

# Fusion plasma velocity-space tomography

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# Outline

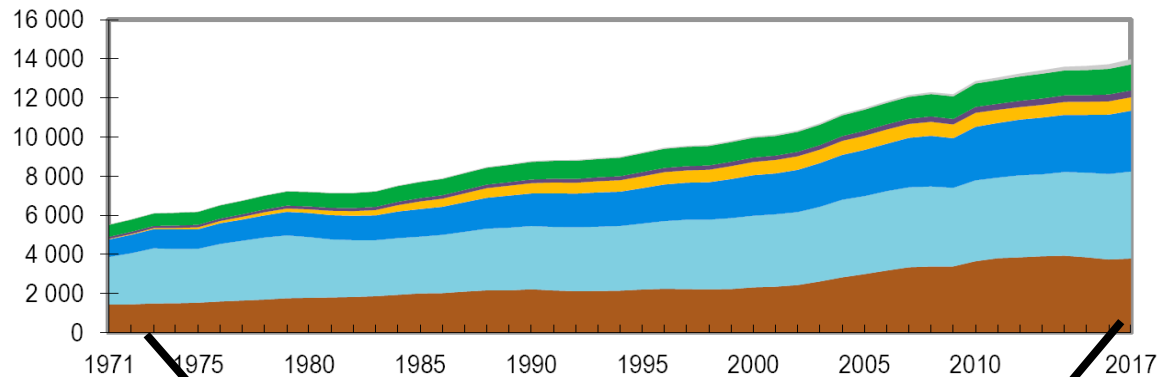
- 1) Introduction to fusion energy, fast ions and **velocity distribution functions**
- 2) Introduction to tomography in position and velocity space -  
The Lucky-Luke-Doppler experiment
- 3) How is Per Christian helping fusion plasma physics?
- 4) A closing riddle

# The energy problem

- Annual worldwide primary energy supply (Mtoe)

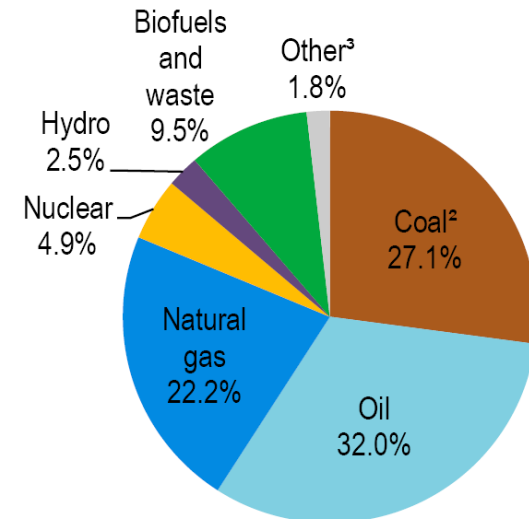
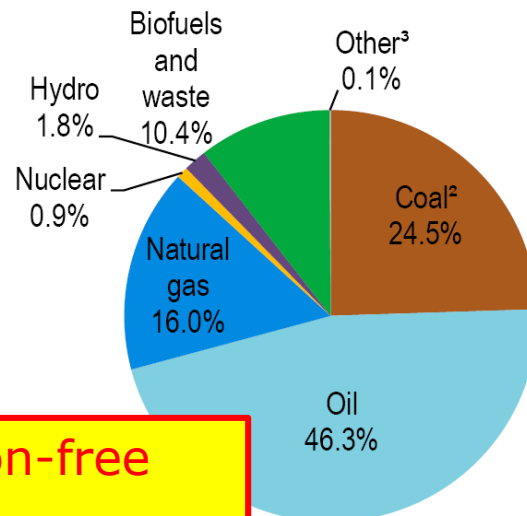
## 2017

- Fossil fuels >80%
- Biofuels 10%
- Fission 5%
- Hydro, wind, solar, tidal, geo <5%



1973

2017

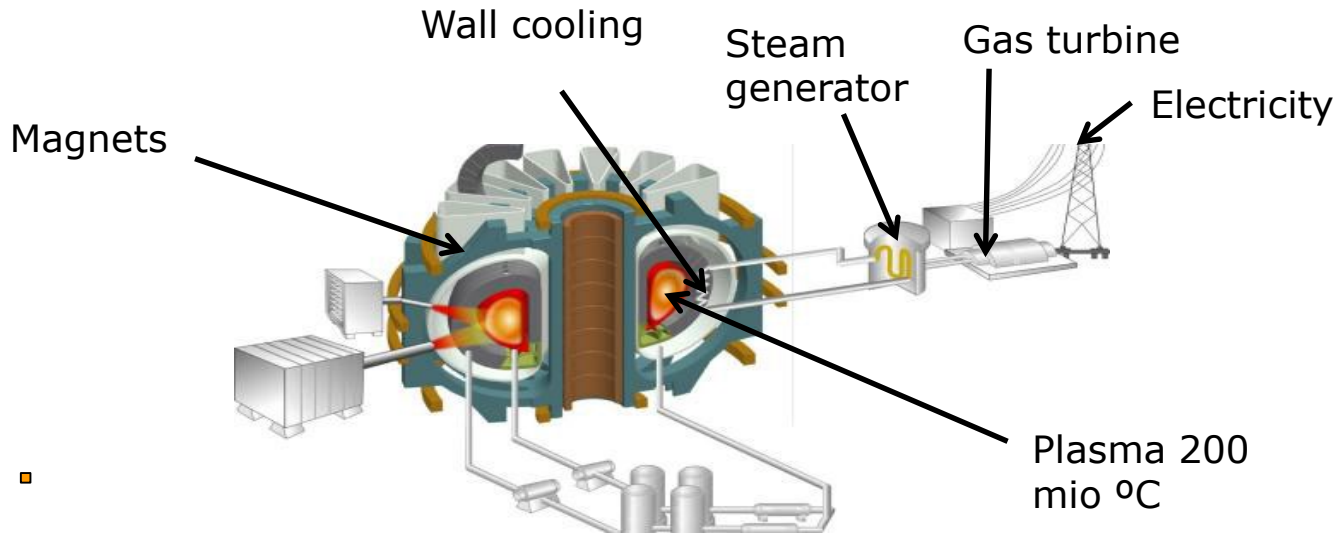


We must switch to emission-free energy sources!

World energy outlook (2019), IAE

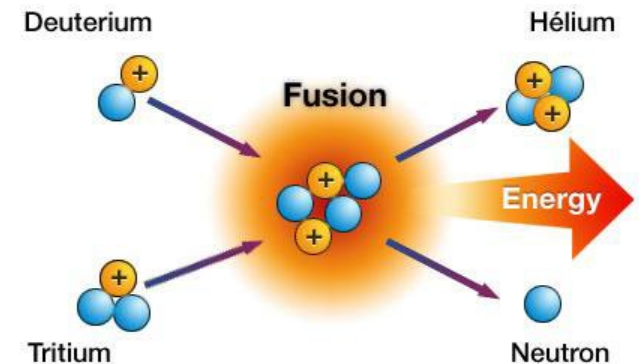
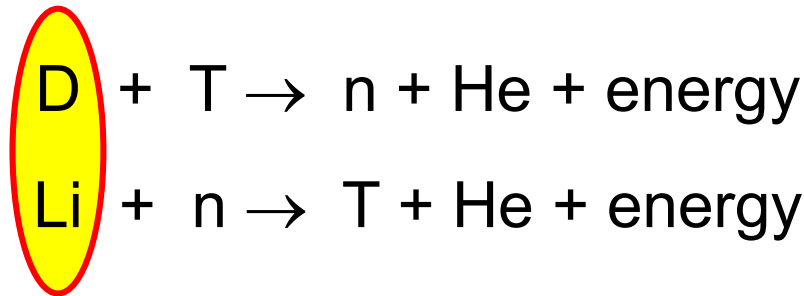
Mirko Salewski

# A fusion power plant



Sun core:  
15 mio °C

Fusion power plant:  
150-200 mio °C



- +No greenhouse gases
- +Safe, no risk of meltdown
- +Abundant fuels

- +No long-lived radioactive waste
- +Reliable, steady power

# Energy from water and lithium

- Average EU citizen energy consumption over a life

500 t coal

300 t oil

*5 train wagons    1/10th  
swimming pool*

20g deuterium and 60g lithium

*10 bath tubs of water and 10  
laptop batteries*

- Fuel consumption of 1 GW electric power over 1 year



2.500.000 t coal  
*250 2km trains*

1.400.000 t oil  
*4 400m tankers*

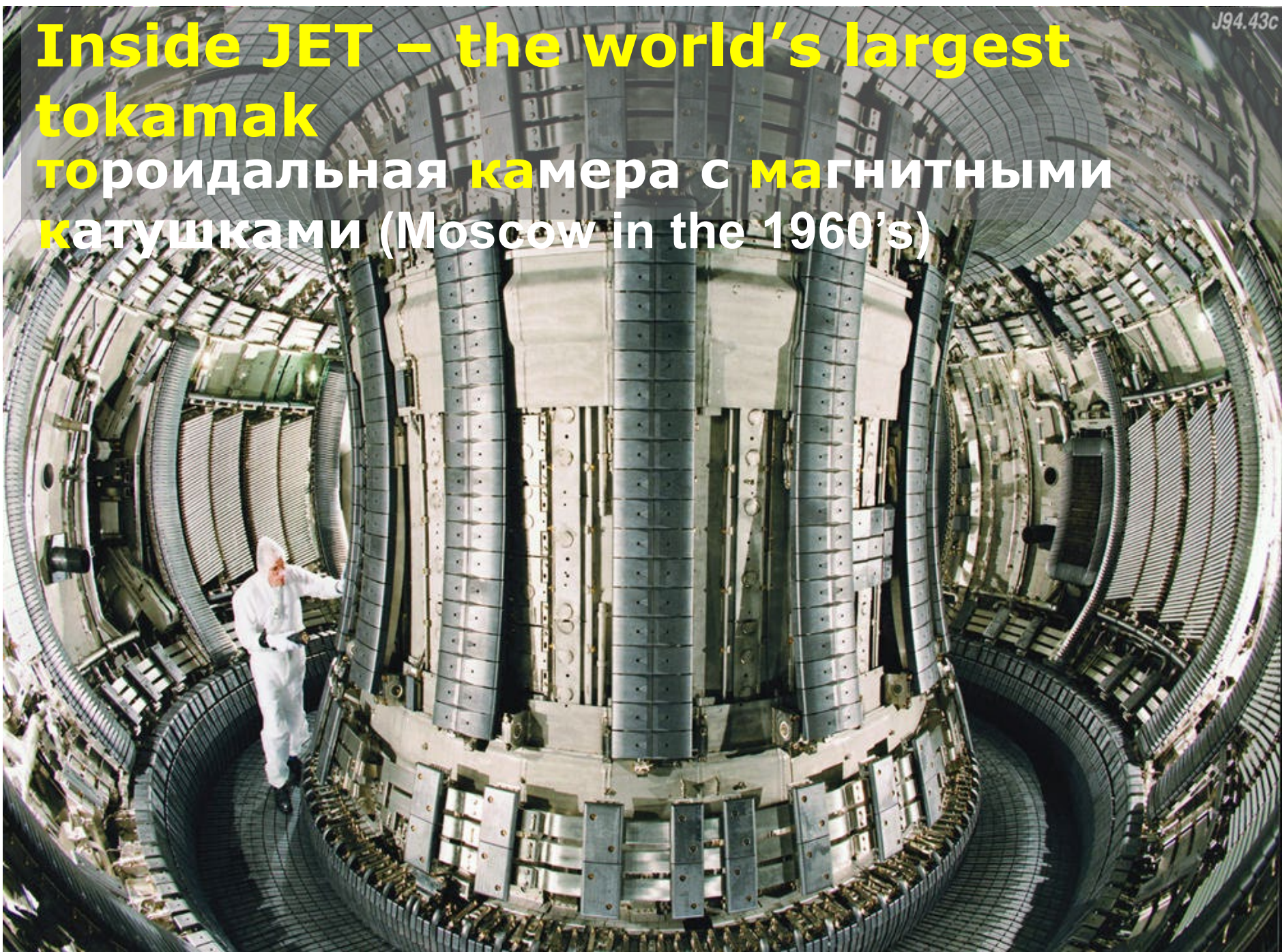
100kg D + 300kg Li  
(5000t water + 10t Li ore)  
*1 swimming pool + 1 truck load*



