From HDTomo

HIGH-DEFINITION TOMOGRAPHY

To CUQI

— Per Christian Hansen's 40th Anniversary —

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DTU Compute

DTU

- HDTOMO (2012 - 2017)

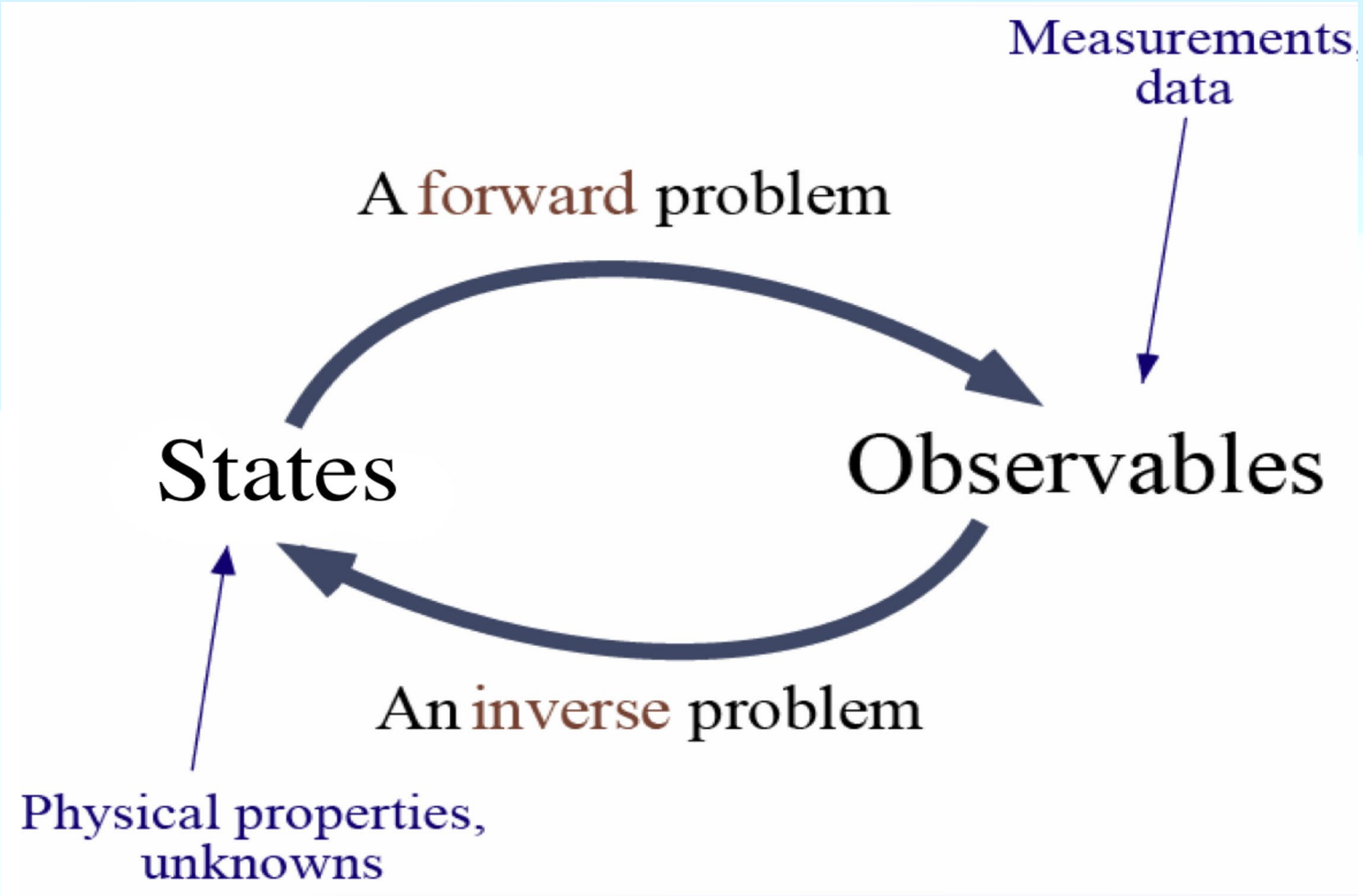
  HIGH - DEFINITION TOMOGRAPHY

  - ERC Advanced Grant
  - 4 senior members, 3 Post Docs, 5 PhDs.

