Inverse Problems

Ill-posed problems

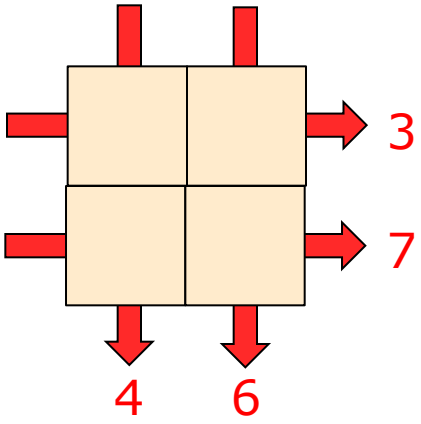
$$\mathcal{K} f = g \,, \qquad \mathcal{K} = \text{compact operator}$$

Typically: they have no solution, or infinitely many solutions.

Example 1:

$$x = x + 5$$

Example 2:



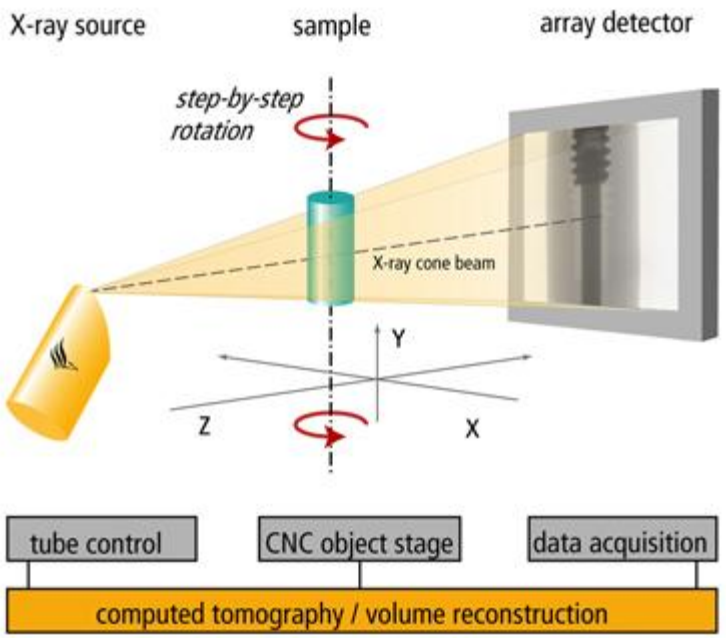
0	3
4	3

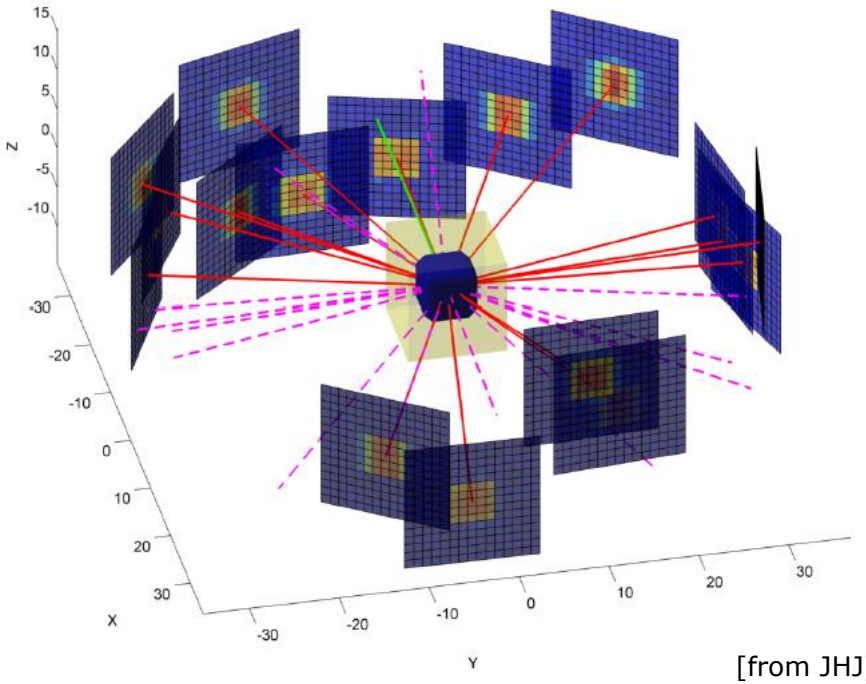
1	2
3	4

2	1
2	5

3	0
1	6

Tomography





[from JHJ]