# Inside JET – the world's largest tokamak

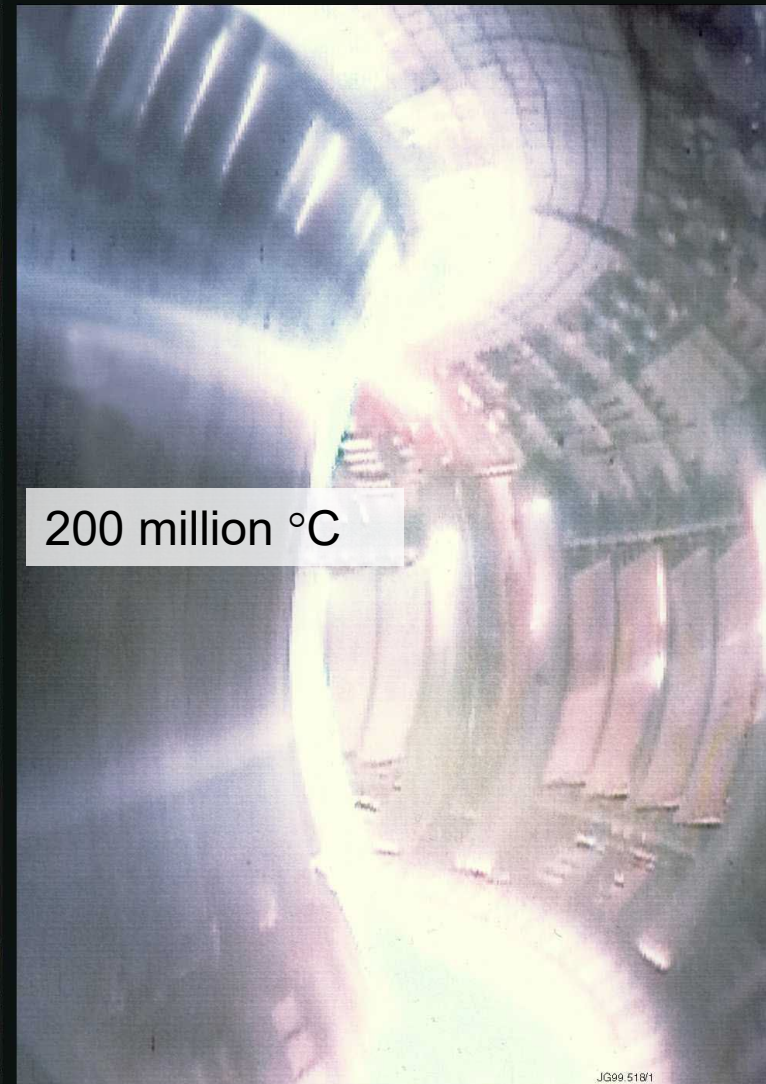
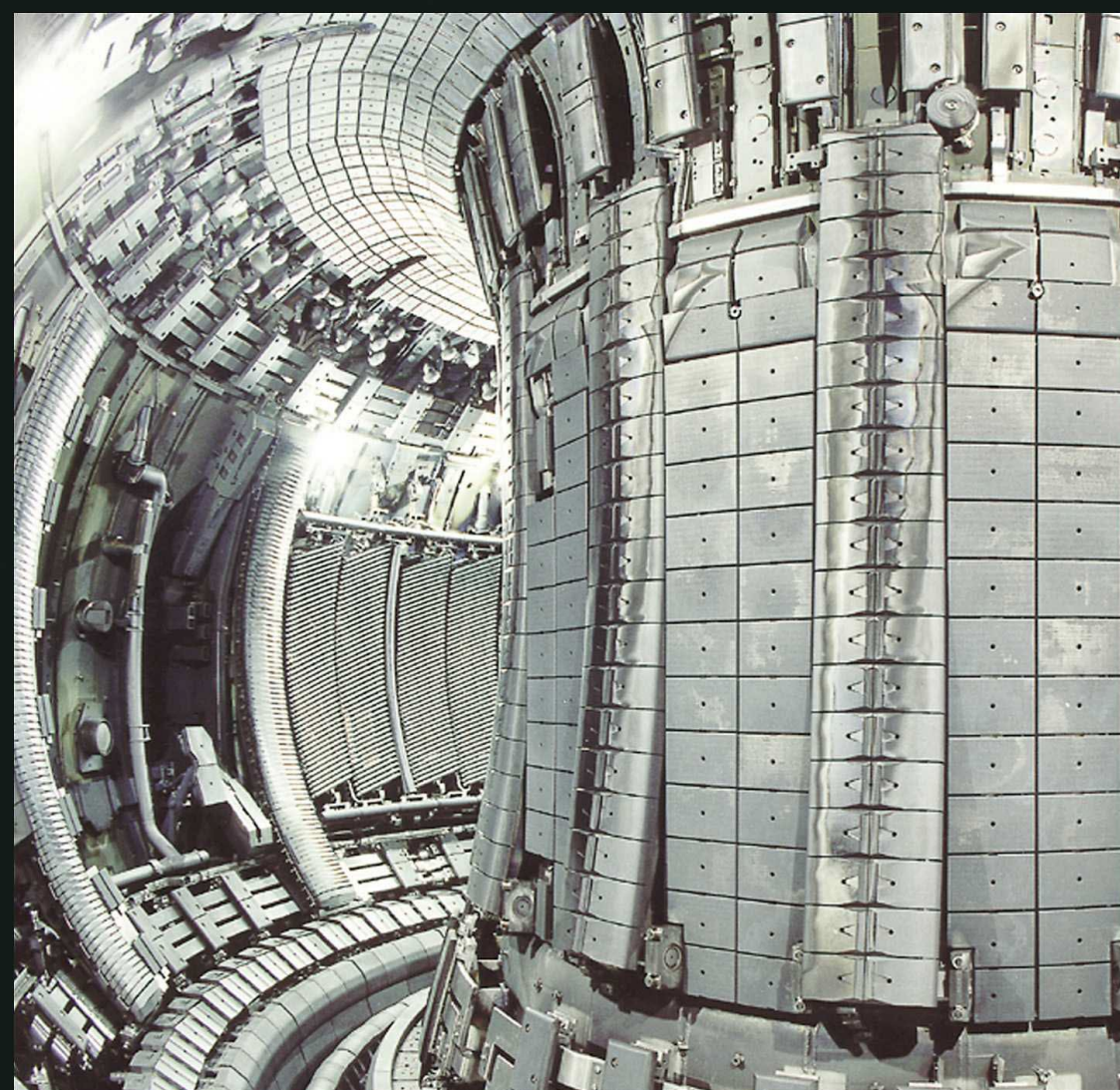
тороидальная камера с магнитными катушками (Moscow in the 1960's)

J94.43c





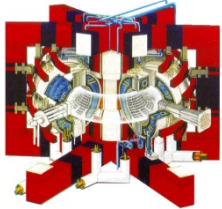
# Fusion plasma in the JET tokamak





# The ITER tokamak and future fusion reactor sizes

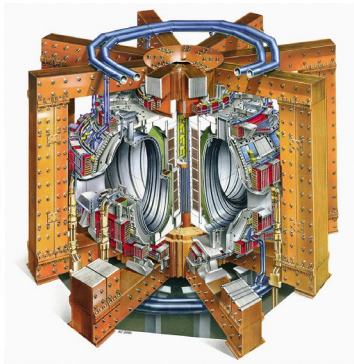
ITER will amplify the injected power by a factor 10.



*Tore Supra*

$25 \text{ m}^3$

$\sim 0 \text{ MW}$

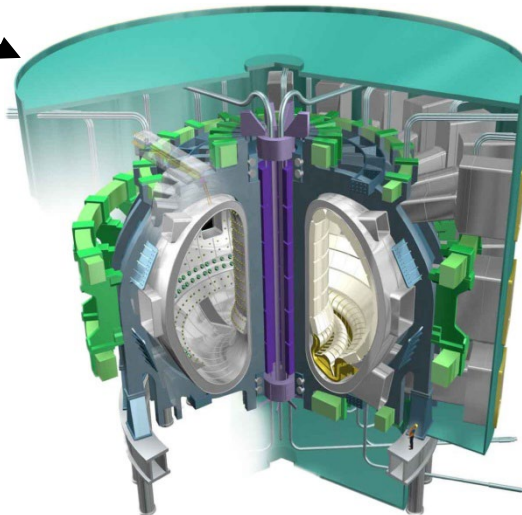


*JET*

$80 \text{ m}^3$

$\sim 16 \text{ MW}_{th}$

$Q=0.64$

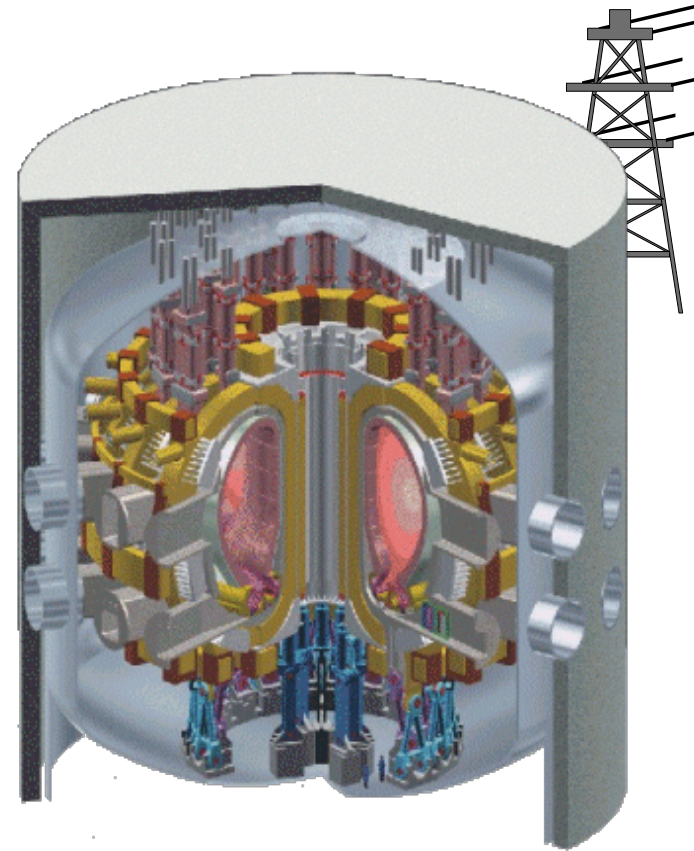


*ITER*

$830 \text{ m}^3$

$\sim 500 \text{ MW}_{th}$

$Q=10$



*Reactor*

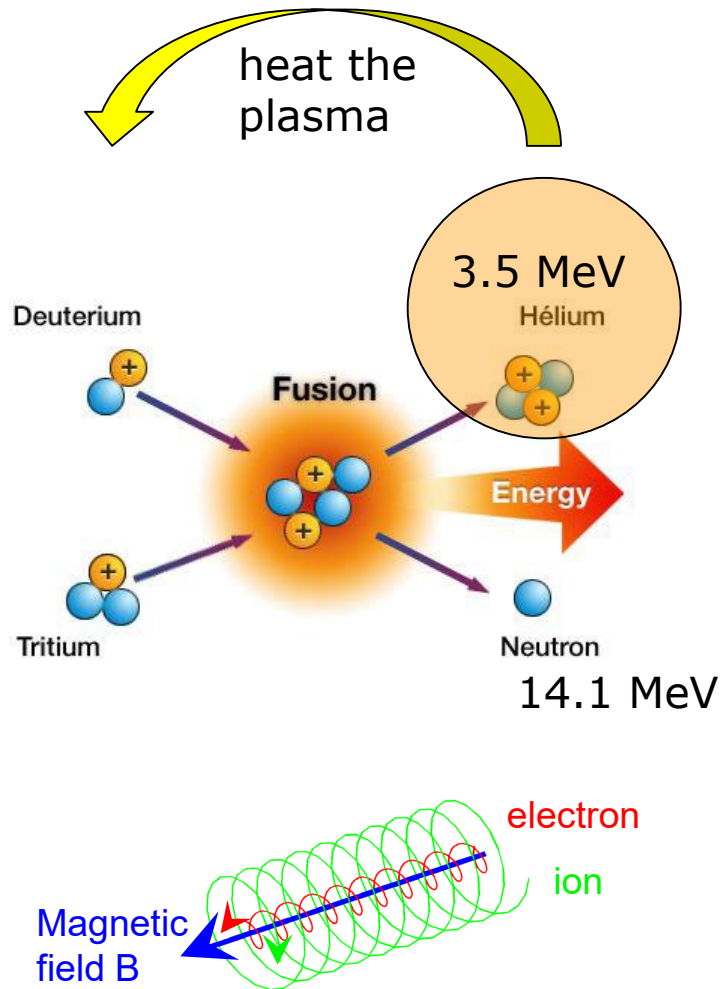
$\sim 1500 - 2000 \text{ m}^3$

$\sim 4500 \text{ MW}_{th}$

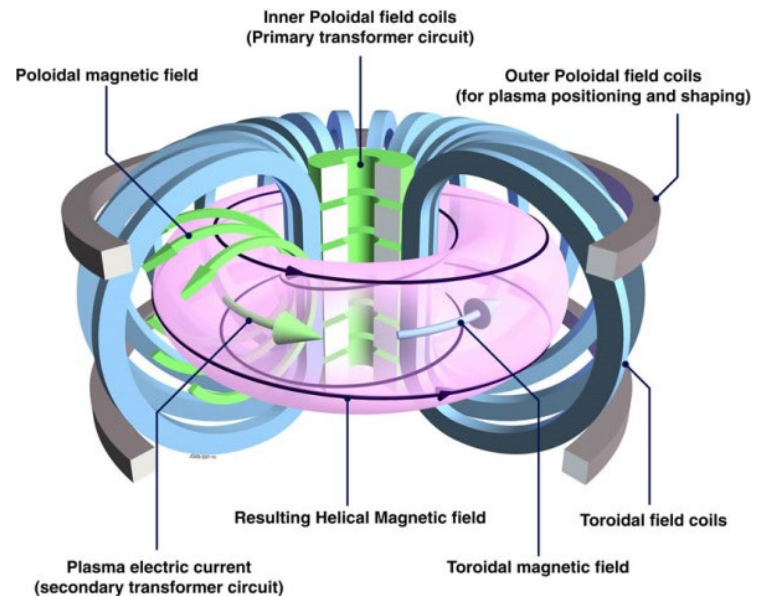
$Q \sim 50$



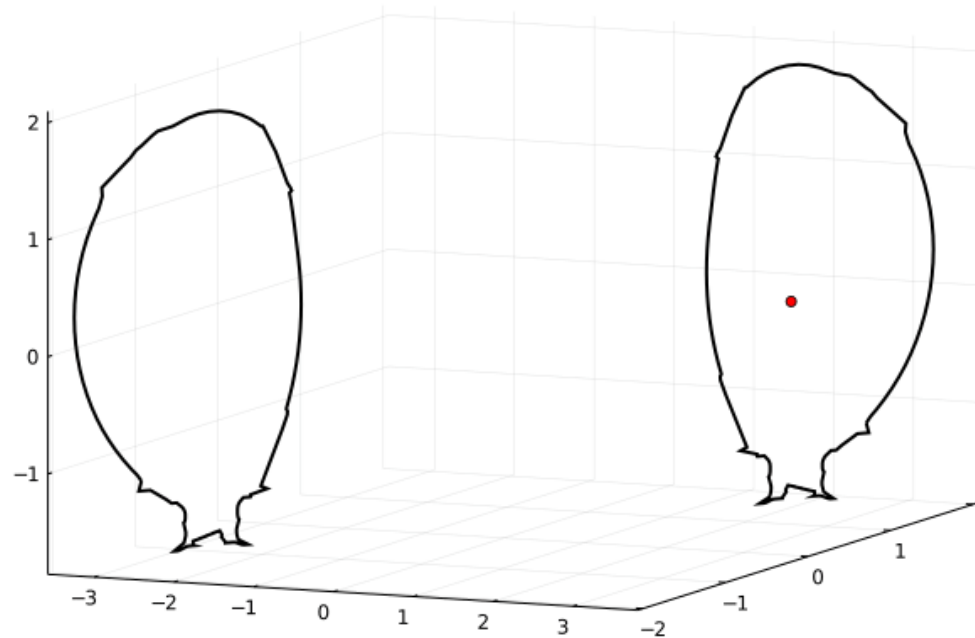
# Fast ions in a tokamak fusion plasma



- Helium is born at 9,000,000 m/s  
~ 3% of the speed of light
- Caught by a donut-shaped magnetic field