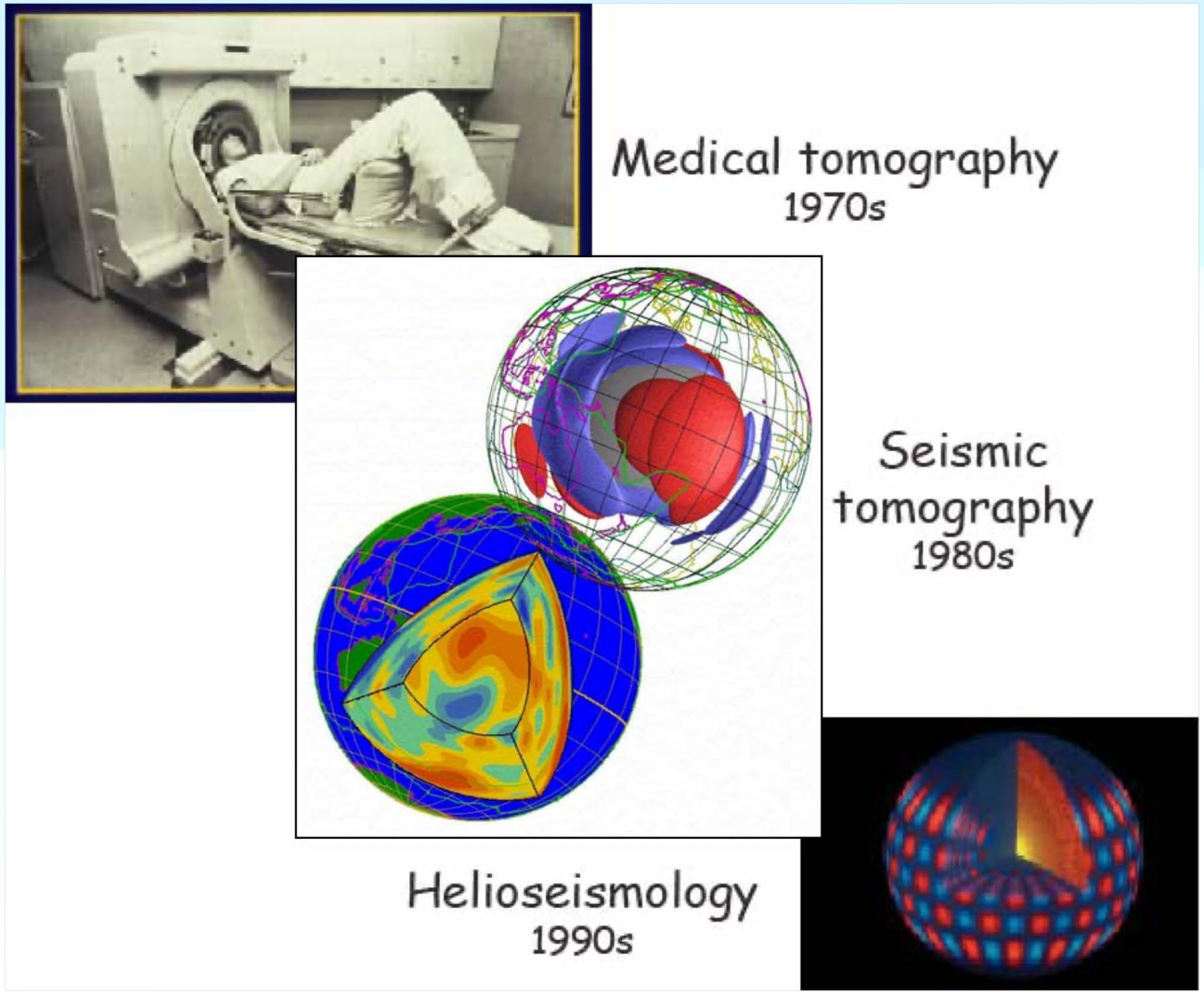
- CUQI (2019 - 2025)

  - **C**omputational **U**ncertainty **Q**uantification for **I**nverse Problems
  - Villum Investigator Grant
  - 6 senior members, 8 Post Docs, 9 PhDs.

What are inverse problems?



Inverse problems are everywhere!



