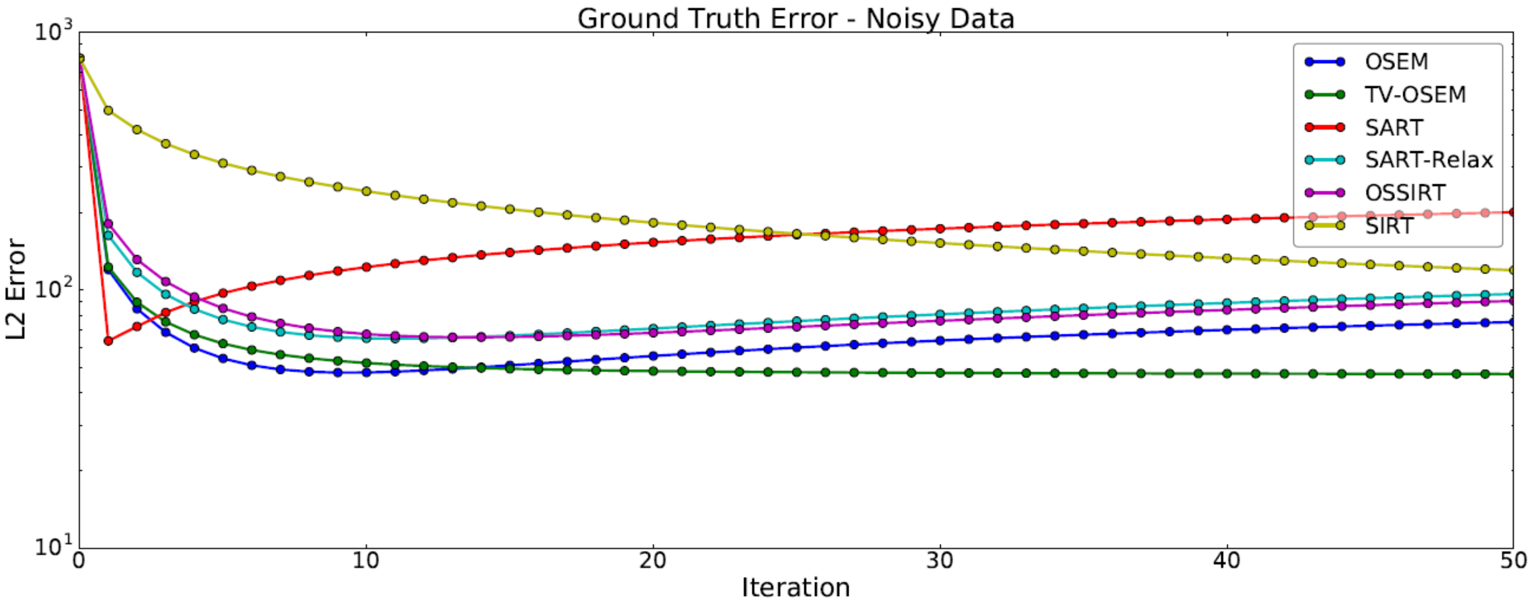
OS-SIRT

OSEM



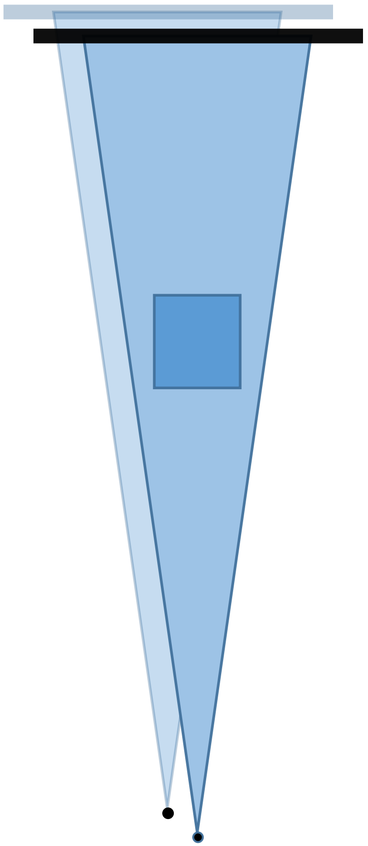
TV-OSEM



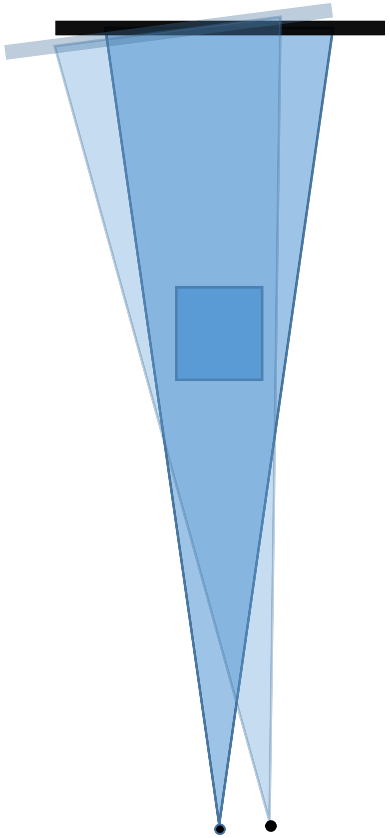


Geometric Parameters (9 per Projection)

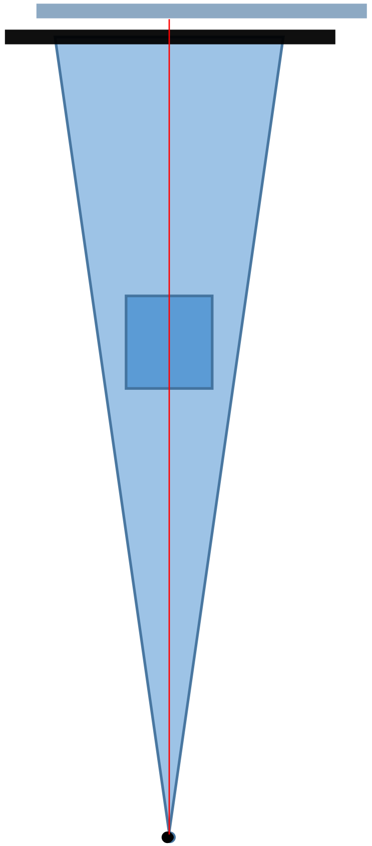
3D translation



3D rotation



Piercing point and SID



Extrinsic parameters:
Rigid transformation of C-arm

Intrinsic parameters:
Internal characteristics of C-arm

Geometry Update

- Maximize correspondence between 2D data f_j^* and projection of the current estimate of 3D image $P\{I;\theta\}$
- Local normalized cross-correlation

$$NCC(f,g,\boldsymbol{u}) = \frac{k*(\bar{f}\bar{g})}{\sqrt{(k*\bar{f}^2)(k*\bar{g}^2)}}$$