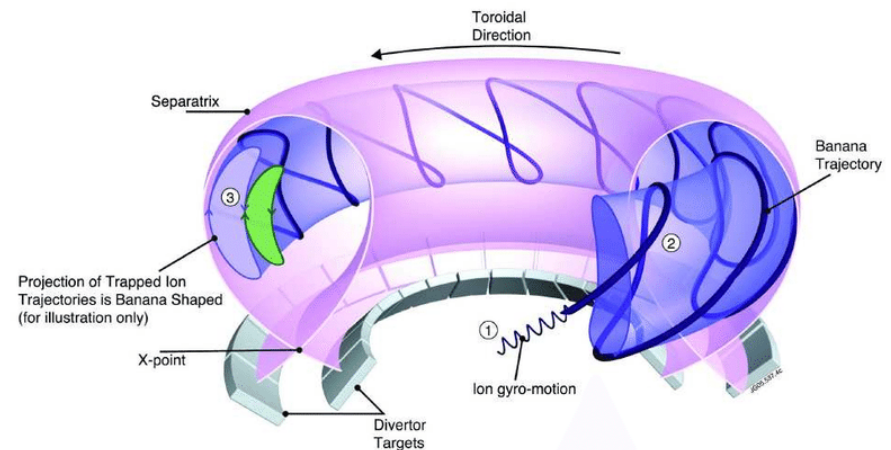
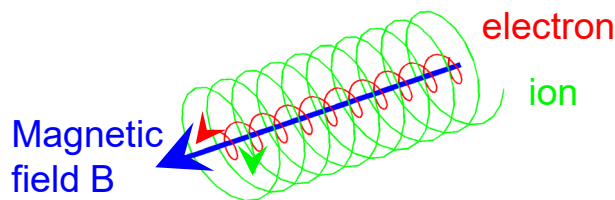


# Fast ions in a tokamak fusion plasma



- Fast ions stay on surfaces in the donut.

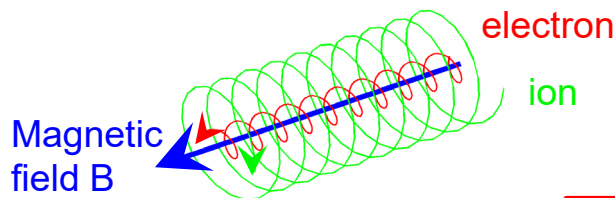
Järleblad et al. (2022) Nucl. Fusion



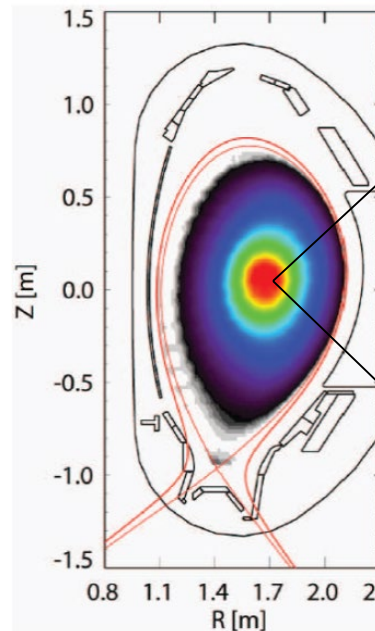
# Fast ions in a tokamak fusion plasma

## Fast ions

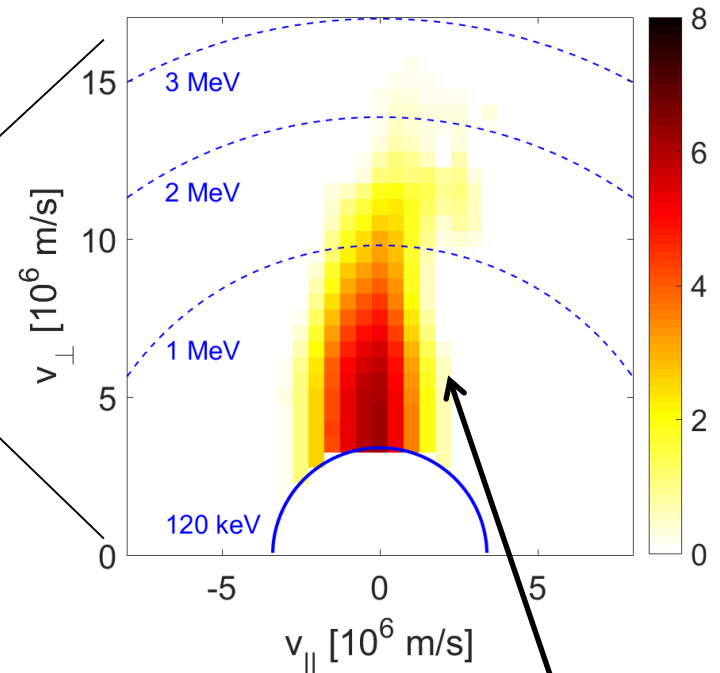
- 👍 heat the plasma
- 🧐 drive instabilities
- 🧐 transported by instabilities



## 2D position distribution



## 2D velocity distribution in the center

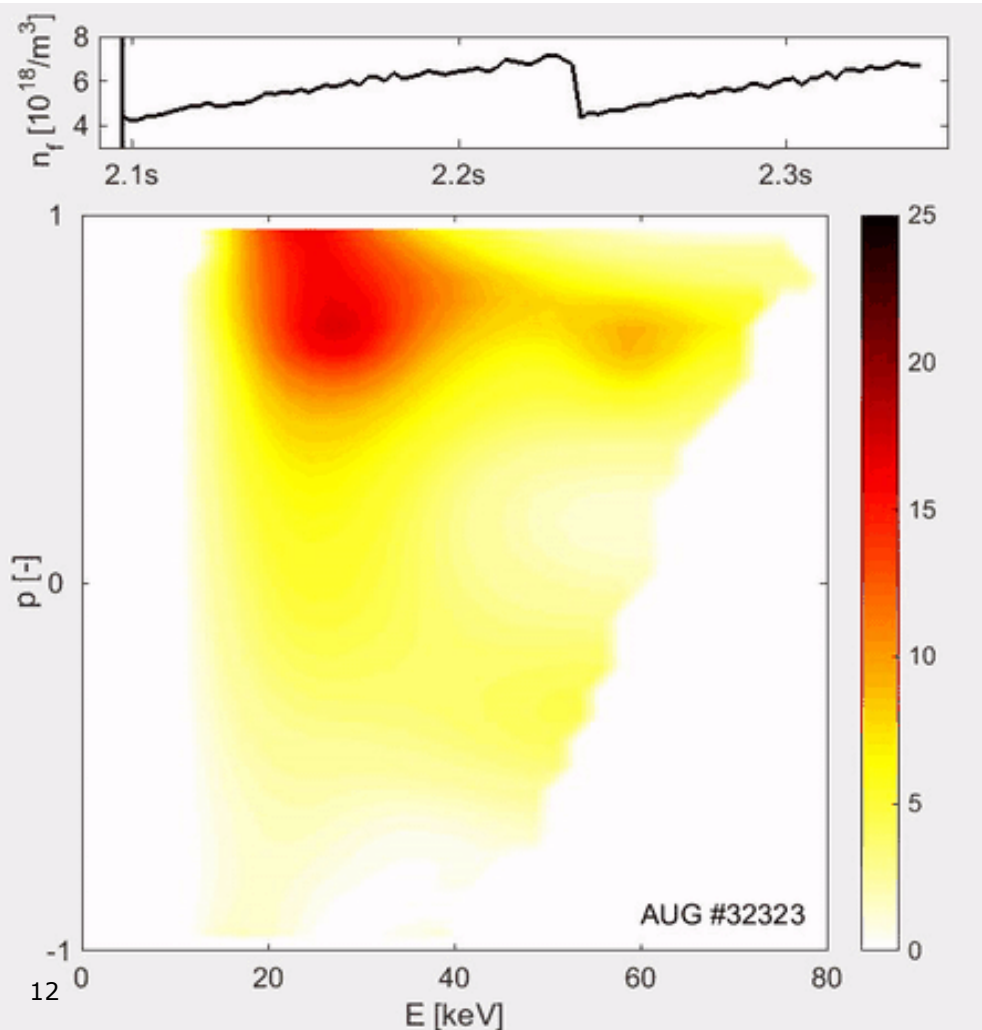


**Velocity-space tomography: Measure this**



# Fast ions can be transported by instabilities

- Sawteeth instability, sawtooth pattern in time traces of  $T$ ,  $n$ ,  $p$
- Measurement of  $f(E, p)$  in a sawtooth plasma at ASDEX Upgrade



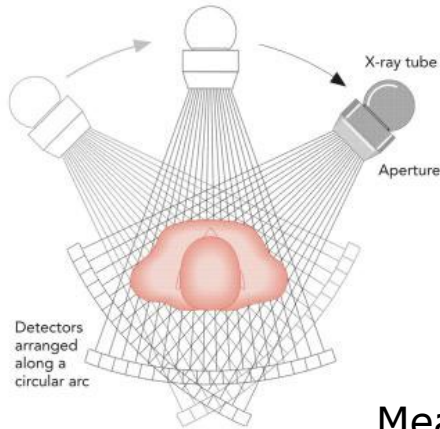
- Upper panel: Measurement of fast ion density (integral of the lower panel)
- Lower panel: Velocity-space tomography movie (100 frames)

Salewski et al (2016b) NF

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- 1) Introduction to fusion energy, fast ions and velocity distribution functions
- 2) Introduction to tomography in position and velocity space - The Lucky-Luke-Doppler experiment**
- 3) How is Per Christian helping fusion plasma physics?
- 4) A closing riddle

# Every-day tomography: CAT scanner



- Red spot in sample traces  
S-curve in data

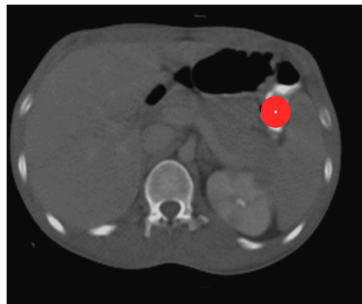
- Cormack 1963, 64
- Hounsfield 1968-73  
Nobel Prize Medicine 1979

$$WF = S$$

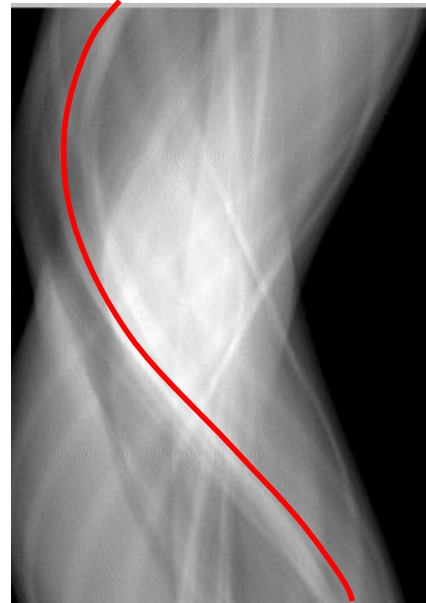
Other medical examples:

- PET – positron emission tomography
- MRI – magnetic resonance imaging
- Ultrasound imaging
- Breast mammography
- ...

Slice through patient

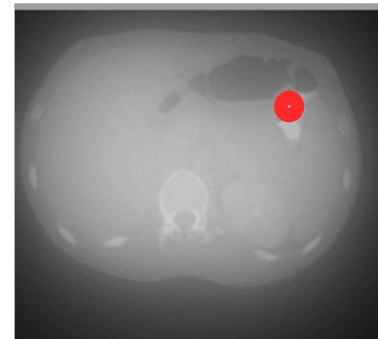


Phase angle



Detector position

Image





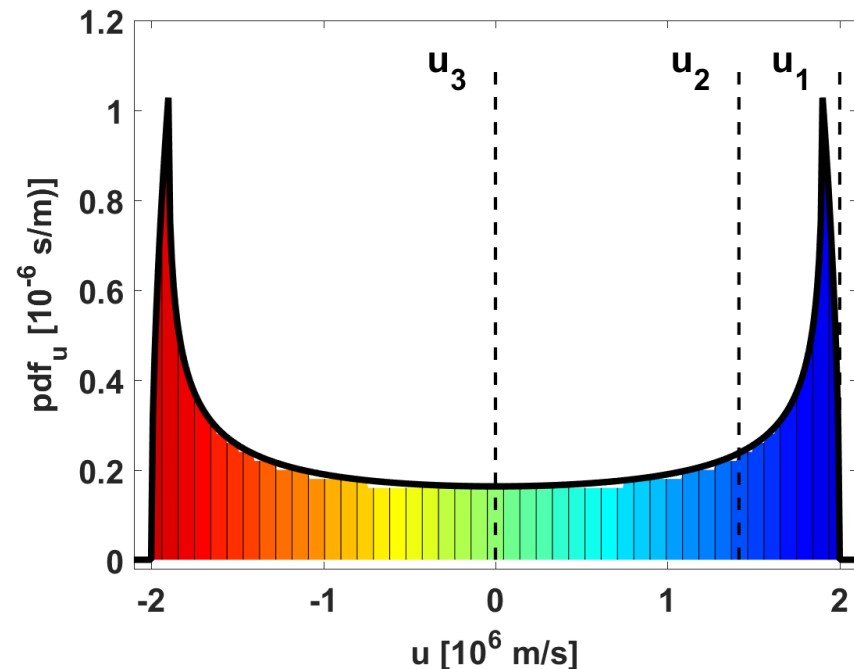
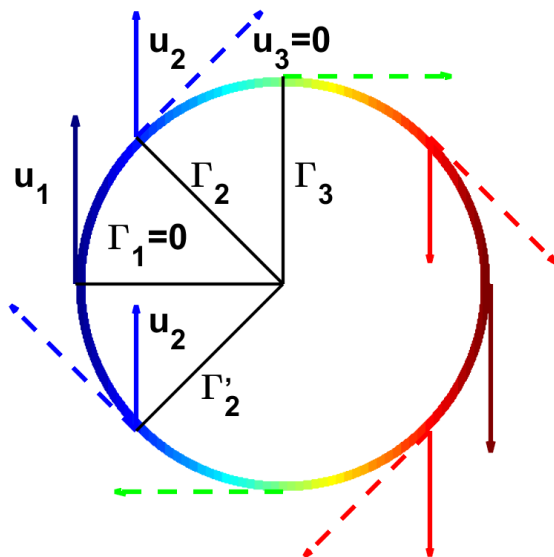
# The Lucky-Luke-Doppler experiment