Example of a linear inverse problem

$$Ax = b \iff \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \iff \begin{cases} x_1 + x_2 = 2 \\ x_1 + 2x_2 = 3 \end{cases}$$

- Forward problem: Given and , calculate .
- Inverse problem: Given and , solve .

Example of a linear inverse problem

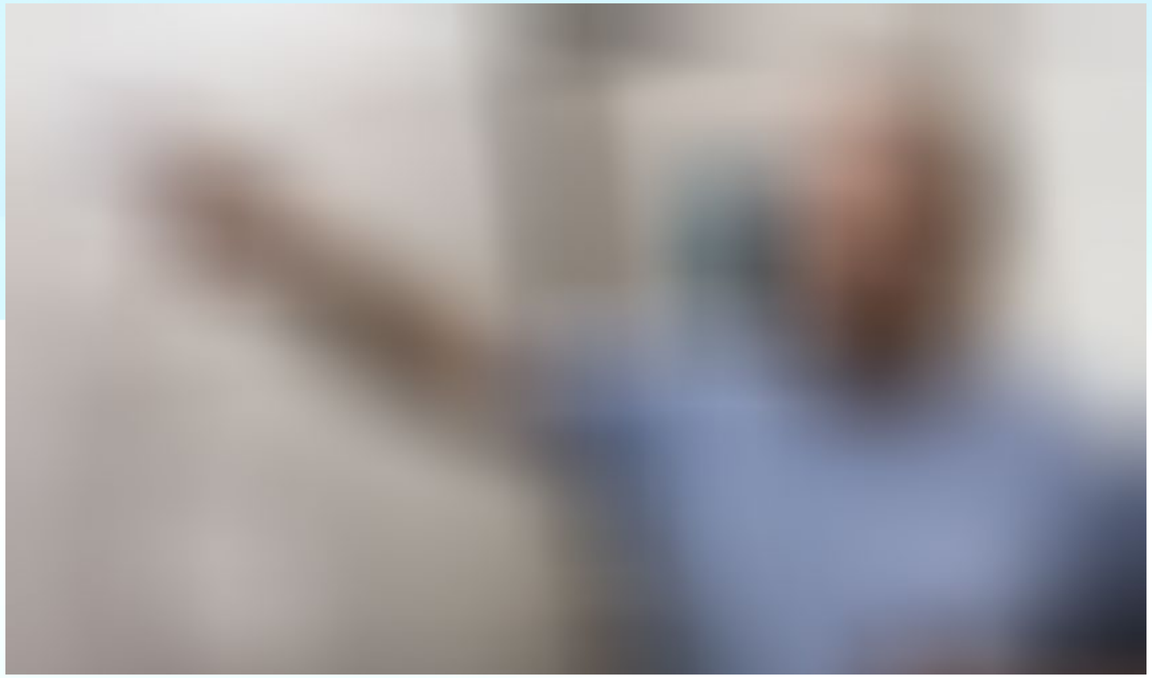
- Case 1: $$Ax = b \iff \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix} \iff \begin{cases} x_1 + x_2 & = 2 \\ x_1 + 2x_2 & = 3 \\ x_1 + 3x_2 & = 3 \end{cases}$$
- Case 2: $$Ax = b \iff \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2 \iff x_1 + x_2 = 2$$
- Case 3: $$Ax = b \iff \begin{pmatrix} 1 & 1 \\ 1 & 1.001 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \iff \begin{cases} x_1 + x_2 & = 2 \\ x_1 + 1.001x_2 & = 2 \end{cases}$$