- Analytically solve for gradient of projection operator with respect to all geometric parameters

$$E_j(\theta) = \int_{\Omega_d} NCC(P\{I;\theta\}, f_j^*, \boldsymbol{u})\, d\boldsymbol{u}$$

$$\delta E_j(\theta) = \int_{\Omega_d} \nabla_\theta NCC(P\{I;\theta\}, f_j^*, \boldsymbol{u})\, d\boldsymbol{u}$$

- Update parameters by taking a gradient ascent step

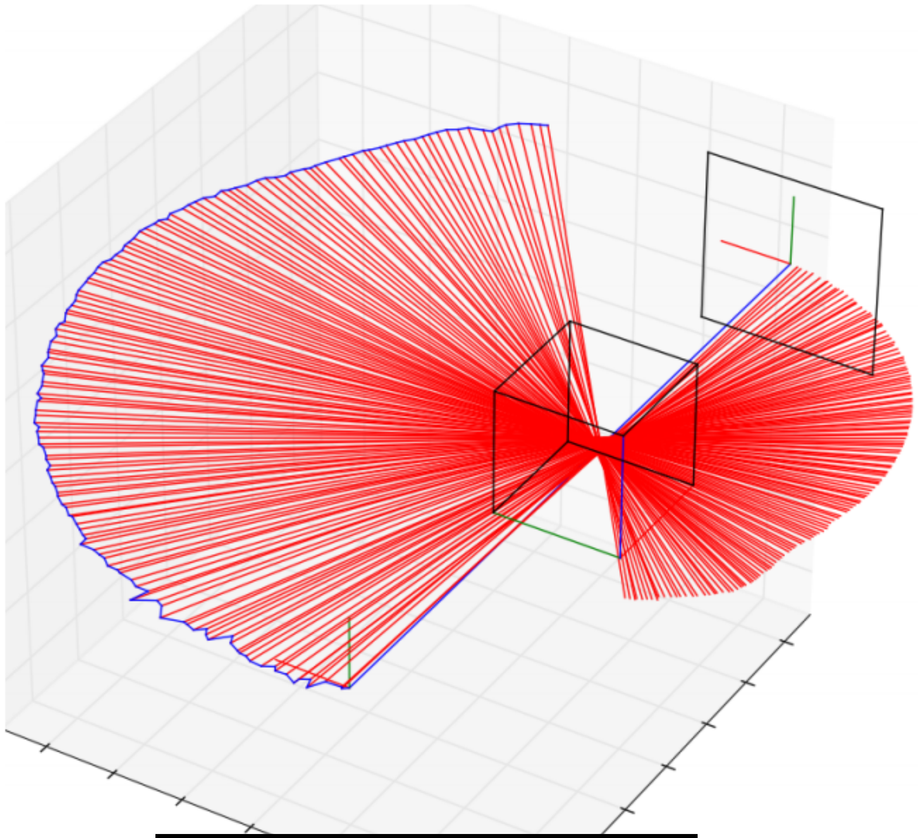
Implementation

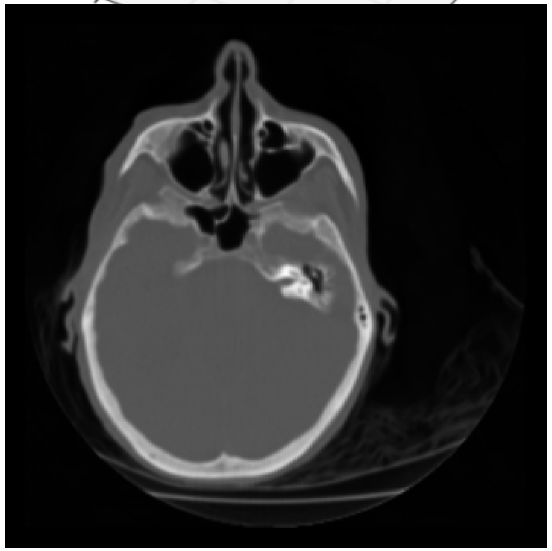
- Challenges
  - Gradient ascent: no guarantee of global convergence
  - Implementing gradient operations is computationally expensive
- Multiscale
  - Start estimation on downsampled data, progress to full-resolution data
  - Lower scales: estimate image and parameters
  - Full-resolution scale: estimate image only
- GPU implementation
  - Parallelization of forward and backward projections as well as parameter gradient calculations

Algorithm Validation

Ground truth parameter dataset

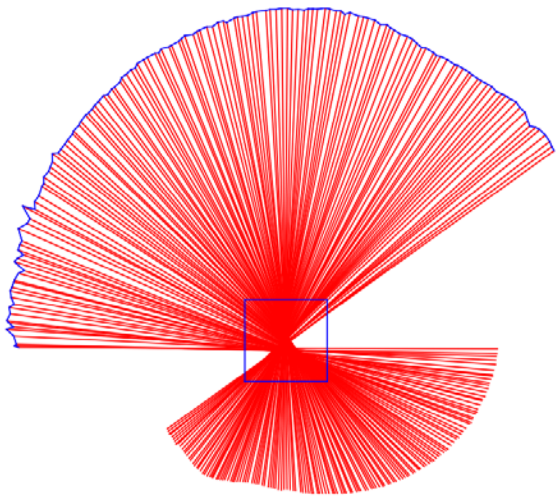
- Used RF tracking to get "ground truth" extrinsic/intrinsic parameters of a full C-arm scan
- Using these parameters, I created a dataset using a digital skull phantom
  - University of North Carolina Volume Rendering Test Data Set