- Doppler-shifted sound from a gyrating pitchfork at 258 Hz.
- Doppler shift:

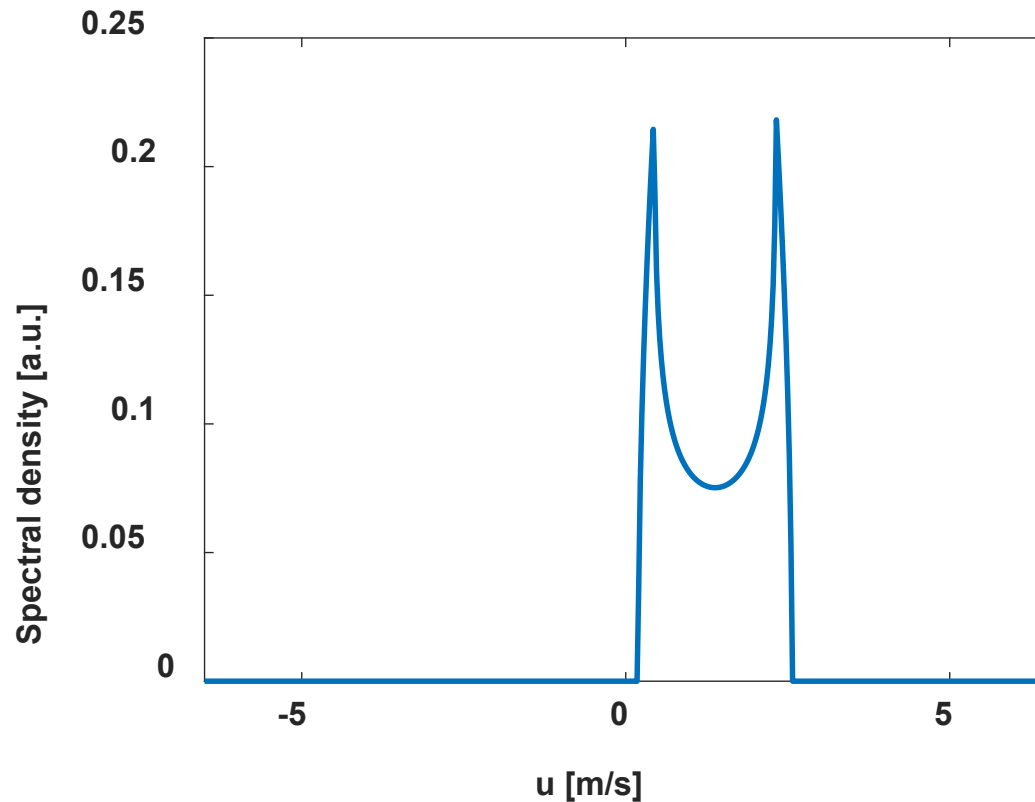
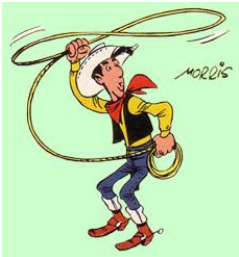
$$u = c \frac{\Delta f}{f}$$

- Fast ions generate Doppler shifted radiation: light, gamma-rays and similar Doppler-shifted signals

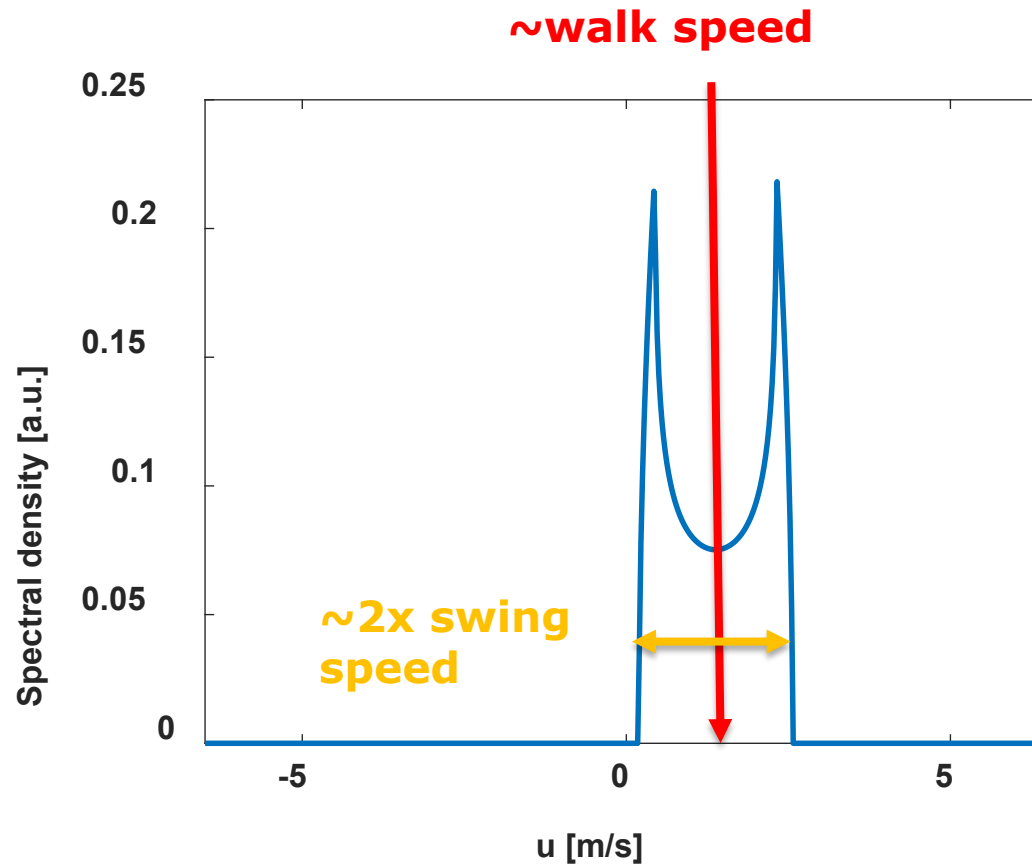


# Spectrum for one walking person with gyrating emitter

How fast am I walking and swinging the pitch fork?



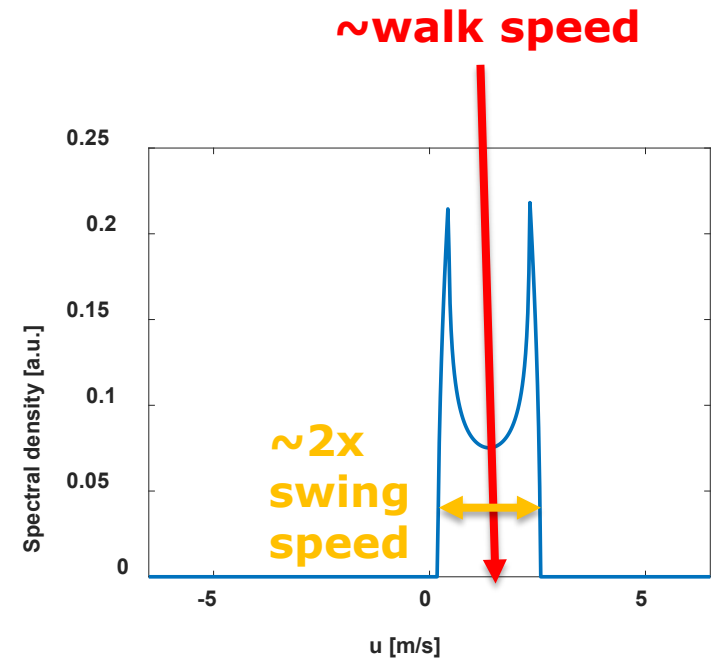
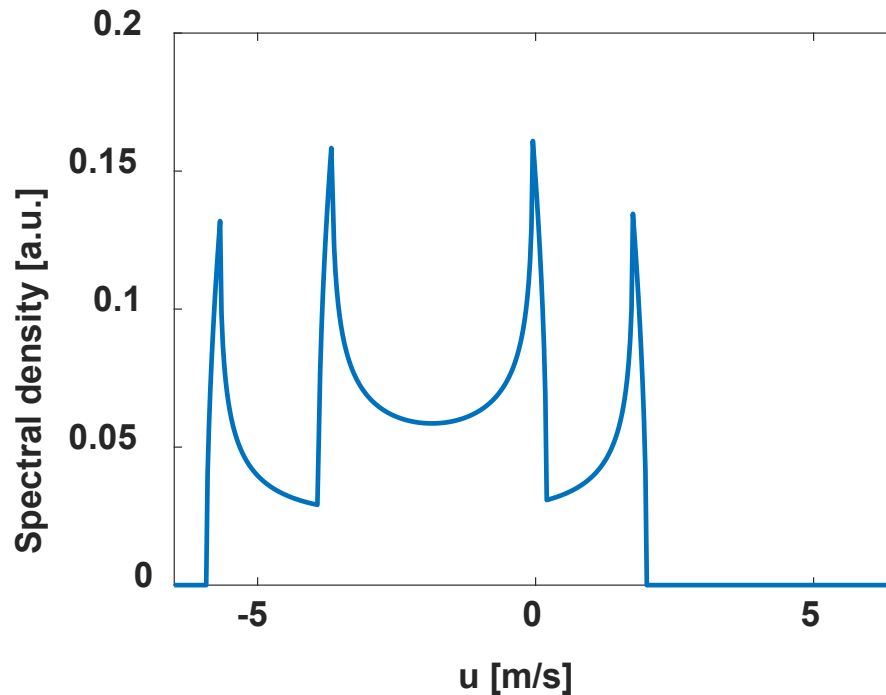
# Spectrum for one walking person with a gyrating emitter



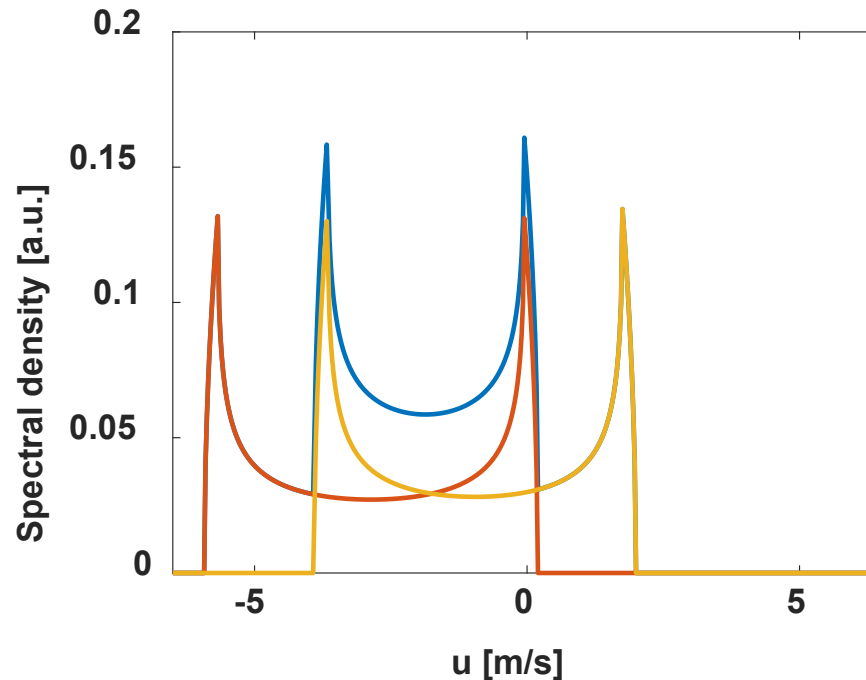


# Spectrum for two walking people with gyrating emitters

How fast are the two people walking and swinging?



