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The Concurrent Calculi Formalisation Benchmark

at COORDINATION 2024

Introduction

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Realisation: mechanising concurrent systems is a big effort.

The Benchmark Approach

Mixing concurrent calculi with the POPLMark spirit!

We want to encourage:

- comparison of different approaches
- the development of guidelines, tutorials, techniques, libraries...
- reusability

`https://concurrentbenchmark.github.io/`

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Three fundamental challenges on concurrency and session types:

1. linearity and behavioural type systems
2. name passing and scope extrusion
3. coinduction and infinite processes

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Linearity and behavioural type systems

Processes:

$$
\begin{array}{rcl}
v, w & ::= & a \mid l \\
P, Q & ::= & \mathbf{0} \mid x!v.P \mid x?(l).P \mid (P \mid Q) \mid (\nu xy)\, P
\end{array}
$$

Linearity and behavioural type systems

Processes:

$$\begin{array}{rcllllll} v, w & ::= & a & \mid & l \\ P, Q & ::= & \mathbf{0} & \mid & x!v.P & \mid & x?(l).P & \mid & (P \mid Q) & \mid & (\nu xy)\ P \end{array}$$

Semantics:

R-Com

$$\frac{}{(\nu xy)\ (x!a.P \mid y?(l).Q \mid R) \rightarrow (\nu xy)\ (P \mid Q\{a/l\} \mid R)}$$

R-Res

$$\frac{P \rightarrow Q}{(\nu xy)\ P \rightarrow (\nu xy)\ Q}$$

R-Par

$$\frac{P \rightarrow Q}{P \mid R \rightarrow Q \mid R}$$

R-Struct

$$\frac{P \equiv P' \qquad P' \rightarrow Q' \qquad Q \equiv Q'}{P \rightarrow Q}$$

Linearity and behavioural type systems

1. No endpoint is used simultaneously by parallel processes.
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Types:

$$S, T \quad ::= \quad \mathbf{end} \quad | \quad \mathbf{base} \quad | \quad ?.S \quad | \quad !.S$$

$$\Gamma \quad ::= \quad \cdot \quad | \quad \Gamma, l \qquad \Delta \quad ::= \quad \cdot \quad | \quad \Delta, x : S$$

Typing rules:

$$\text{T-INACT}\ \frac{\mathbf{end}(\Delta)}{\Gamma; \Delta \vdash \mathbf{0}} \qquad \text{T-PAR}\ \frac{\Gamma; \Delta_1 \vdash P \qquad \Gamma; \Delta_2 \vdash Q}{\Gamma; \Delta_1, \Delta_2 \vdash P \mid Q} \qquad \text{T-RES}\ \frac{\Gamma; (\Delta, x : T, y : \overline{T} \vdash P)}{\Gamma \vdash (\nu xy)\ P}$$

$$\text{T-OUT}\ \frac{\Gamma \vdash_v v : \mathbf{base} \qquad \Gamma; \Delta, x : T \vdash P}{\Gamma; (\Delta, x : !.T) \vdash x!v.P} \qquad \text{T-IN}\ \frac{(\Gamma, l); (\Delta, x : T) \vdash P}{\Gamma; (\Delta, x : ?.T) \vdash x?(l).P}$$

Linearity and behavioural type systems

Challenge:

Theorem (Subject reduction)

If $\Gamma; \Delta \vdash P$ *and* $P \rightarrow Q$ *then* $\Gamma; \Delta \vdash Q$

Theorem (Type safety)

If $\Gamma; \cdot \vdash P$*, then* P *is well formed*

Name passing and scope extrusion

Processes:

$$P, Q := \mathbf{0} \quad | \quad (P \mid Q) \quad | \quad x!y.P \quad | \quad x?(y).P \quad | \quad (\nu x)\ P$$