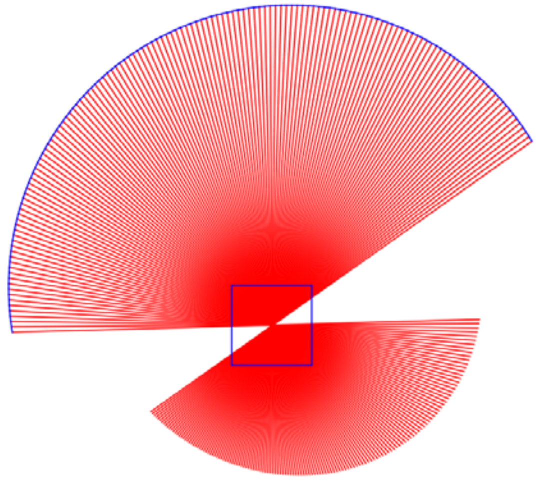


Testing

- Created nominal trajectory (initial estimate of geometry)
  - Circular, equal angular spacing, fixed intrinsic parameters
- 3 Scenarios
  - Ground truth static image reconstruction
  - Nominal trajectory static image reconstruction
  - Joint image and geometry estimation (given nominal trajectory)



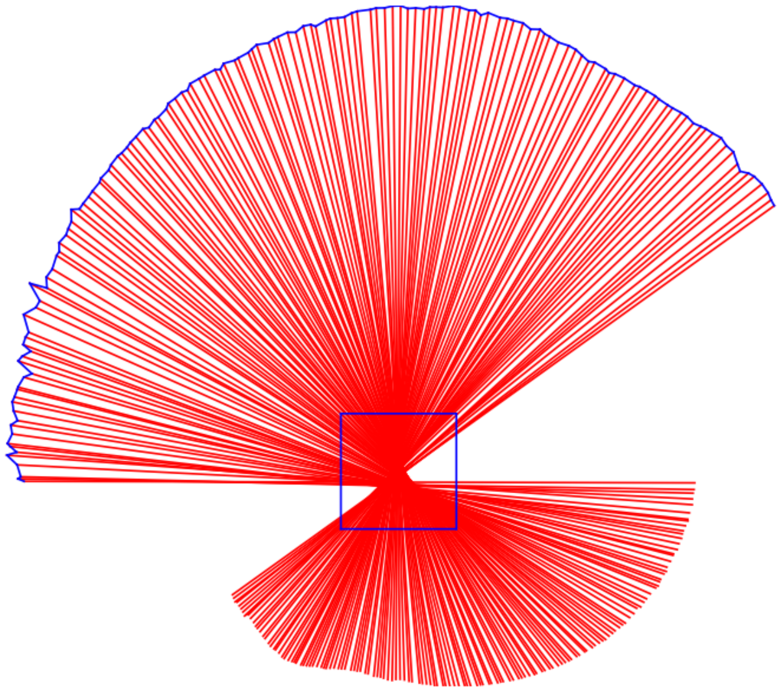
Ground Truth



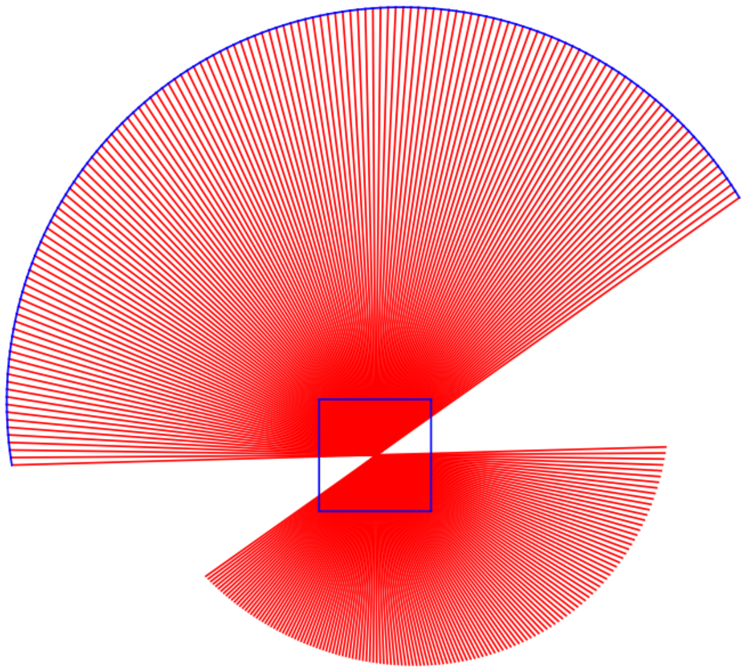
Nominal Trajectory

Results

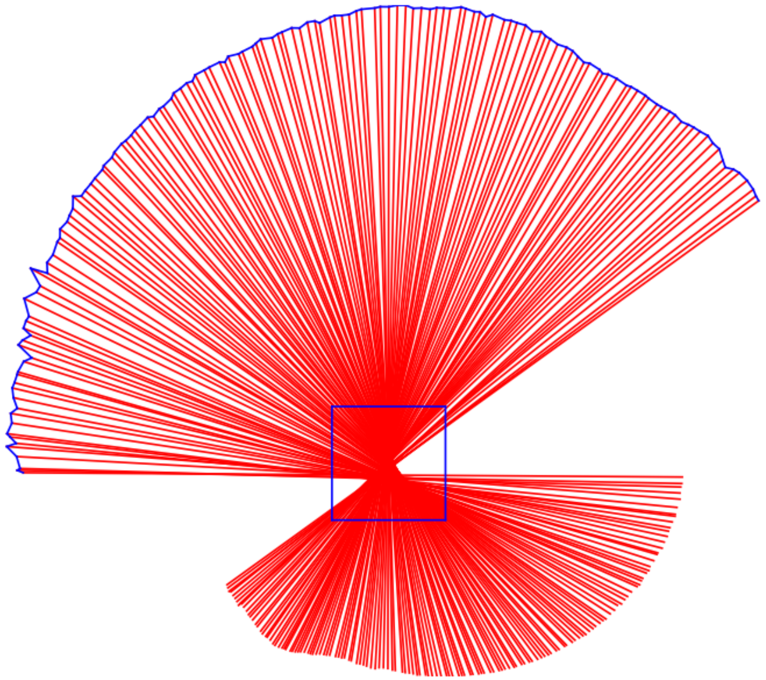
Ground Truth



Nominal Trajectory
(Initial Estimate)