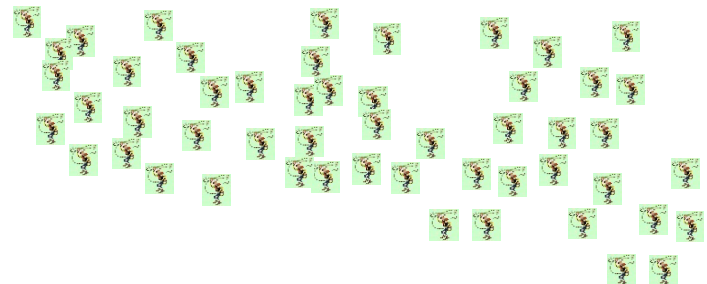
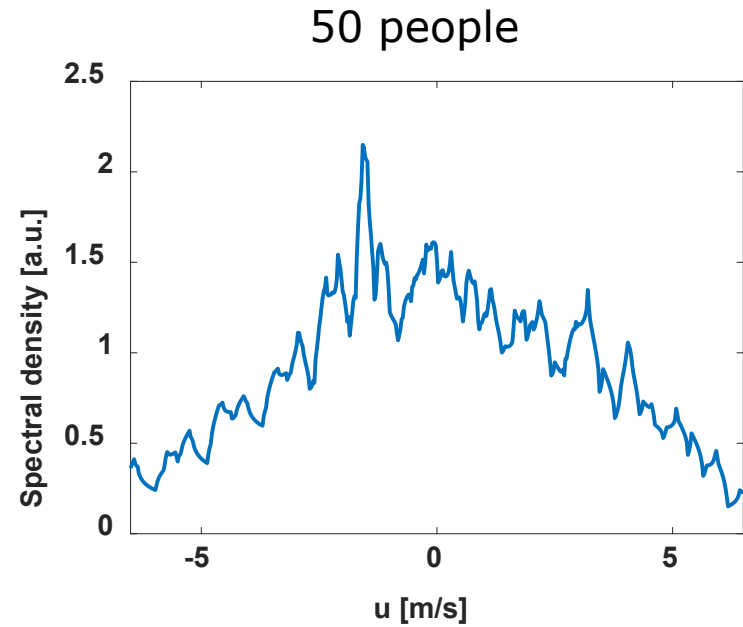
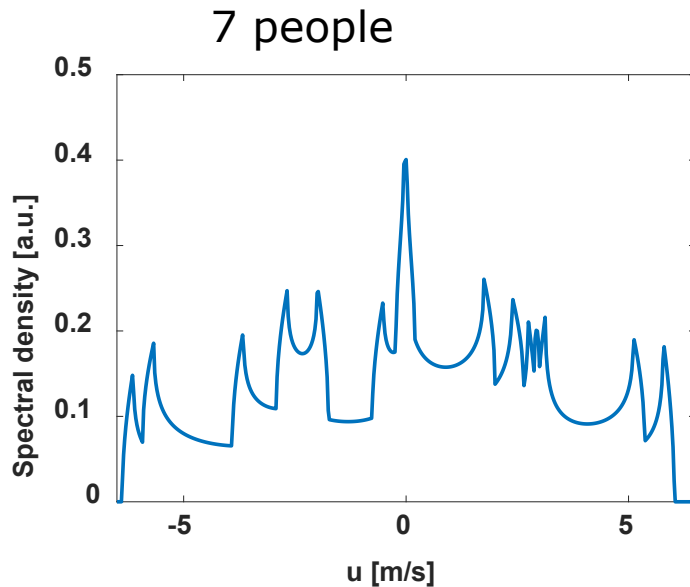
# Spectrum for two walking people with gyrating emitters



- Both: Swing about 3m/s (width/2)
- Red: Walks at -3m/s (center)
- Yellow: Walks at -1m/s (center)

- Both: Walk at about -2m/s (centers)
- Person 1 swings at 2m/s (width/2)
- Person 2 swings at 4m/s (width/2)

# Spectrum for many walking people with gyrating emitters





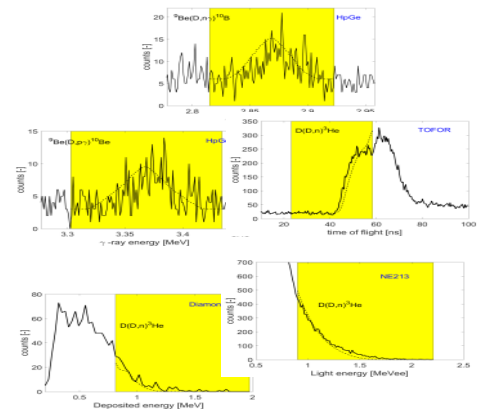
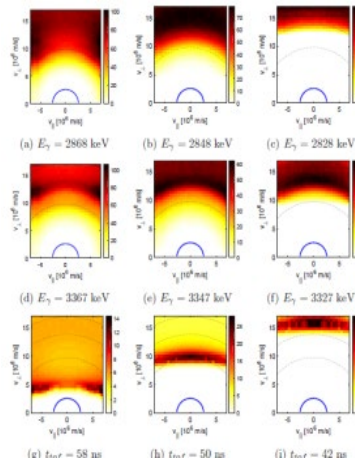
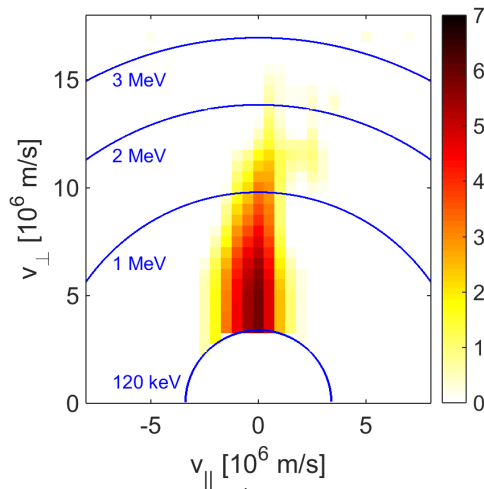
# Velocity-space tomography

Forward problem  
 $WF = S$

Distribution function  $F$

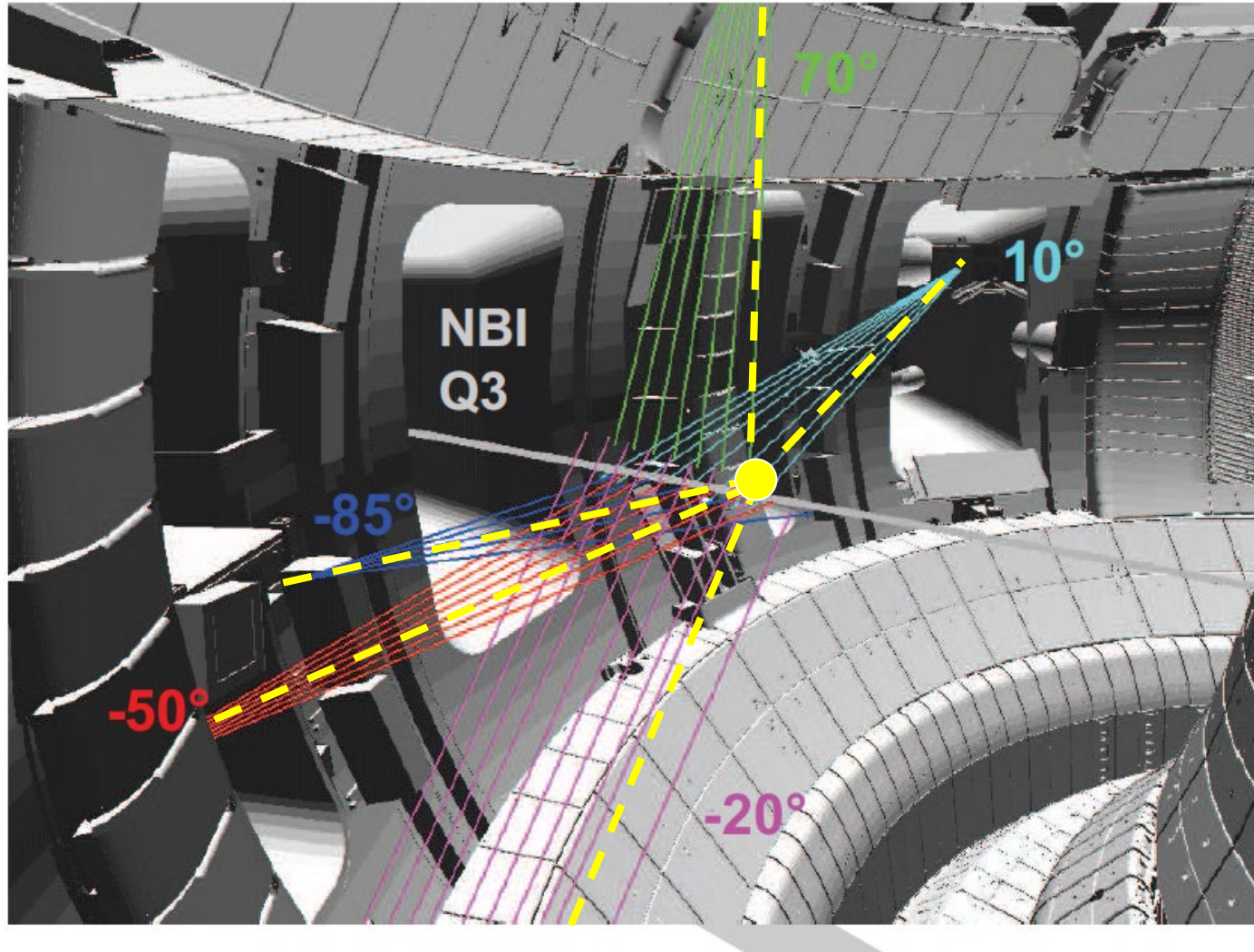
Matrix  $W$

Measurements  $S$

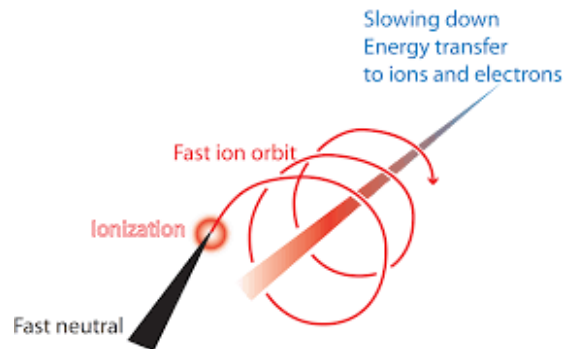
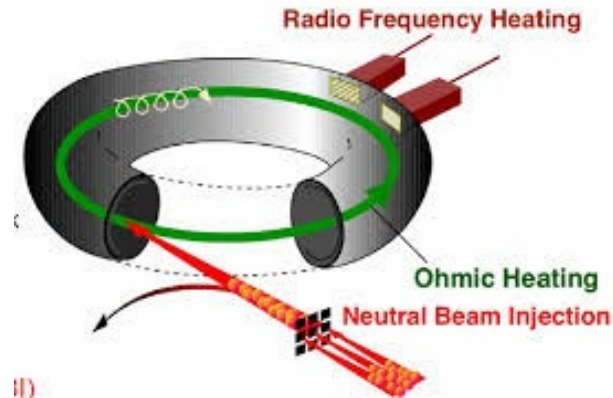


Inverse problem

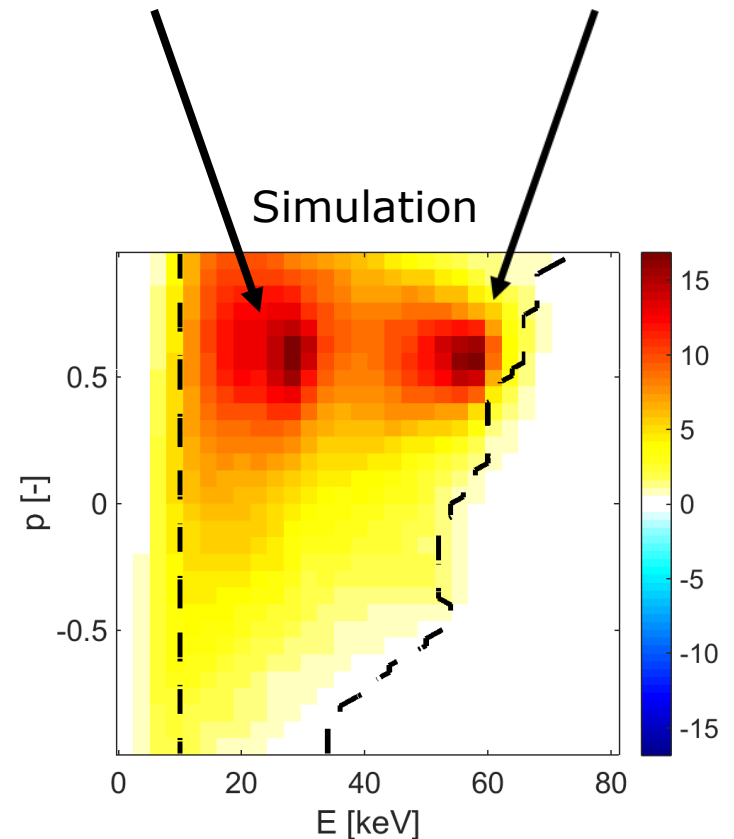
# Fast ion D-alpha spectroscopy at ASDEX Upgrade



# Energetic particle injection at ASDEX Upgrade

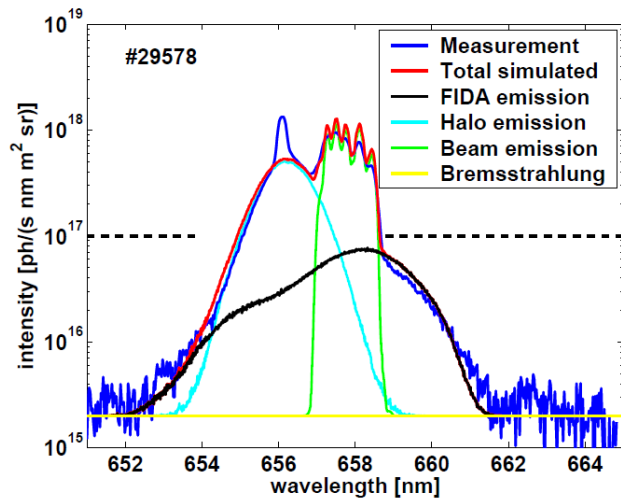


We know well at what energies and direction particles are born



Salewski et al (2016b) NF

# Velocity tomography without extra prior information



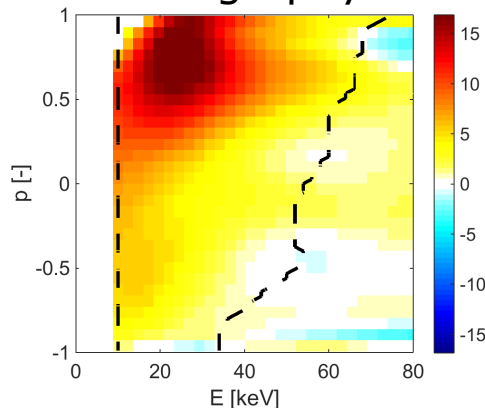
- 5 measured spectra like this
- Truncated singular value decomposition
- Standard Tikhonov regularization

Artifacts appear when no further prior information is used

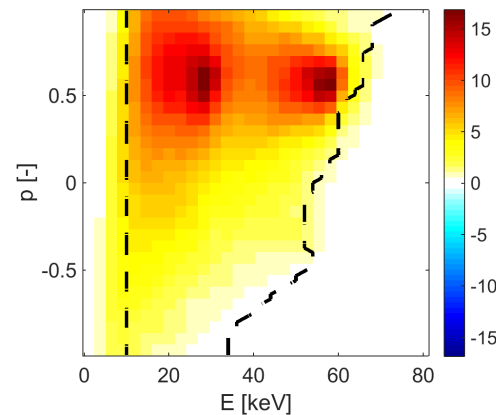
- 1) Negative values!
- 2) Particles at too large energies
- 3) Not good at finding peaks