Tomography is the science of *seeing inside objects*. Physical signals – waves, particles, currents – are sent through an object from many different angles, the response of the object to the signal is measured, and an image of the object's interior is reconstructed via sophisticated mathematical techniques.

Tomography is behind important scientific discoveries: The interior structure and processes of the Earth, Moon and Sun and the first maps showing the location of simple mental processes in the human brain are notable examples.

European Research Council

Supporting top researchers from anywhere in the world

Proposals should rise to pioneering and far-reaching challenges at the frontiers of the field(s) addressed. They should involve new, ground-breaking or unconventional methodologies, whose risky outlook is justified by the possibility of a major breakthrough with an impact beyond a specific research domain/discipline.

The Principal Investigators should be exceptional leaders in terms of originality and significance of their research contributions.

Funding: up to € 2.5 million per grant

Duration: up to 5 years

Indicative Statistics

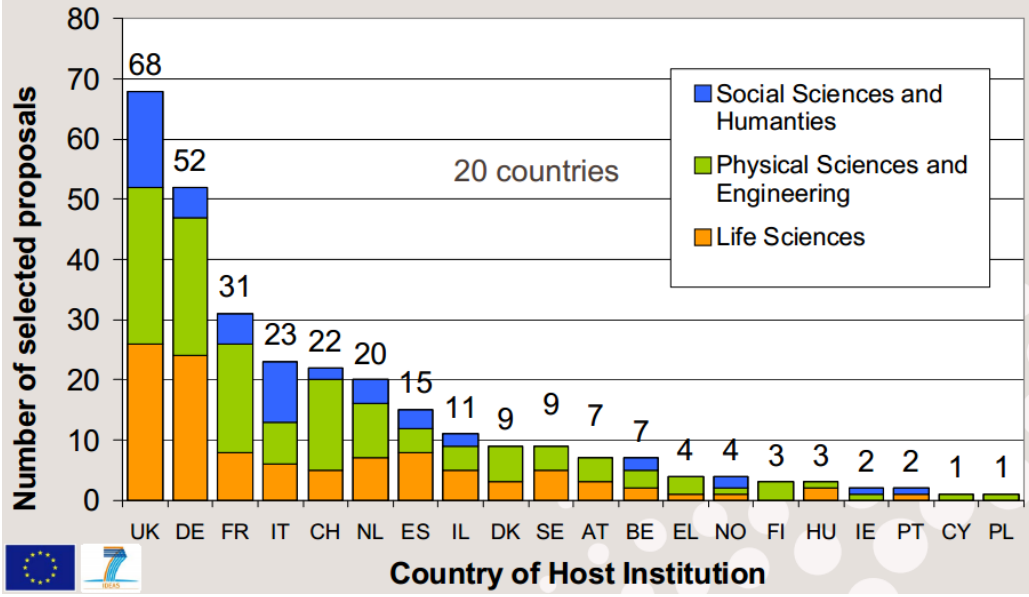
erc.europa.eu/statistics

Physical Sciences & Engineering

- PE1 Mathematics
- PE2 Fundamental constituents of matter
- PE3 Condensed matter physics
- PE4 Physical & Analytical Chemical sciences
- PE5 Materials & Synthesis
- PE6 Computer science & informatics
- PE7 Systems & communication engineering
- PE8 Products & process engineering
- PE9 Universe sciences
- PE10 Earth system science

	Submitted proposals	Selected proposals
PE	917	134 (45,6%)
LS	789	107 (36,4%)
SH	578	53 (18%)
	Total = 2284	Total = 294