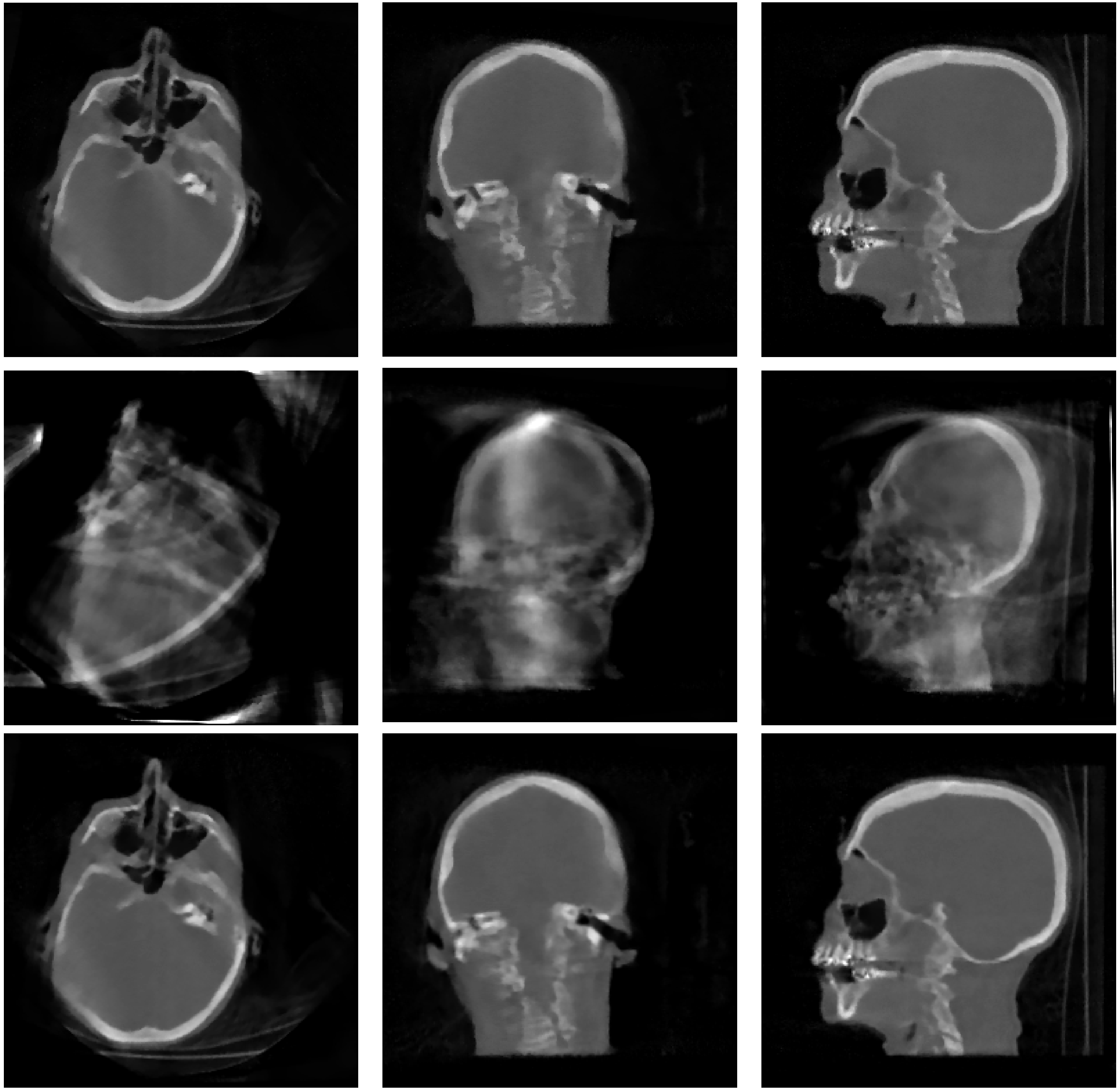
Estimated

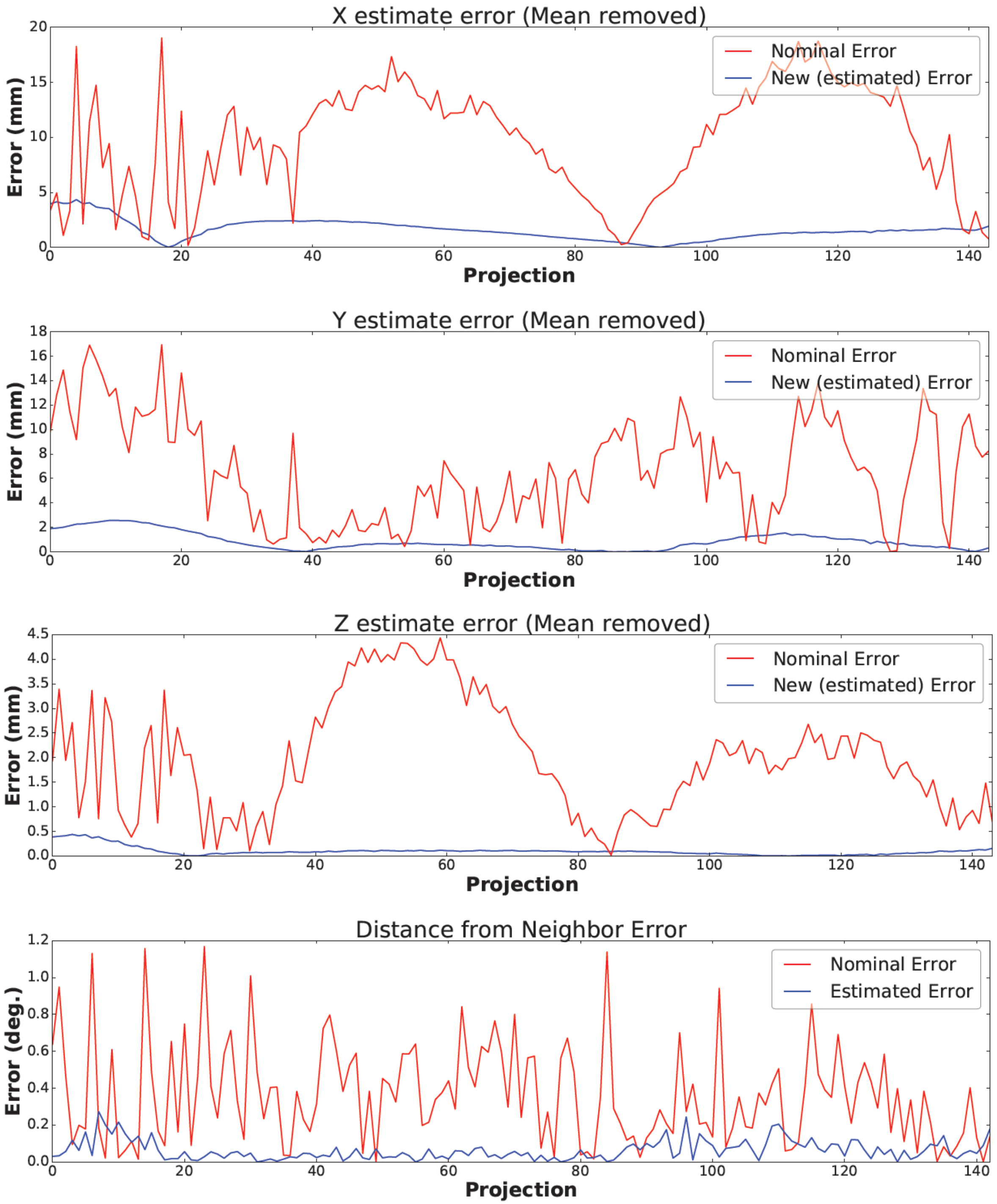


Static reconstruction
(given ground truth parameters)
L^2 error: 10.5

Static reconstruction
(given nominal parameters)
L^2 error: 233.2

Joint image and geometry estimation
(given nominal parameters)
L^2 error: 13.0




X estimate error (Mean removed)
Nominal Error
New (estimated) Error
Error (mm)
Projection
Y estimate error (Mean removed)
Nominal Error
New (estimated) Error