Measurement by tomography



Simulation



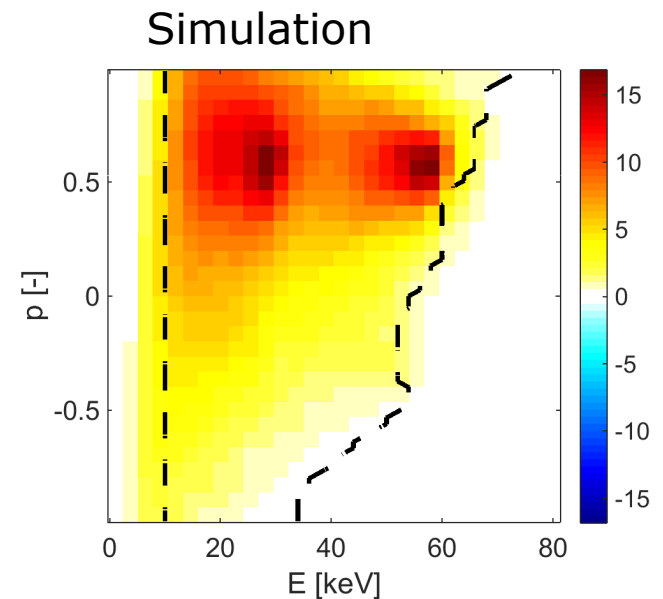
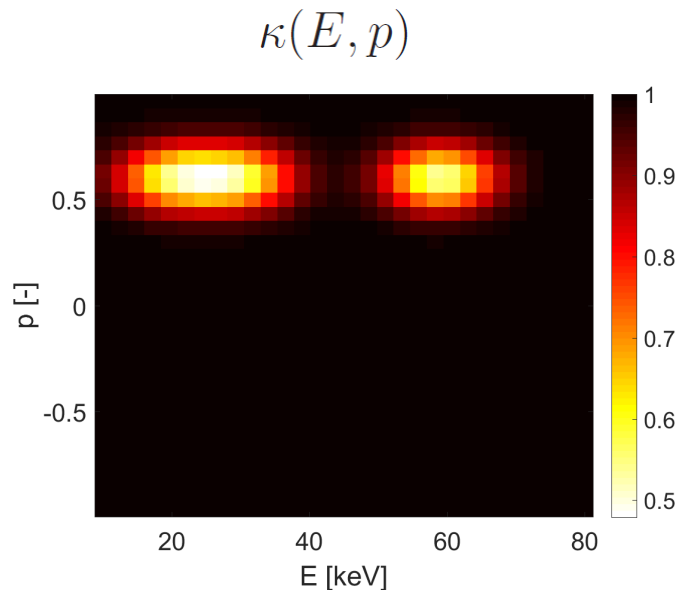
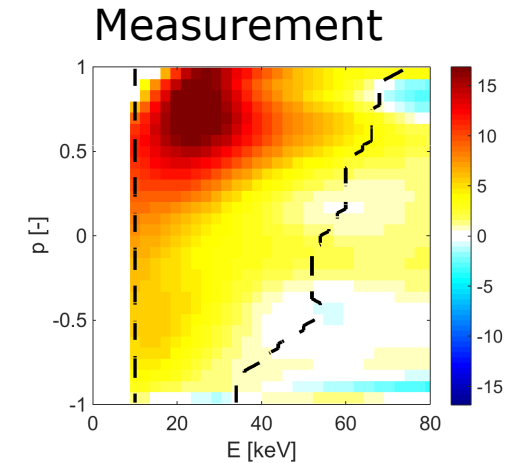


# Outline

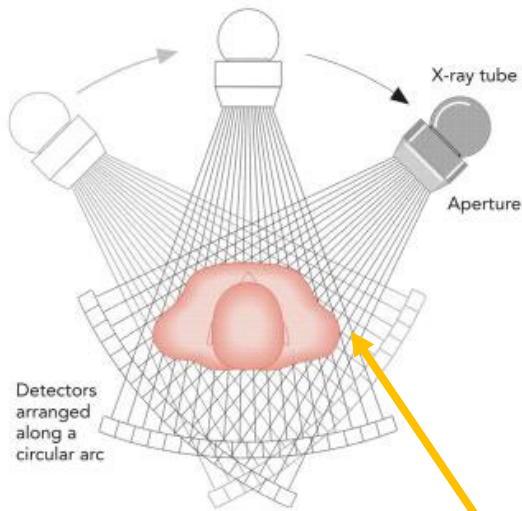
- 1) Introduction to fusion energy, fast ions and velocity distribution functions
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The Lucky-Luke-Doppler experiment
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# Prior information: What do we know?

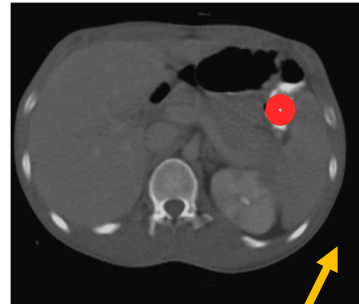
- The number of particles in a box is non-negative.
- Energies and direction of injected high-energy particles are known.



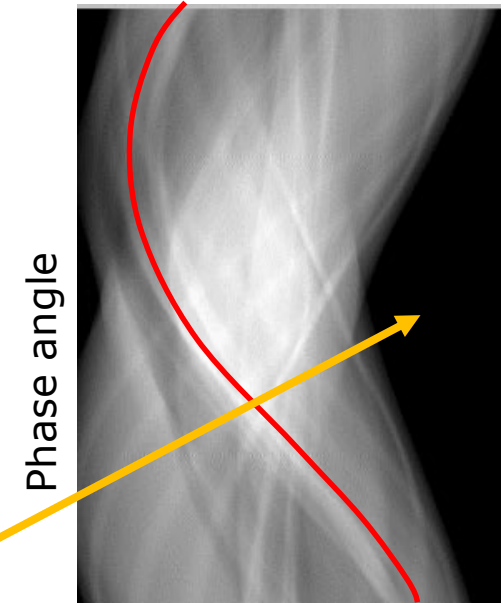
# Null measurements in a CAT scanner



Slice through patient



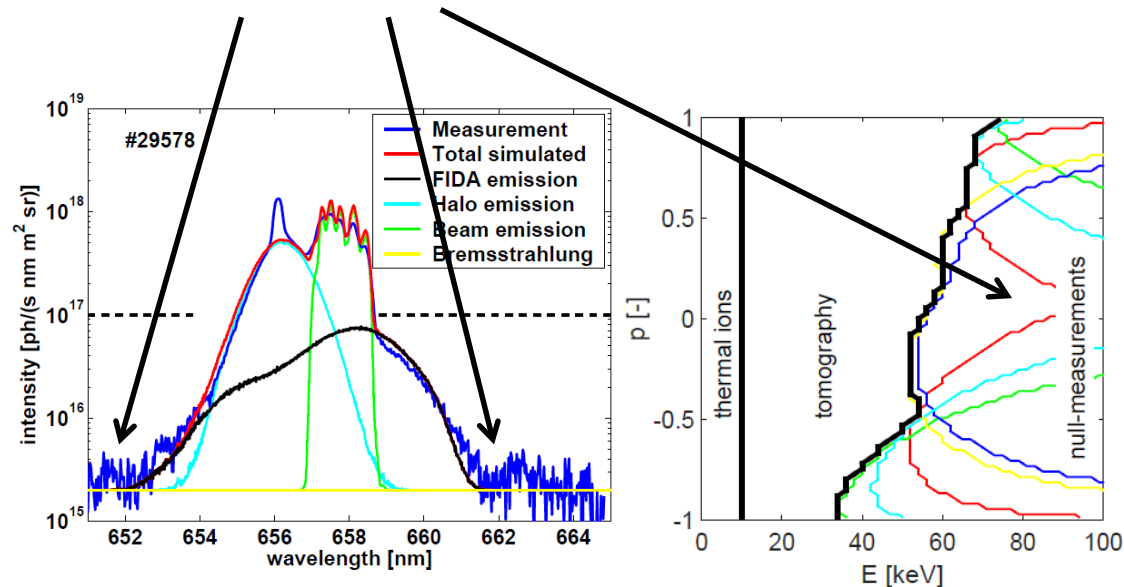
Measurement data



The outer beams measure the absence of the patient!

# Prior information: What do we know?

- Null-measurements: no ions here

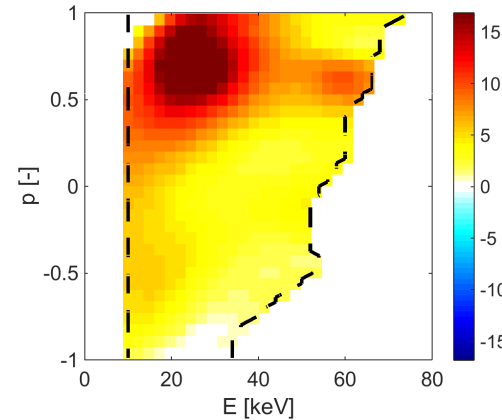
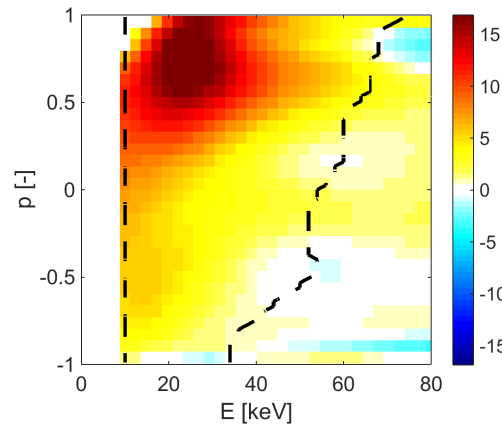
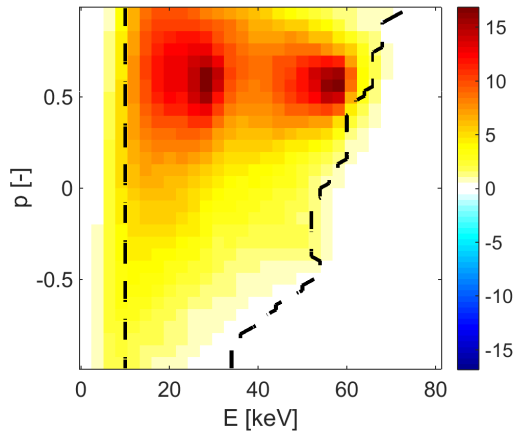


$$F^* = \arg \min_F \left\| \begin{pmatrix} W \\ \lambda \kappa(E, p) L \end{pmatrix} F - \begin{pmatrix} S \\ 0 \end{pmatrix} \right\|_2 \quad \text{subject to} \quad \begin{cases} F^*(E_0, p_0) = 0 \\ F^* \geq 0 \end{cases}$$

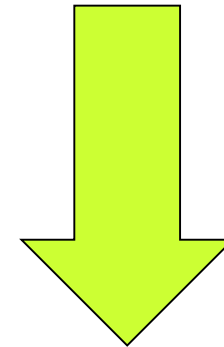


# High-definition tomography: Prior information

Simulation



Truncated SVD  
Standard Tikhonov  
Without prior information:  
Artifacts



With prior information:  
No visible artifacts

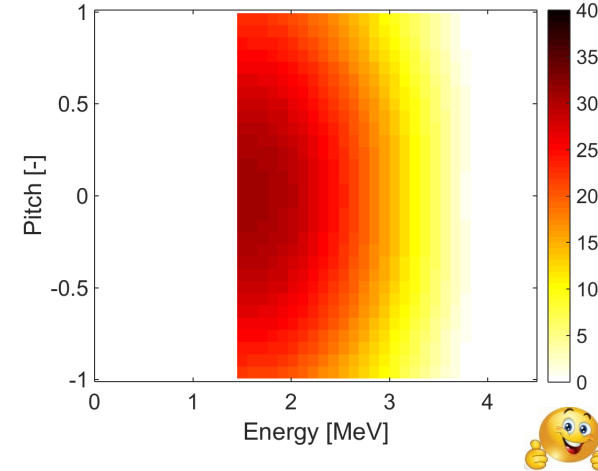
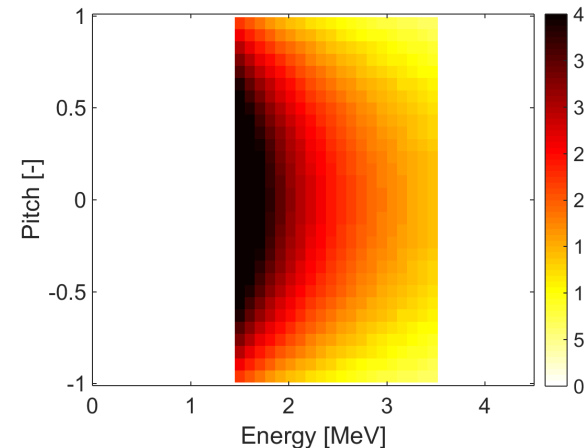
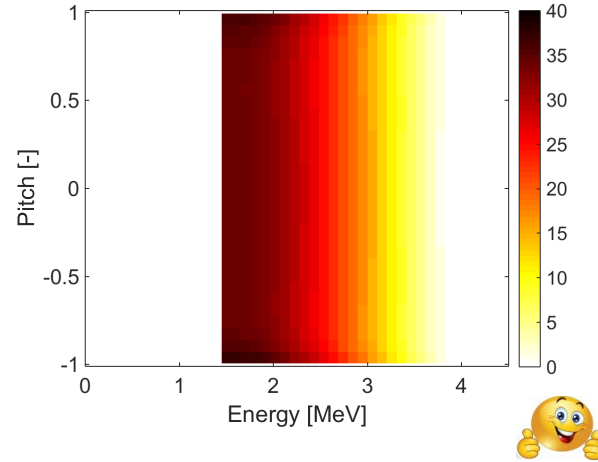
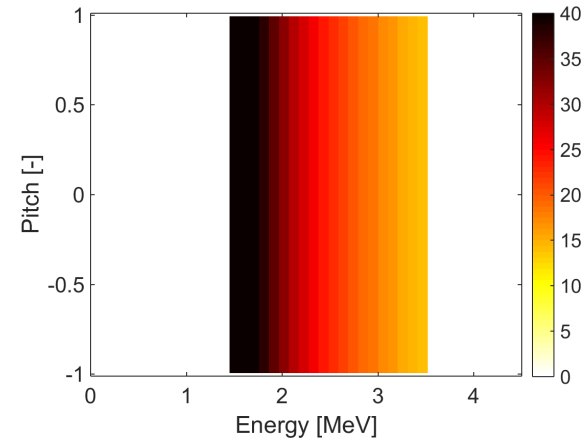


$$F^* = \arg \min_F \left\| \begin{pmatrix} W \\ \lambda \kappa(E, p) L \end{pmatrix} F - \begin{pmatrix} S \\ 0 \end{pmatrix} \right\|_2 \quad \text{subject to} \quad \begin{cases} F^*(E_0, p_0) = 0 \\ F^* \geq 0 \end{cases}$$