Success rate 13%



	2008	2009	2010	2011	TOT
UK	58	61	53	68	240
DE	26	33	45	52	156
FR	35	34	32	31	132
CH	28	29	21	22	100
IT	20	14	21	23	78
NL	19	19	17	20	75
ES	14	12	13	15	54
IL	15	11	13	11	50
SE	16	12	11	9	48
AT	8	7	6	7	28
DK	4	3	5	9	21
BE	5	2	5	7	19

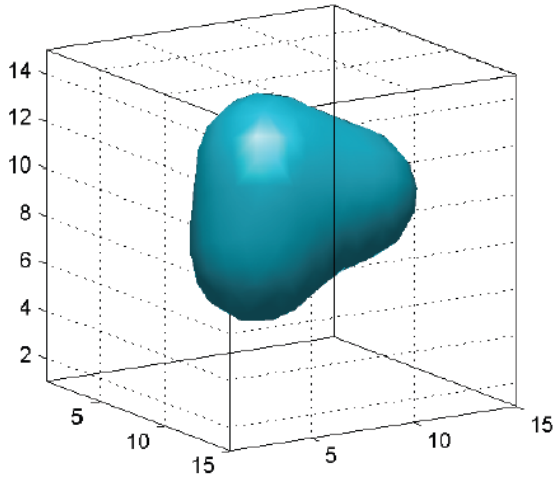
HD tomo
HIGH - DEFINITION TOMOGRAPHY

- **The Goal.** Utilize and develop the mathematical technology and computational algorithms that can incorporate many different kinds of available prior information in order to produce *high-definition reconstructions*, i.e., sharper images with more reliable details.
- **The Challenges.** Prior information comes in many different forms (e.g., constraints, statistical priors, or "catalogues" of trustful images) – design methods that incorporate all this information in an optimal way.
- **The Ingredients.** Linear/nonlinear models, integral/differential equations, analytical methods, variational formulations, sampling methods.
- **The Focus.** Mathematical and numerical algorithm development with close ties to high-performance computing and application scientists.
- **The Impact.** Advance the use of tomographic methods in a wide range of applications, e.g.:
  - security scanners for passengers and cargo,
  - oil/gas/ geothermal energy exploration,
  - process and production monitoring for safety and quality,
  - X-ray and neutron scattering in materials science,
  - medical applications, dementia diagnostics, screening, surgery aid.

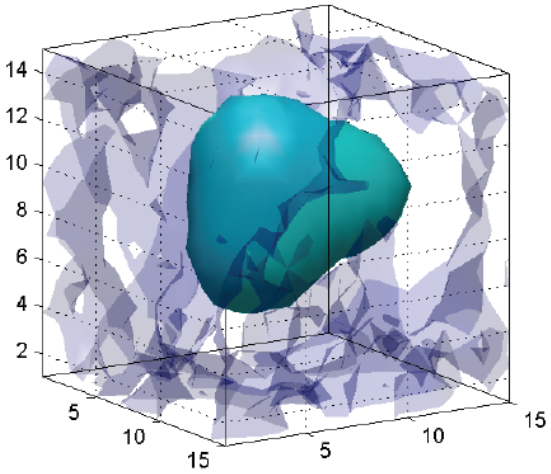
Using Prior Information

Improved reconstructions through the use of prior information!

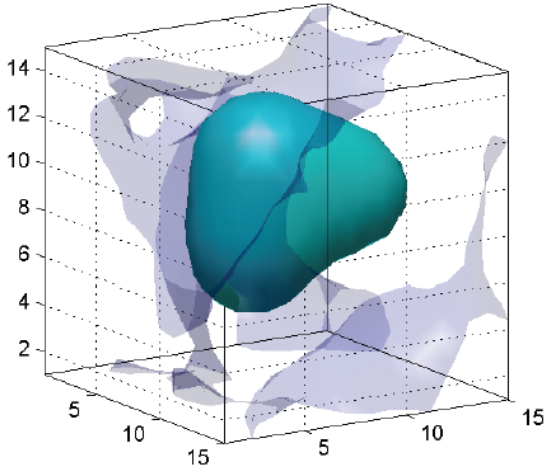
Example: ODF reconstruction in materials science (with DTU Physics):



Exact model.