One relevant example:

$$((\nu y)\ x!y.P) \mid (x?(z).Q)$$

Name passing and scope extrusion

First approach: structural congruence and reduction

$$((\nu y)\ x!y.P) \mid (x?(z).Q) \quad \equiv$$

$$(\nu y)\ (x!y.P \mid x?(z).Q) \quad \rightarrow$$

$$(\nu y)\ (P \mid Q\{y/z\})$$

Name passing and scope extrusion

Second approach: labelled transition system

$$\dfrac{\dfrac{x!y.P \xrightarrow{x!y} P \qquad x \neq y}{(\nu y)\ x!y.P \xrightarrow{x!(y)} P} \qquad x?(z).Q \xrightarrow{x?y} Q\{y/z\} \qquad y \notin \mathrm{fn}(Q)}{((\nu y)\ x!y.P) \mid (x?(z).Q) \xrightarrow{\tau} (\nu y)\ (P \mid Q\{y/z\})}$$

Open

$$\dfrac{P \xrightarrow{x!z} P' \qquad z \neq x}{(\nu z)\ P \xrightarrow{x!(z)} P'}$$

Close-L

$$\dfrac{P \xrightarrow{x!(z)} P' \qquad Q \xrightarrow{x?z} Q' \qquad z \notin \mathrm{fn}(Q)}{P \mid Q \xrightarrow{\tau} (\nu z)\ P' \mid Q'}$$

Name passing and scope extrusion

Challenge:

Theorem

$P \xrightarrow{\tau} Q$ implies $P \to Q$.

Theorem

$P \to Q$ implies the existence of a Q' such that $P \xrightarrow{\tau} Q'$ and $Q \equiv Q'$.

Coinduction and infinite processes

Describing the behaviour of recursive loops in programs.

$$v, w \quad ::= \quad a \mid l$$
$$P, Q \quad ::= \quad \mathbf{0} \mid x!v.P \mid x?(l).P \mid (P \mid Q) \mid (\nu x)\ P \mid\ !P$$

REP
$$\frac{P \xrightarrow{\alpha} P'}{!P \xrightarrow{\alpha} P' \mid !P}$$

$$\alpha \ ::= \ x!a \mid x?a \mid \tau$$

Coinduction and infinite processes

Observability predicate:

$$P \downarrow_{x?} \quad \text{if } P \text{ can perform an input action via } x.$$
$$P \downarrow_{x!} \quad \text{if } P \text{ can perform an output action via } x.$$

Strong barbed bisimilarity:

the *largest* symmetric relation such that, whenever $P \stackrel{\bullet}{\sim} Q$:

$$P \downarrow_{\mu} \text{ implies } Q \downarrow_{\mu} \quad (1)$$

$$P \xrightarrow{\tau} P' \text{ implies } Q \xrightarrow{\tau} \stackrel{\bullet}{\sim} P' \quad (2)$$

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NOT a congruence:

$$x!a.y!b.\mathbf{0} \quad \stackrel{\bullet}{\sim} \quad x!a.\mathbf{0}$$
$$x!a.y!b.\mathbf{0} \mid x?(l).\mathbf{0} \quad \stackrel{\bullet}{\not\sim} \quad x!a.\mathbf{0} \mid x?(l).\mathbf{0}$$

Coinduction and infinite processes

Strong barbed congruence:
$P \simeq^c Q$, if $C[P] \overset{\bullet}{\sim} C[Q]$ for every context C.

Lemma

$\simeq^c$ is the largest congruence included in $\overset{\bullet}{\sim}$.

Coinduction and infinite processes

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Lemma

$\simeq^c$ is the largest congruence included in $\stackrel{\bullet}{\sim}$.

Challenge:

Theorem

$P \simeq^c Q$ if, for any process R and substitution σ, $P\sigma \mid R \stackrel{\bullet}{\sim} Q\sigma \mid R$.

Was this tedious?

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A community effort towards:

- tutorial formalisations for different approaches
- comparing different approaches
- establishing “best practices”
- investigating strengths and weaknesses of proof assistants
- suggesting and developing new features of proof assistants

Why contribute and how to get involved

Why:

- solving your problems (and other people's)
- connecting different parts of the community
- conducting your own mechanisation
- publication, both experience reports/tutorials and novelties
- learn a new proof assistant with cool features

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The long and winding road

“How close are we to a world where every paper on programming languages is accompanied by an electronic appendix with machine-checked proofs?”

The long and winding road

"How close are we to a world where every paper on **concurrency** is accompanied by an electronic appendix with machine-checked proofs?"