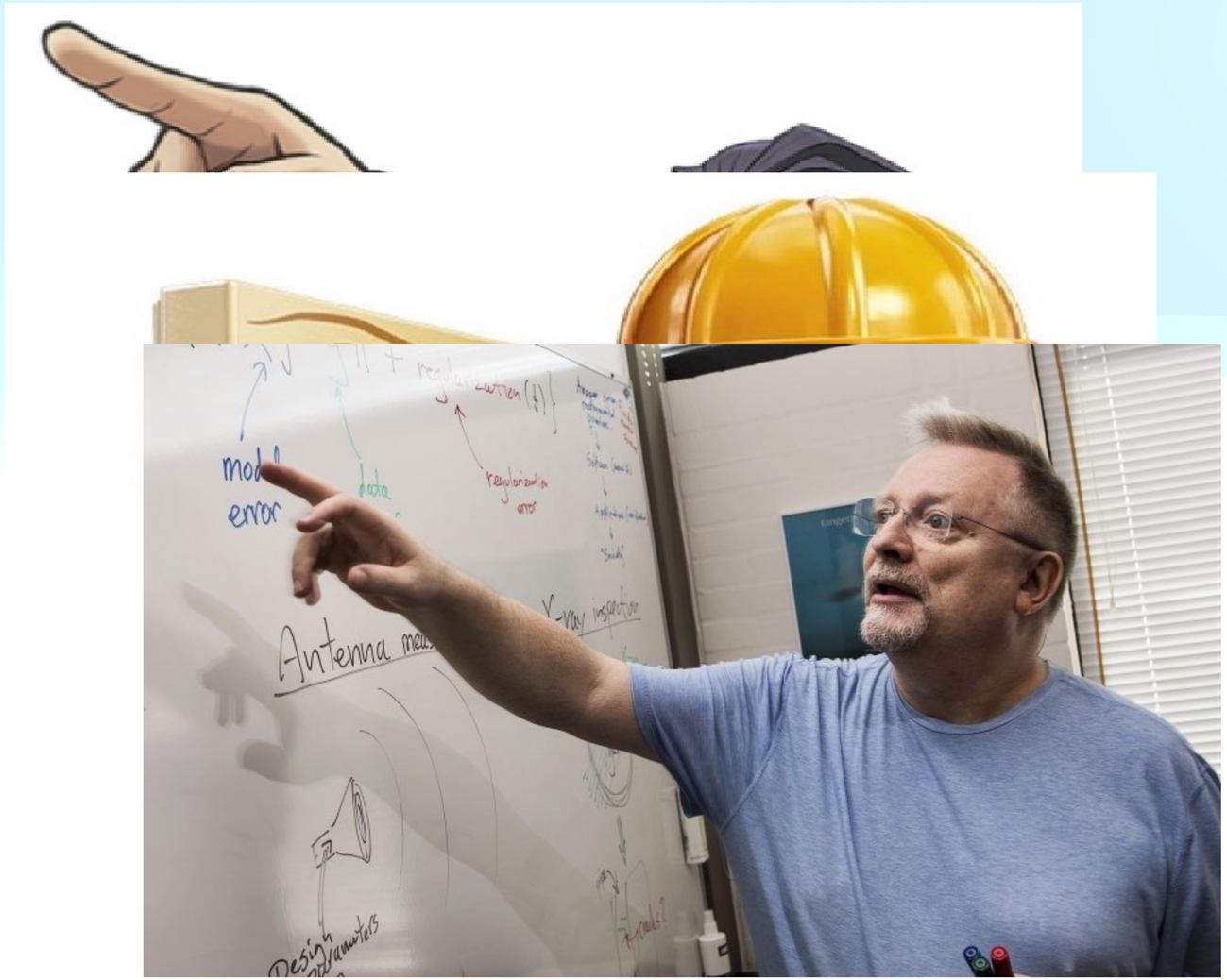
How can we solve IPs?

What can we tell about who/ whatever made it?

- Collect data: measure size and depth, check the properties of the ground …
- Use *prior information*: who lives around here? …

Prior information is important!





HDtomo
HIGH - DEFINITION TOMOGRAPHY

Objective: Optimal use prior information in inverse problems.

