▸ ANDREAS HALKJÆR FROM, *Formalized soundness and completeness of natural deduction for first-order logic.*
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We present a soundness and completeness proof of a natural deduction calculus for first-order logic, formalized in the interactive proof assistant Isabelle/HOL [1].

Our formalization is based on previous work by Stefan Berghofer [2]. The proof formalized by Berghofer uses Hintikka sets and only considers completeness for closed formulas [3]. We build on this proof to cover formulas with free variables via the following steps. First we universally close the formula, obtaining a derivation of its closure. Since we consider entailment in general we turn any judgment premises into implications as part of this. Then we eliminate each added quantifier with a fresh constant using the universal elimination rule from the calculus. Thereafter we use our own admissible rule to substitute the original variables for the fresh constants. Finally we show that the premises can always be weakened and use this to turn the implications back into premises, obtaining a derivation of the original formula.

We eliminate the universal closure with fresh constants instead of the free variables directly because we represent variables with de Bruijn indices; this makes reasoning about a chain of substitutions for free variables tricky, as each new substitution adjusts the variables from the previous ones.

Furthermore, we have updated Berghofer's formalization to use Isabelle's declarative proof style Isar [4]. Our formalization is available online.
URL Address: https://bitbucket.org/isafol/isafol/src/master/FOL_Berghofer/.

A further development of the calculus is used for teaching at DTU [5].

[1] Tobias Nipkow, Lawrence C. Paulson and Markus Wenzel, ***Isabelle/HOL — A Proof Assistant for Higher-Order Logic***, vol. 2283, Lecture Notes in Computer Science, Springer, 2002.

[2] Stefan Berghofer, *First-Order Logic According to Fitting*, ***Archive of Formal Proofs***, August 2007.
URL Address: http://isa-afp.org/entries/FOL-Fitting.html.

[3] Melvin Fitting, ***First-Order Logic and Automated Theorem Proving, Second Edition***, Graduate Texts in Computer Science, Springer, 1996.

[4] Markus Wenzel, *Isar — A Generic Interpretative Approach to Readable Formal Proof Documents*, ***Theorem Proving in Higher Order Logics, 12th International Conference, TPHOLs'99, September, Proceedings*** (Nice, France), (Yves Bertot, Gilles Dowek, André Hirschowitz, Christine Paulin-Mohring and Laurent Théry, editors), vol. 1690, Lecture Notes in Computer Science, Springer, 1999, pp. 167–184.

[5] Jørgen Villadsen, Andreas Halkjær From and Anders Schlichtkrull, *Natural Deduction and the Isabelle Proof Assistant*, Proceedings 6th International Workshop on ***Theorem proving components for Educational software*** (Gothenburg, Sweden), (Pedro Quaresma and Walther Neuper, editors), vol. 267, Electronic Proceedings in Theoretical Computer Science, Open Publishing Association, 2018, pp. 140–155.
URL Address: http://eptcs.org/paper.cgi?ThEdu17.9.