# Velocity-space tomography at the ITER tokamak

Ground truth

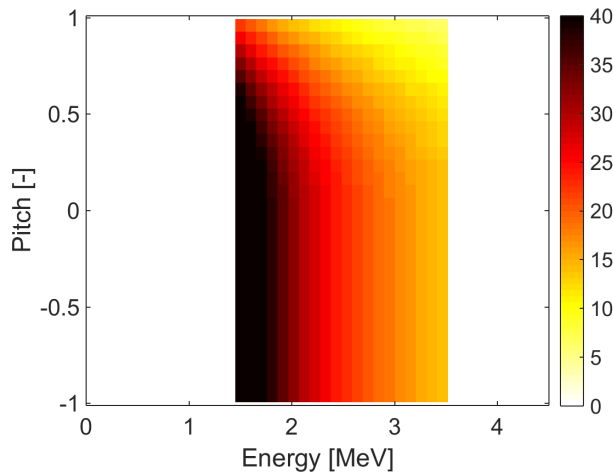
Tomography



- First plasmas in ITER in >2025
- Fusion plasmas in ITER in >2035
- We can reconstruct up-down symmetric functions

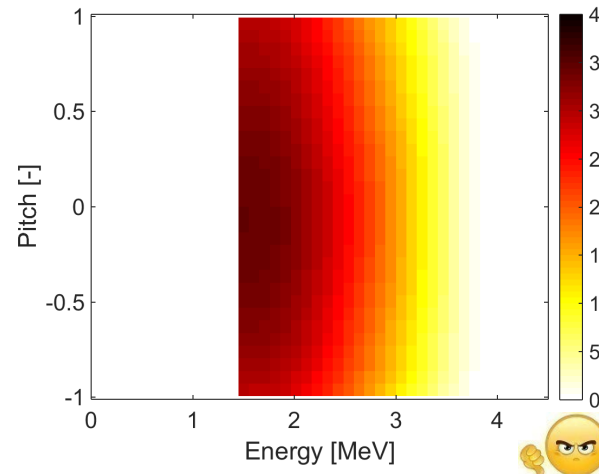
# Velocity-space tomography at ITER

Ground truth



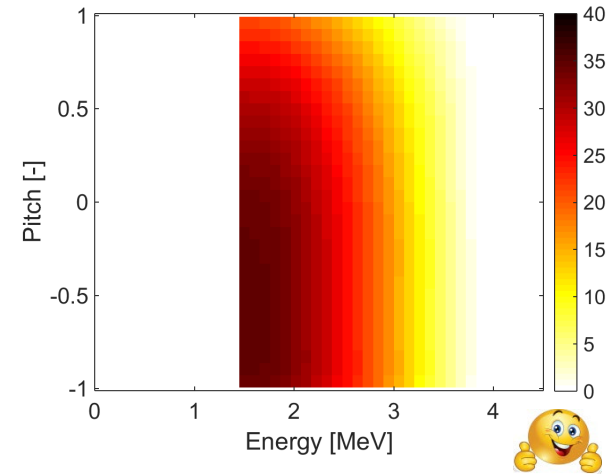
Up-down asymmetric

Tomography with currently foreseen diagnostics



Up-down symmetric

Tomography with an additional gamma-ray detector



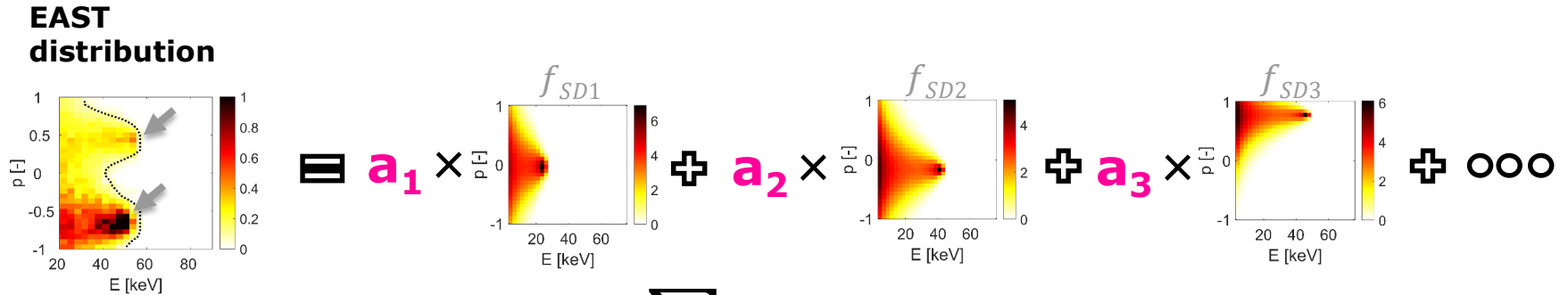
Up-down asymmetric

- For asymmetric ground truth, we get symmetric solutions
- Found reason through maths tools: singular value decomposition

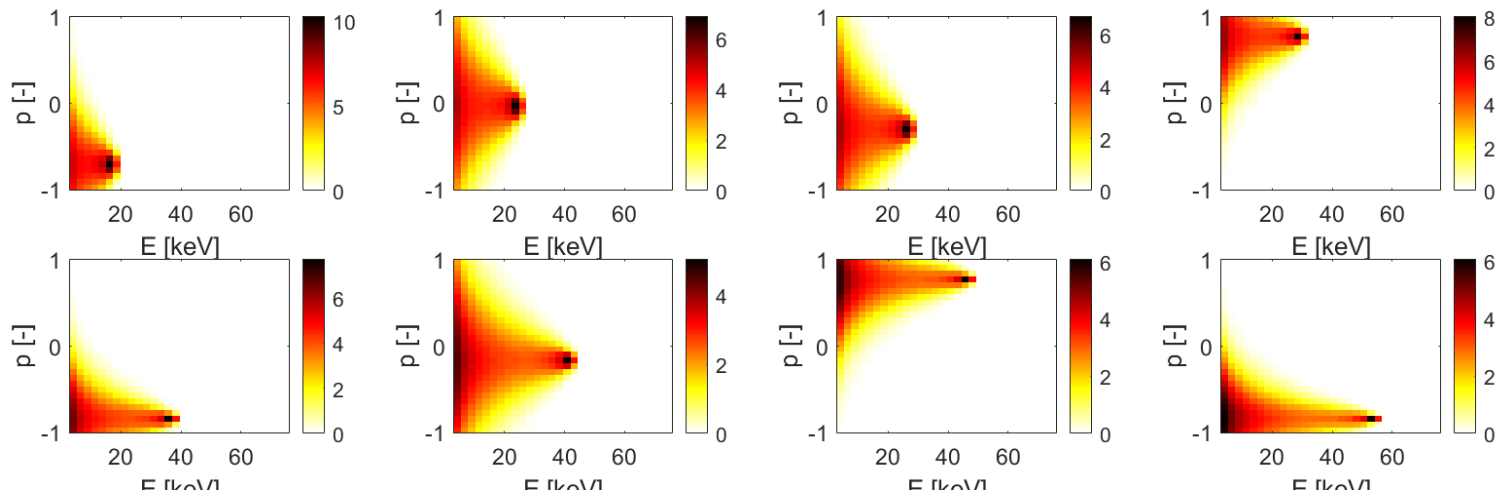
**We need an additional oblique gamma-ray detector at ITER!**

# Collision physics-based basis functions

## Prior information: Collisions in plasmas



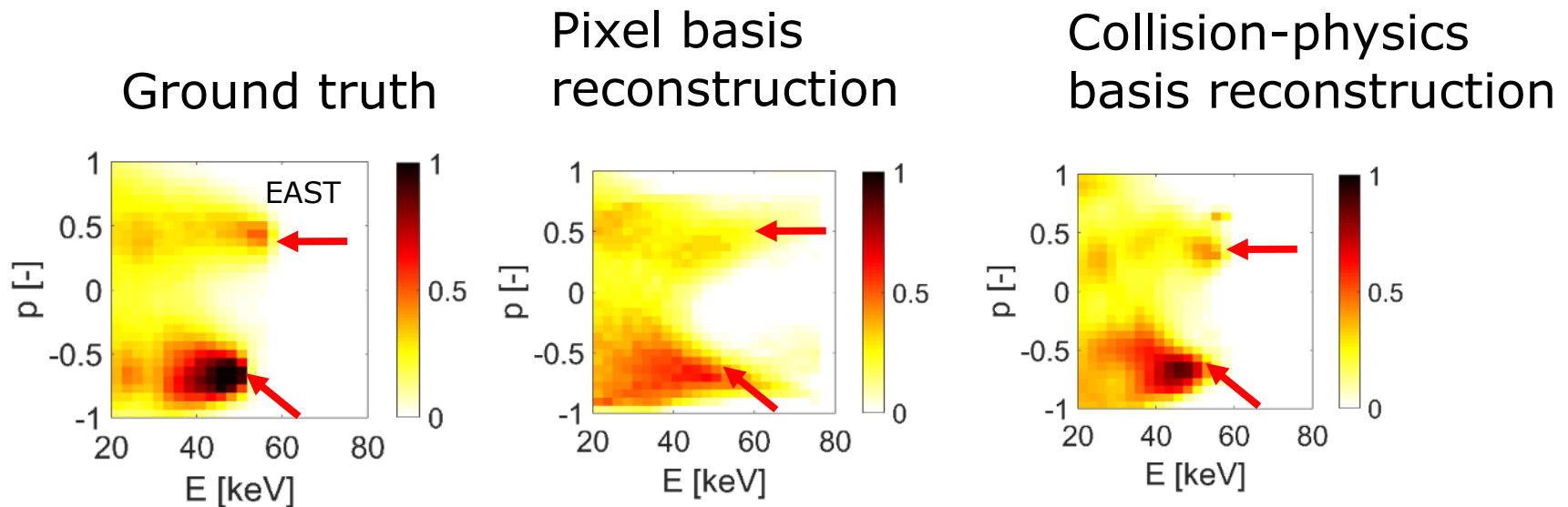
$$f = \sum_i a_i f_{SD,i}$$





# Collision physics-based basis functions

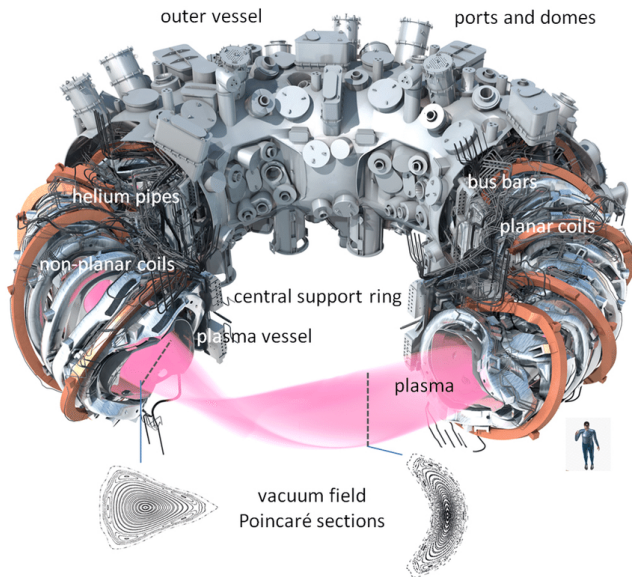
## Prior information: Collisions in plasmas



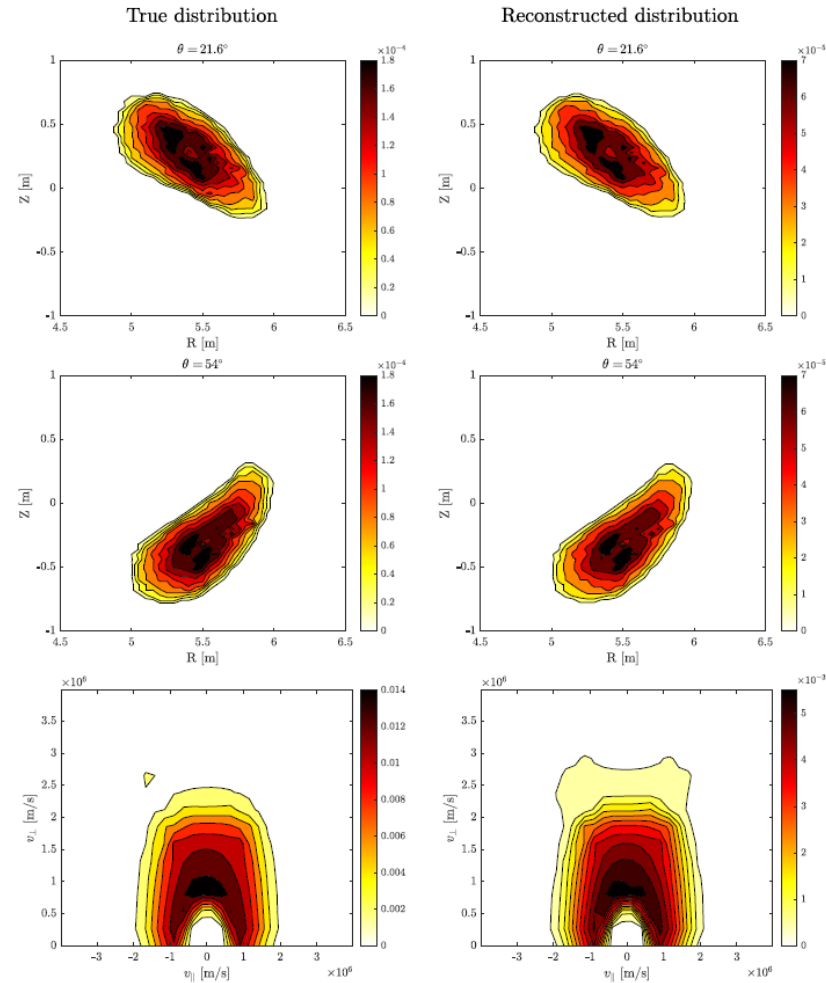
- Collision physics is very strong prior information
- This approach can be extended to 3D, 4D, and 5D tomography