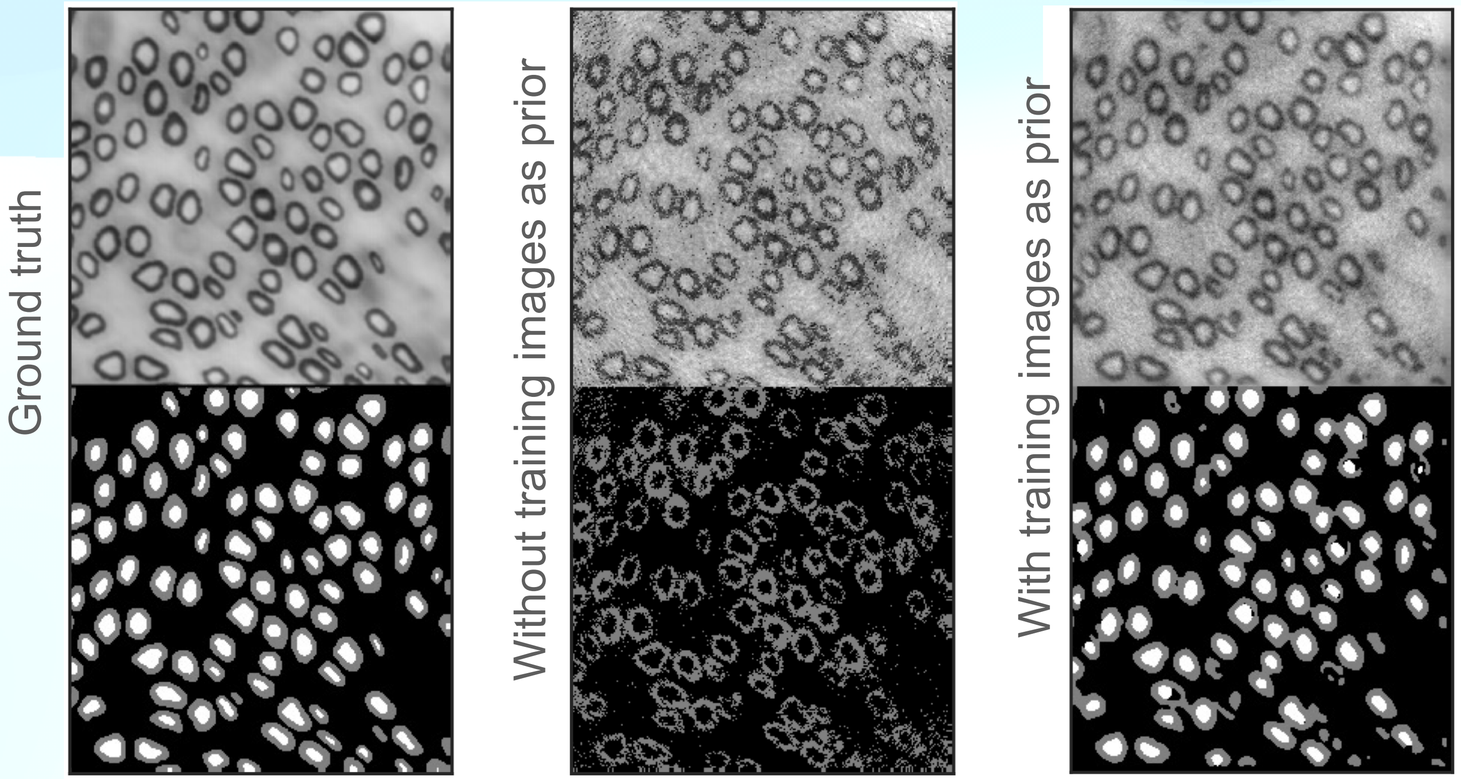
Example: Computed X-ray tomography (CT)

(The video is created by Prof. Samuli Siltanen from University of Helsinki.)

Example: Training images as prior

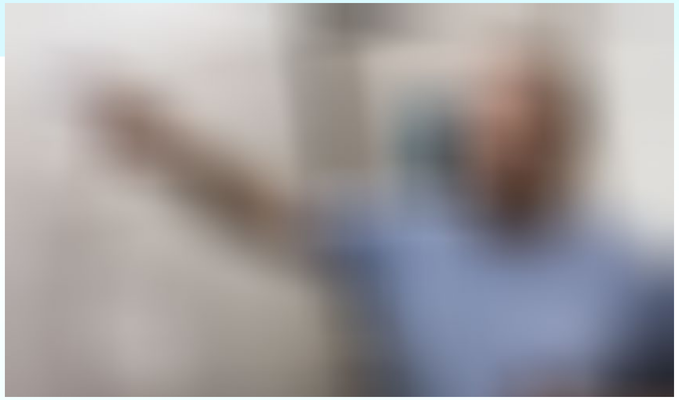
(D., Hansen and Kjer, *IEEE Transactions on Computational Imaging*, 2018)

Goal: In CT problem, we simultaneously computes a reconstruction and a corresponding segmentation by using dictionary learning technique.