Classical reconstruction with a lot of noise.

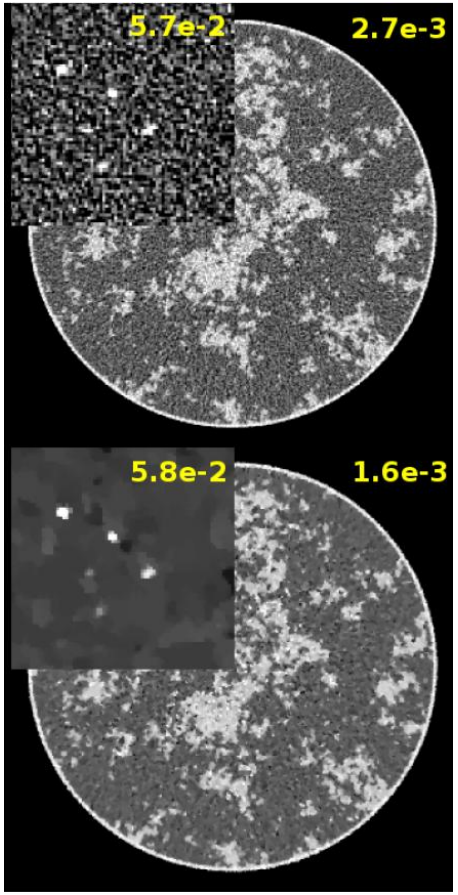


Using the prior that the model is *smooth*.

We developed a new preconditioned iterative Krylov-subspace method.

Total Variation Prior: Sharper Edges

$$\min \int_\Omega |\nabla f| \, \mathrm{d}\Omega \quad \text{s.t.} \quad \|K f - g\|_2 \le \tau\sigma, \qquad \sigma = \text{noise level}$$



[from JHJ]

TVReg (publ. in BIT): optimal first-order methods for 2-D & 3-D tomographic reconstructions.

mxTV (publ. in Numer. Algo.):