Error (mm)
Projection
Z estimate error (Mean removed)
Nominal Error
New (estimated) Error
Error (mm)
Projection
Distance from Neighbor Error
Nominal Error
Estimated Error
Error (deg.)
Projection

Cadaver Hand Dataset

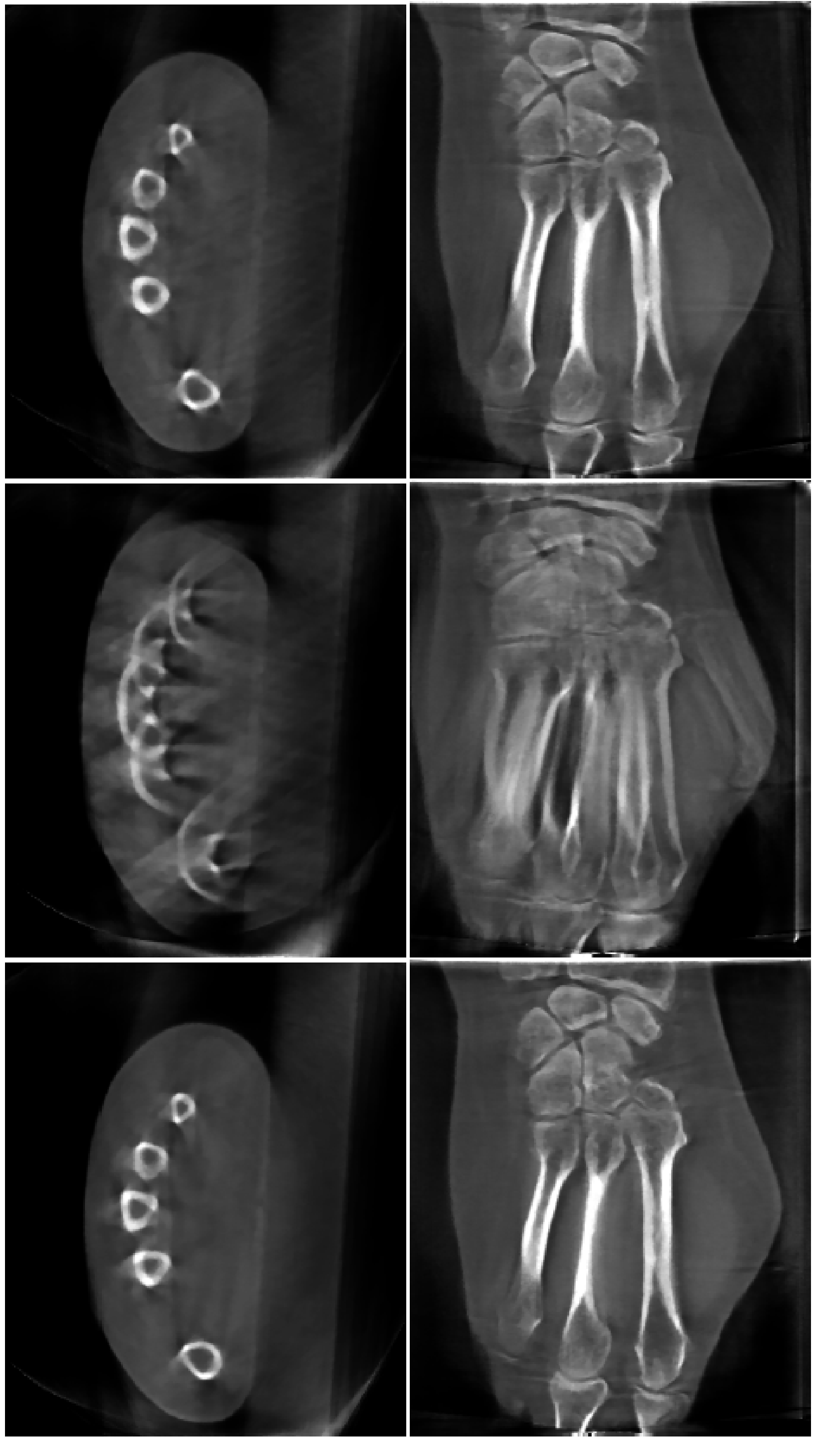
- Acquisition scan using a full-size development mobile C-arm with flat panel detector (GE Healthcare)
- RF trackers placed on table and detector, source location and intrinsic parameters estimated using markers