CUQI (Computational Uncertainty Quantification for Inverse Problems)

Objective: Make UQ an operational tool for inverse problems.



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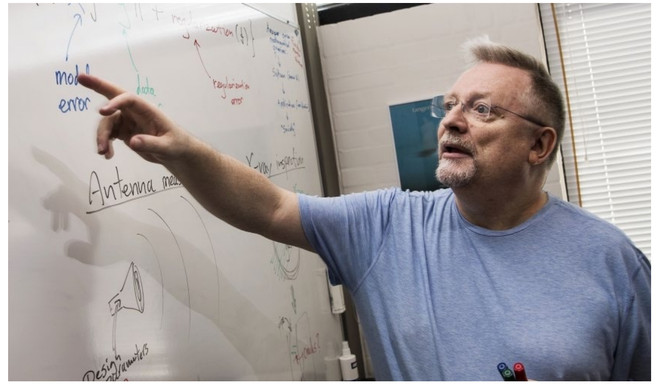
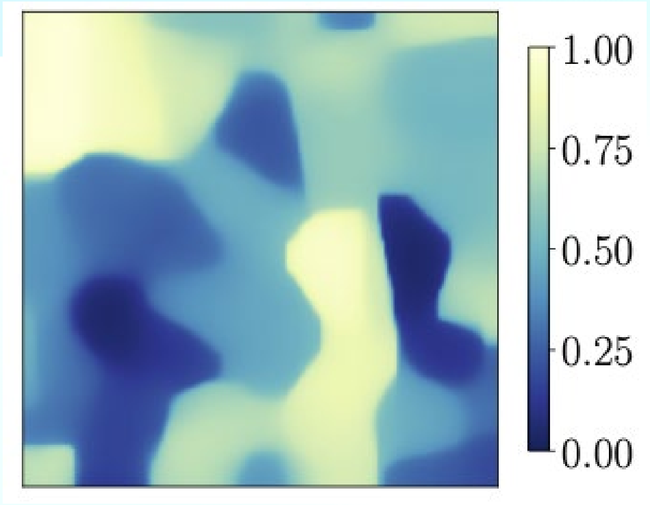


80%

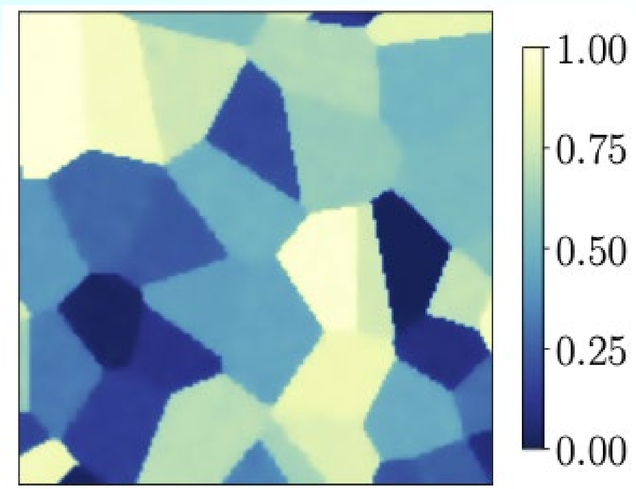
Example: CT with uncertain view angles

(Uribe, Bardsley, D., Hansen and Riis, *SIAM Journal on UQ*, 2022)

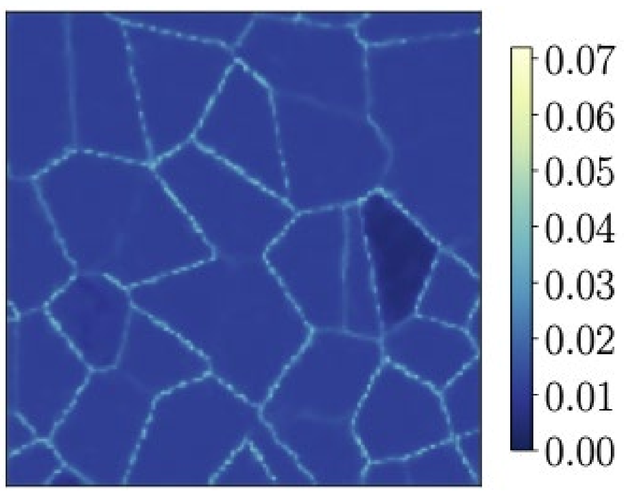
Goal: In the case that the view angles in CT scan are not known accurately, we reconstruct the image and estimate view angles simultaneously. Furthermore, we provide uncertainty estimates of both.