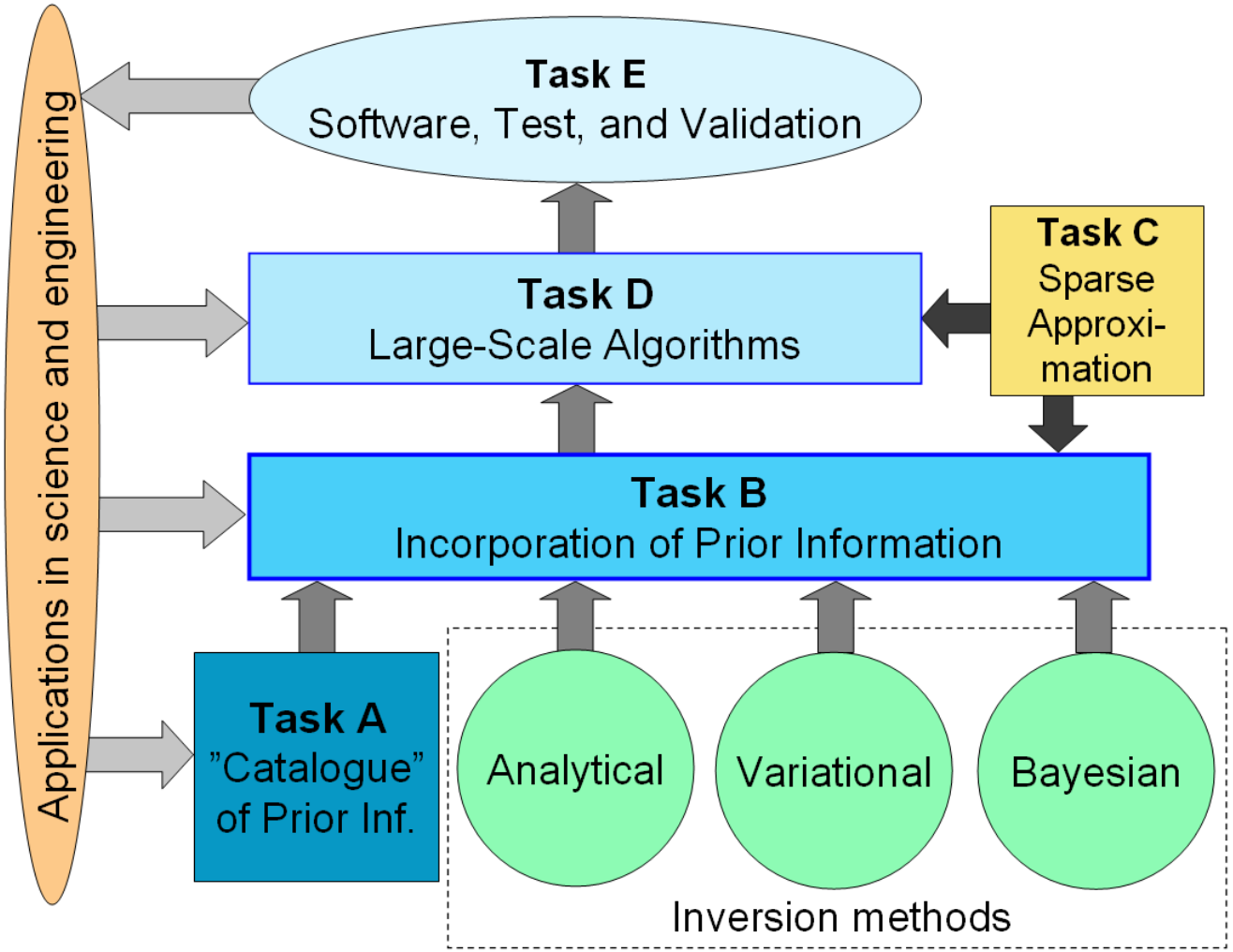
- denoising
- inpainting
- deblurring

CSI Lyngby: DTU Informatics

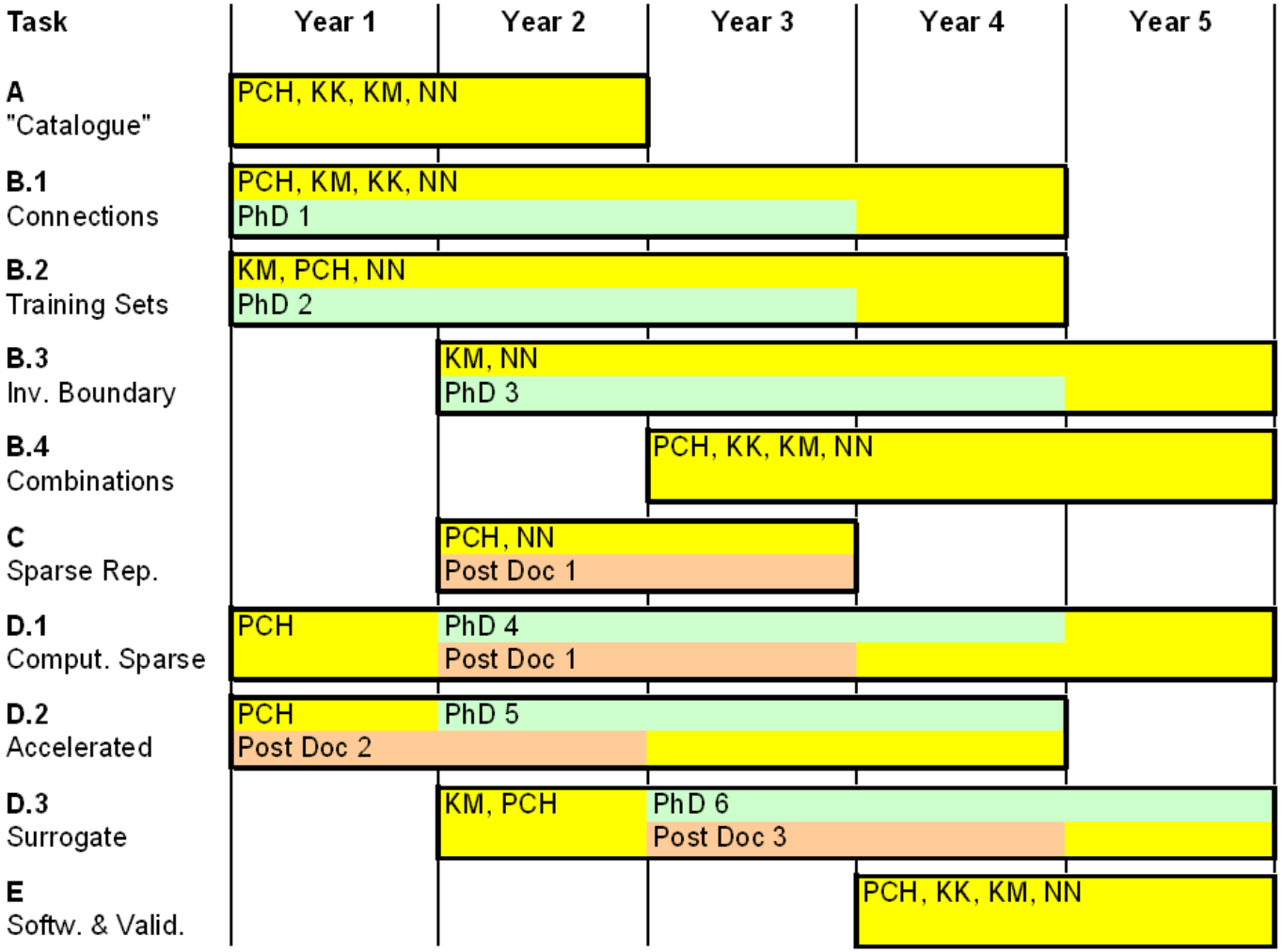
CSI Aalborg: Aalborg Univ.