Cadaver Dataset

RF Tracking

Nominal

Joint Estimation



Physical Knee and Skull Phantoms

No ground truth image/parameters available

Static Reconstruction

Joint Estimation

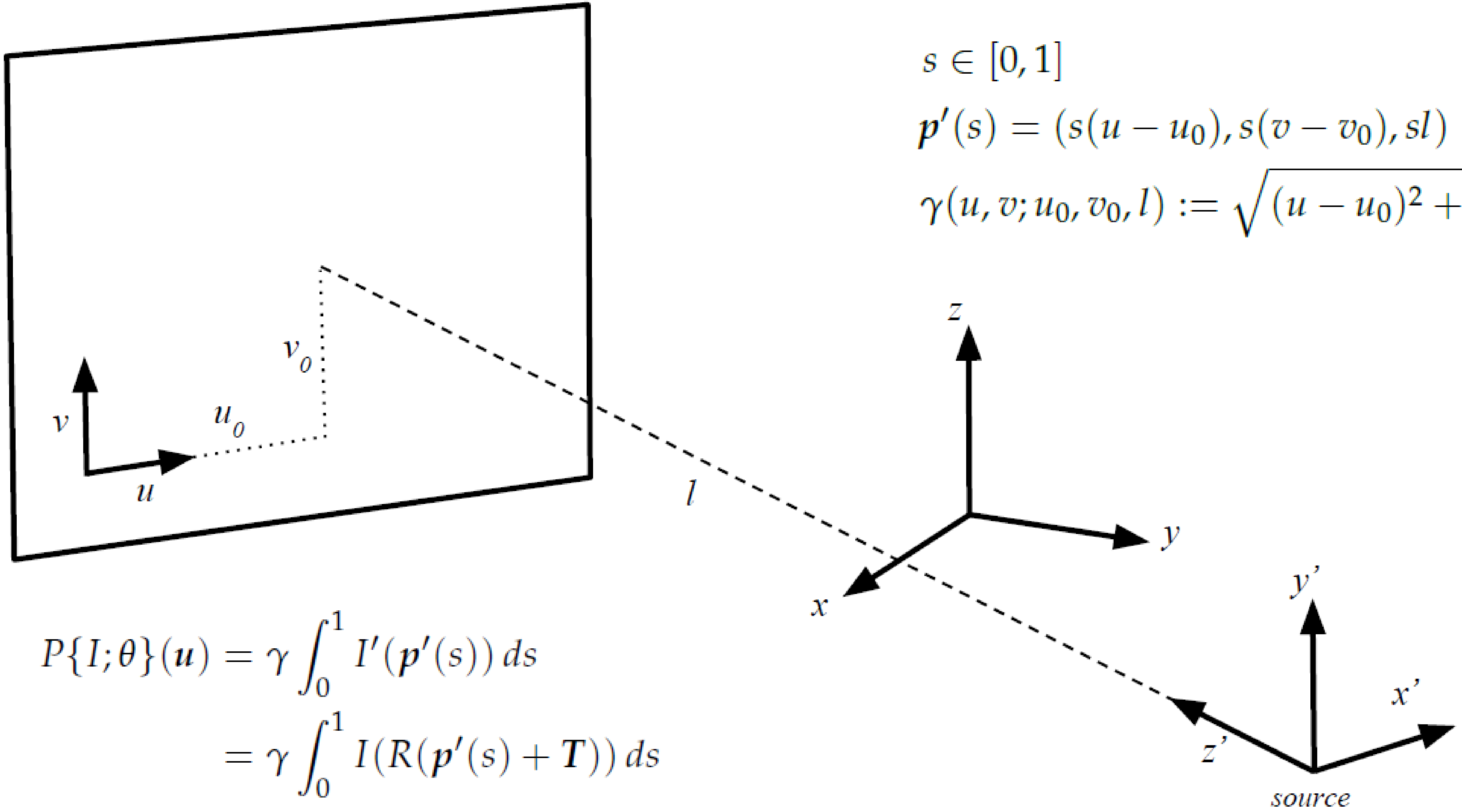
Conclusion

- Joint reconstruction and motion estimation framework allows for 3D reconstruction on mobile C-arms
- Reconstruction results show greatly improved image quality
- GPU/Multiscale implementation allows for clinically feasible reconstruction times
- Reconstruction timing (single Nvidia Titan Z)
  - 256^3 volume: static = 2 min, proposed method = 4 min
  - 512^3 volume: static = 10 min, proposed method = 16 min

Questions?