(a) Without considering angle uncertainty



(b) taking angle uncertainty into account

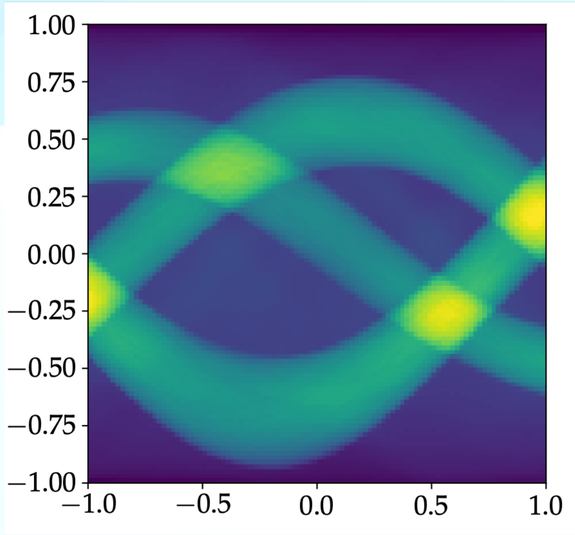


(c) uncertainty in the reconstruction

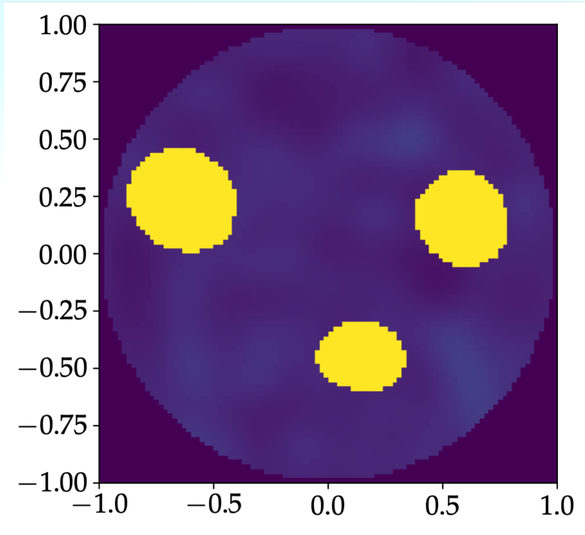
Example: Boundary reconstruction in CT

(Afkham, D. and Hansen, *SIAM Journal on UQ*, 2022)

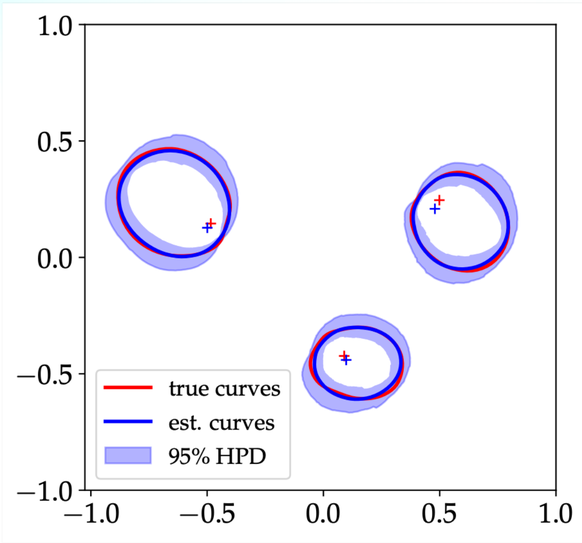
Goal: In CT problem, we reconstruct the boundary of the inclusions directly and quantify the uncertainties of the boundary curves.



(a) sinogram $\boldsymbol{y}$



(b) true image $\alpha(\boldsymbol{\xi})$



(c) predicted boundaries

Congratulations, Per Christian!
Thank you!

For your interest:

- **HD-Tomo project:** http://www2.compute.dtu.dk/~pcha/HDtomo/
- **CUQI project:** https://www.compute.dtu.dk/english/CUQI