mosek ApS

The Ingredients of the Project

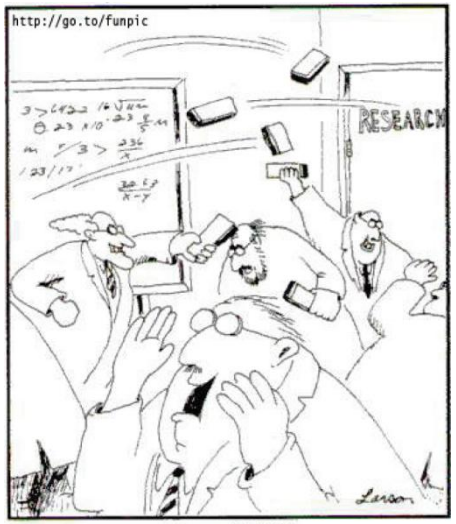


Time Plan


Task
Year 1
Year 2
Year 3
Year 4
Year 5
A
"Catalogue"
PCH, KK, KM, NN
B.1
Connections
PCH, KM, KK, NN
PhD 1
B.2
Training Sets
KM, PCH, NN
PhD 2
B.3
Inv. Boundary
KM, NN
PhD 3
B.4
Combinations
PCH, KK, KM, NN
C
Sparse Rep.
PCH, NN
Post Doc 1
D.1
Comput. Sparse
PCH
PhD 4
Post Doc 1
D.2
Accelerated
PCH
PhD 5
Post Doc 2
D.3
Surrogate
KM, PCH
PhD 6
Post Doc 3
E
Softw. & Valid.
PCH, KK, KM, NN

The Research Team

- **Main scientific team**
  - Prof. Per Christian Hansen
  - Prof. Klaus Mosegaard
  - Assoc. Prof. Kim Knudsen, DTU Mathematics
  - Assoc. Prof. NN, funded by this project
- **Post docs**
  - 3 funded by this project (starting 2012, 2013, 2014)
  - morefunded by other resources ...
- **PhD students**
  - 6 funded by this project (with 1/3 co-funding by DTU)
  - morefunded by other resources!
- **Closest collaborator**
  - DTU's 2nd ERC project: Diffraction-Based Transmission X-Ray Tomography



"Eraser fight!!"