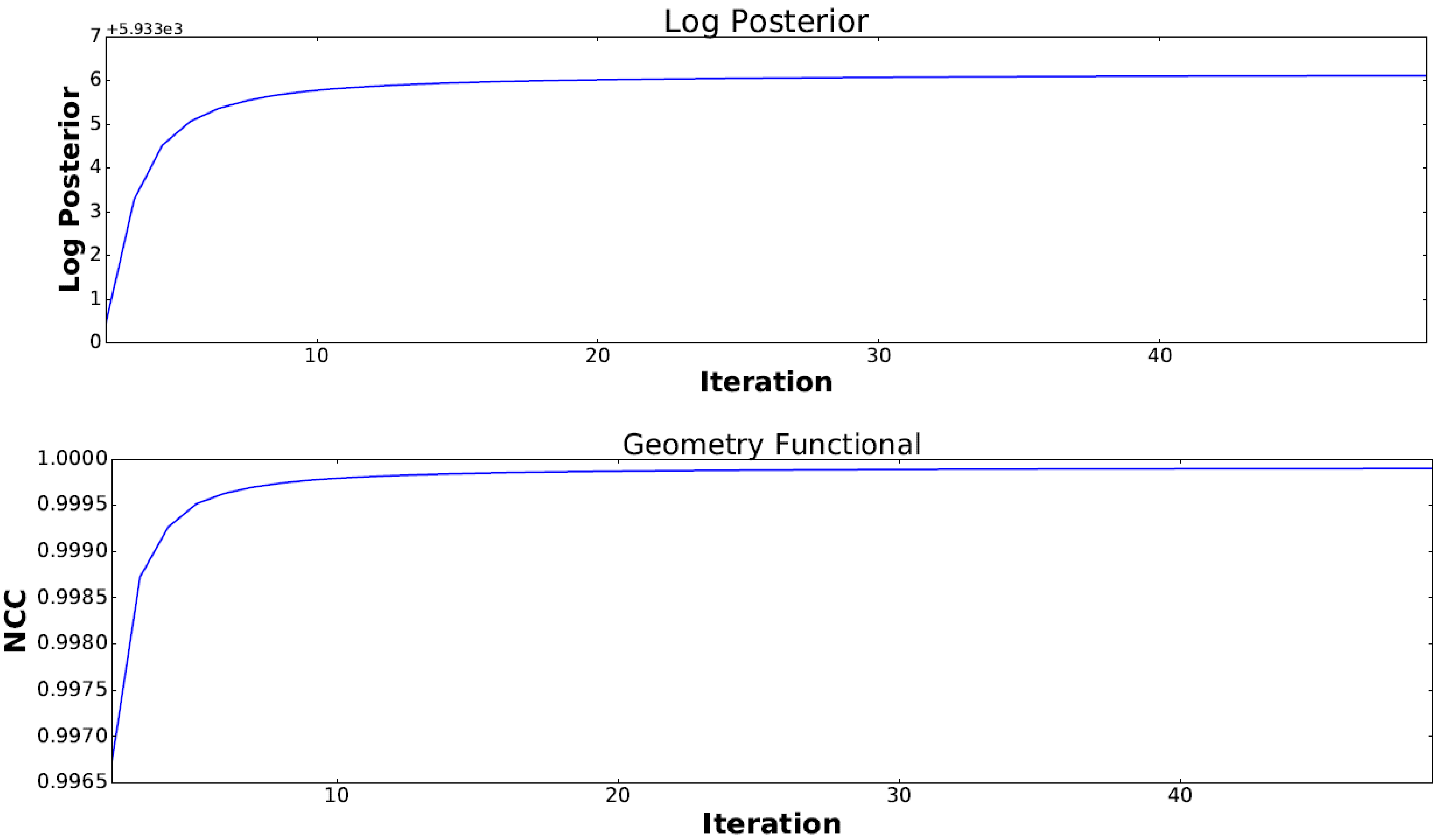
Cone-beam Coordinate Systems

$$s \in [0, 1]$$
$$\boldsymbol{p}'(s) = (s(u - u_0), s(v - v_0), sl)$$
$$\gamma(u, v; u_0, v_0, l) := \sqrt{(u - u_0)^2 + (v - v_0)^2 + l^2}$$

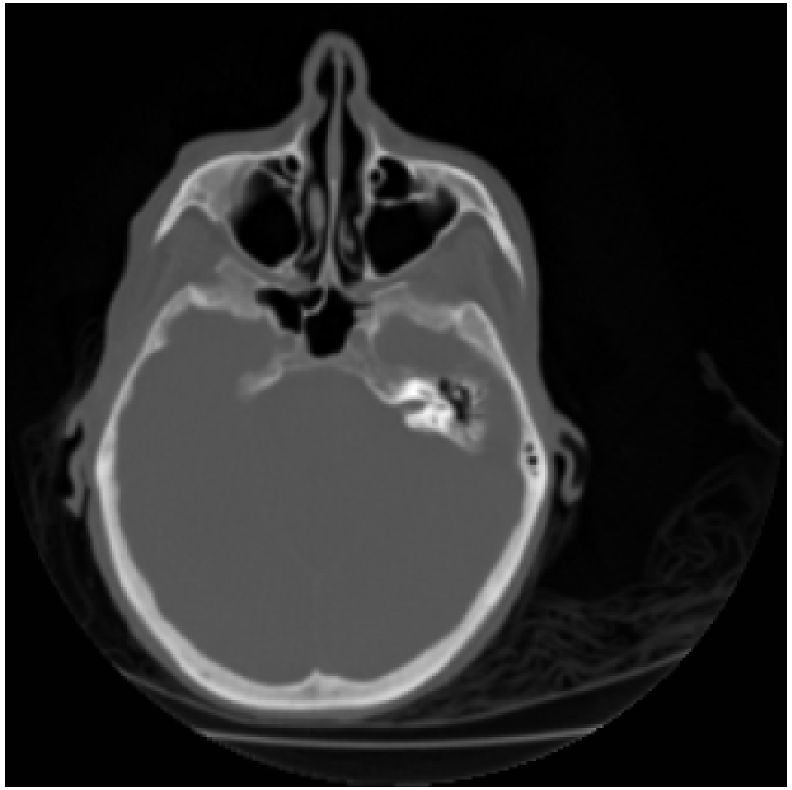


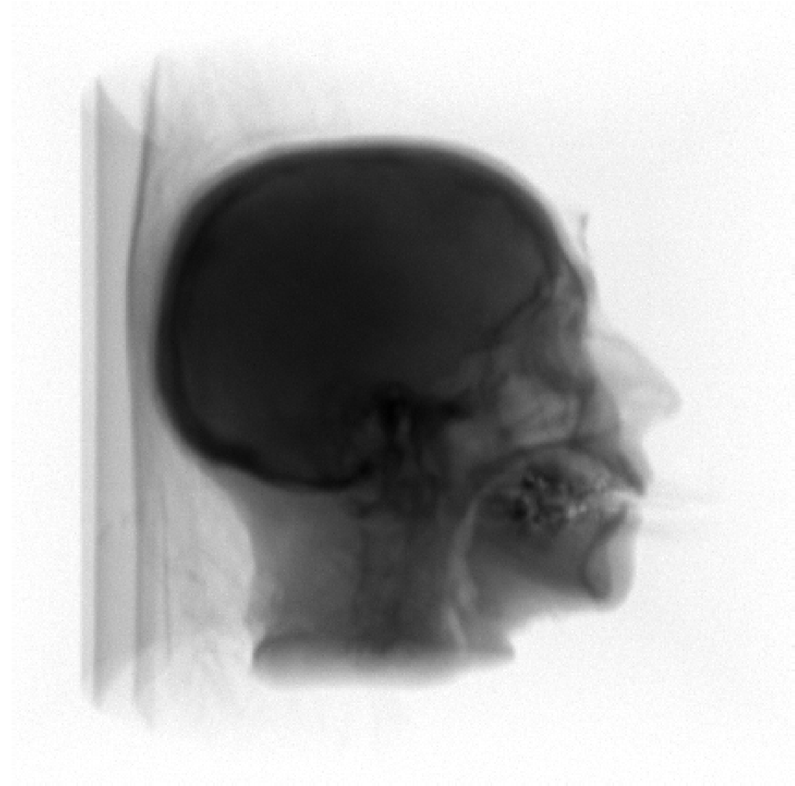
$$\begin{aligned} P\{I;\theta\}(\boldsymbol{u}) &= \gamma \int_0^1 I'(\boldsymbol{p}'(s))\, ds \\ &= \gamma \int_0^1 I(R(\boldsymbol{p}'(s) + \boldsymbol{T}))\, ds \end{aligned}$$