*Madsen et al. 2020 ppcf*

# The 4D/5D tomography problem



- At each point in 3D position space, there is a 2D velocity distribution function.
- Our first baby steps: inference based on 24 base functions

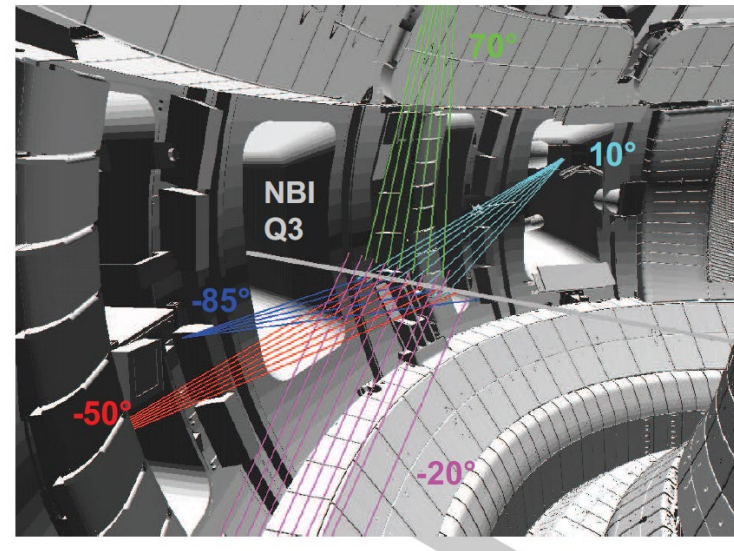
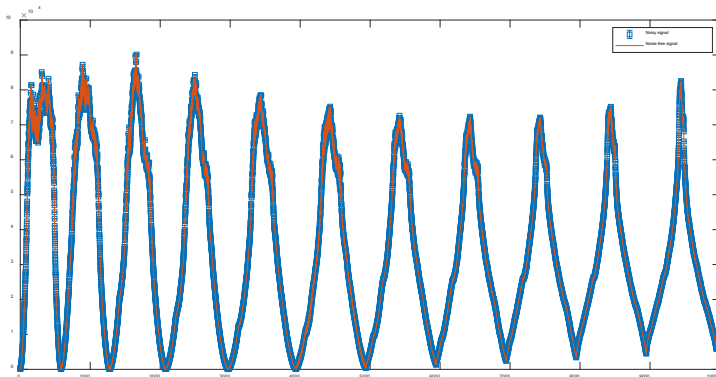


*Simmendefeldt et al. (to be submitted)*

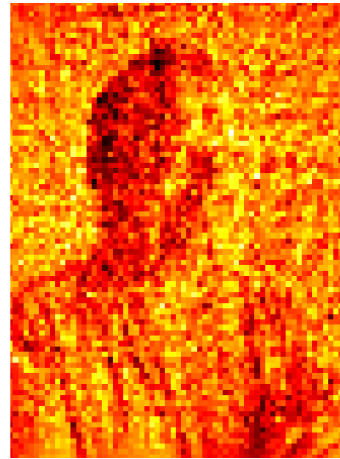
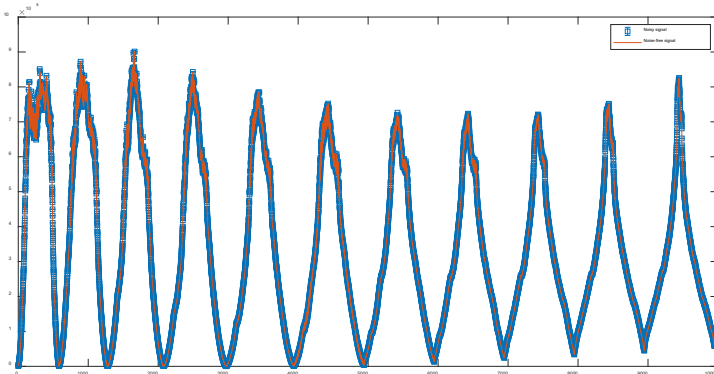
# Outline

- 1) Introduction to fusion energy, fast ions and velocity distribution functions
- 2) Introduction to tomography in position and velocity space -  
The Lucky-Luke-Doppler experiment
- 3) How is Per Christian helping fusion plasma physics?
- 4) A closing riddle**

What velocity distribution function in a fusion plasma produces these 11 spectra of projected velocities ?

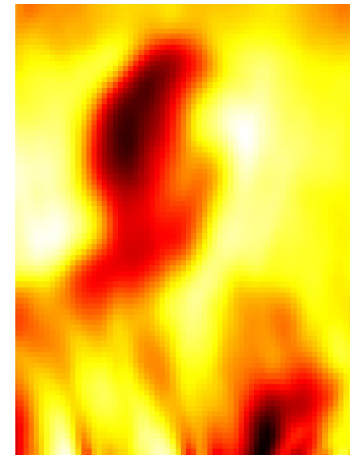


First two trials of  
velocity-space  
tomography



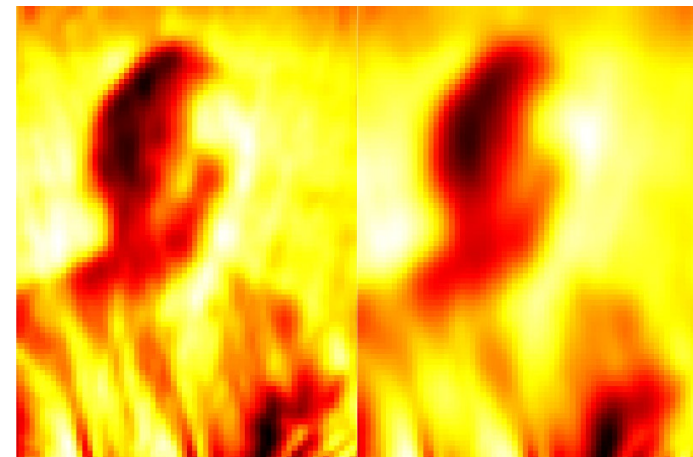
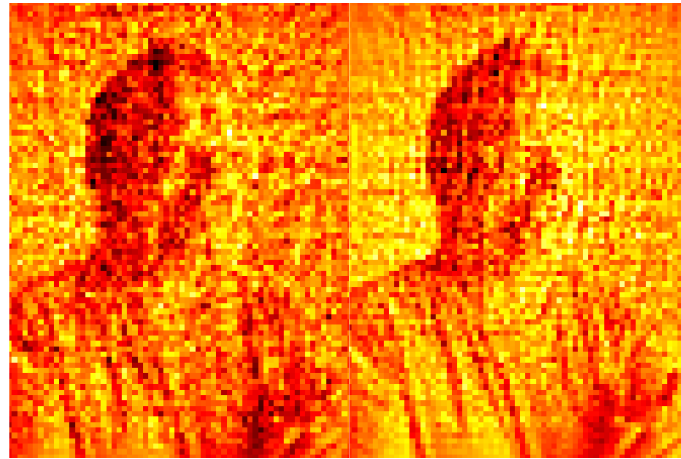
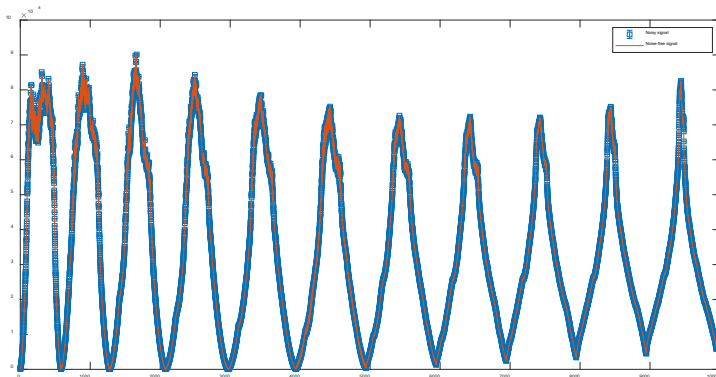
Too spiky / jittery.  
Need more  
smoothing.

Too smooth / blobby.  
Need less smoothing.

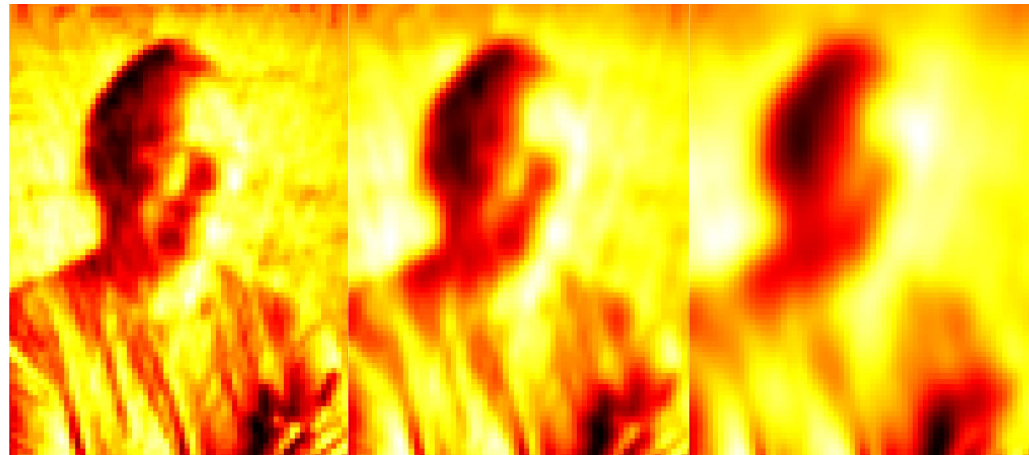
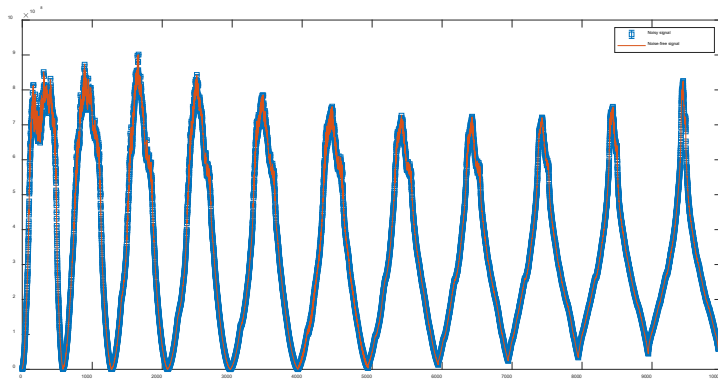




# Next two trials of velocity-space tomography



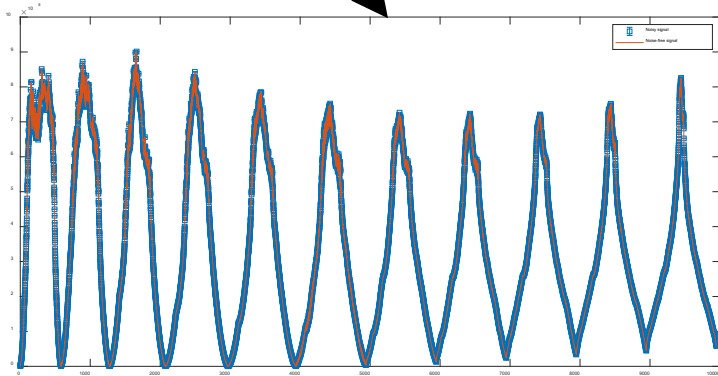
# Next two trials of velocity-space tomography



Exploded Per  
Christian:  
Too spiky

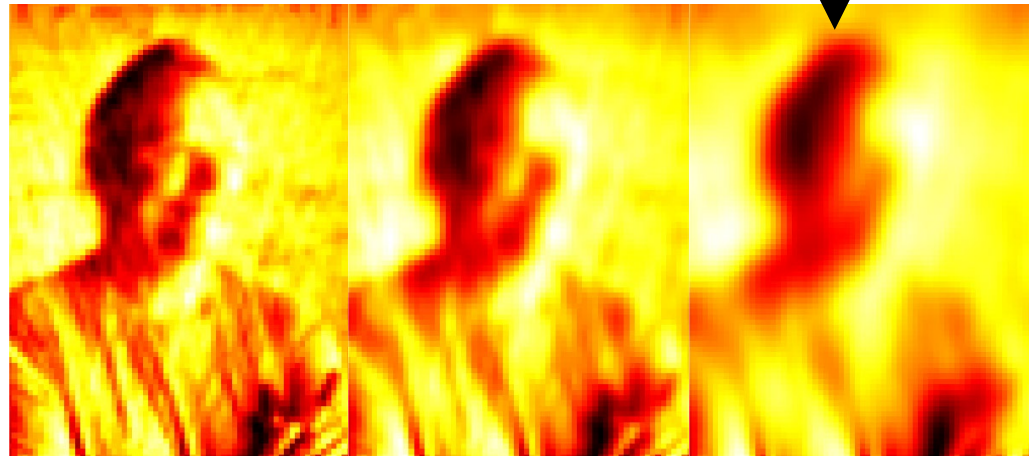


Per Christian projected  
on 11 spectrometers  
Very hard to tell!



**Now find the *right*  
amount of smoothing,  
e.g. by the "L-curve"  
Hansen 1992, 1993, 2000**

Bloppy Per  
Christian:  
Too smooth





# Congratulations on 40 years at DTU!

Ground truth