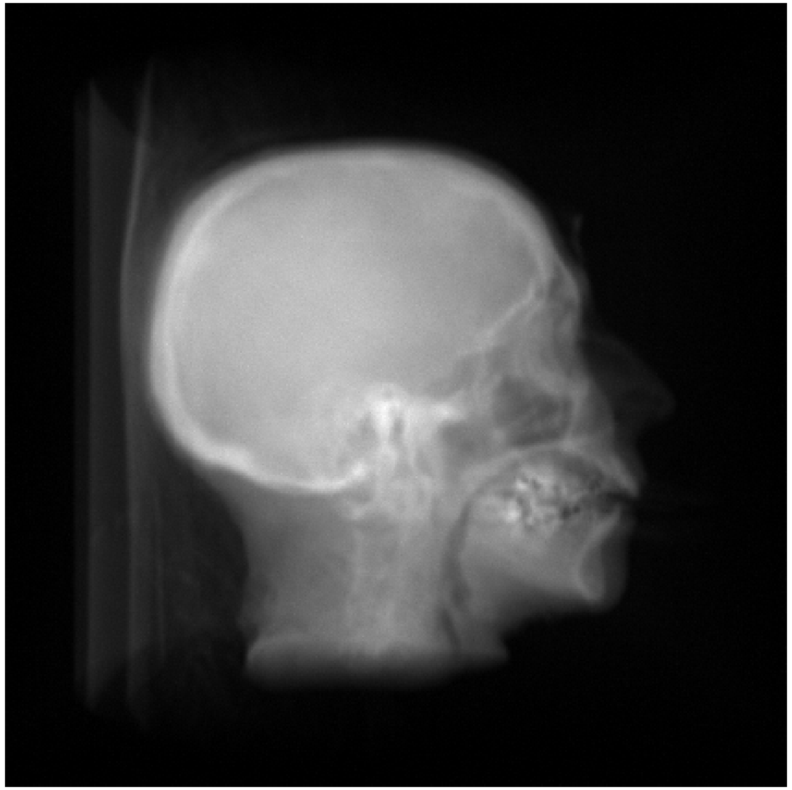
Joint Reconstruction Convergence



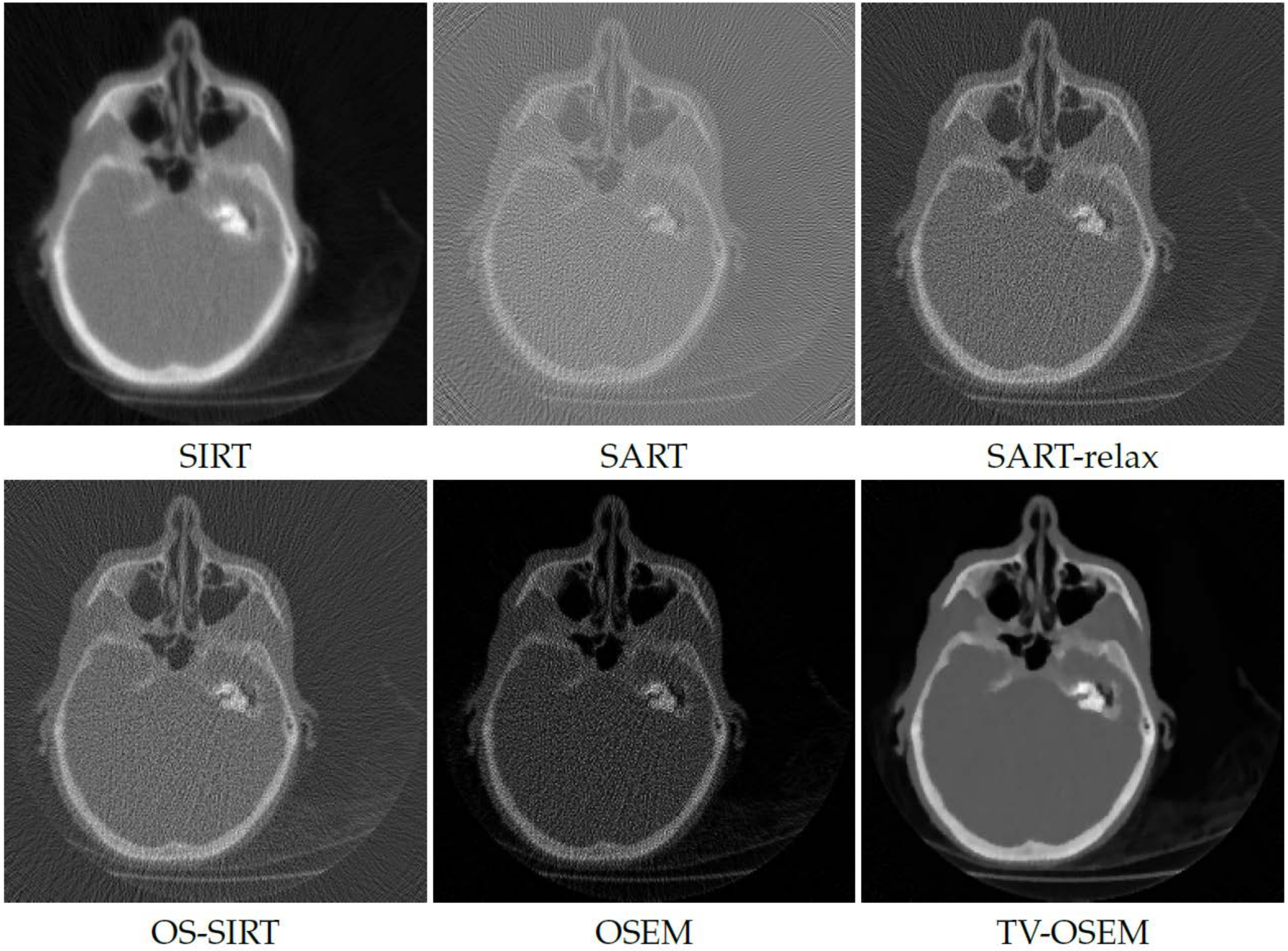
Simulated Cone-beam Dataset







Iterative Reconstruction



Physical Knee and Skull Phantoms

No ground truth image/parameters available

Static Reconstruction

Joint Estimation