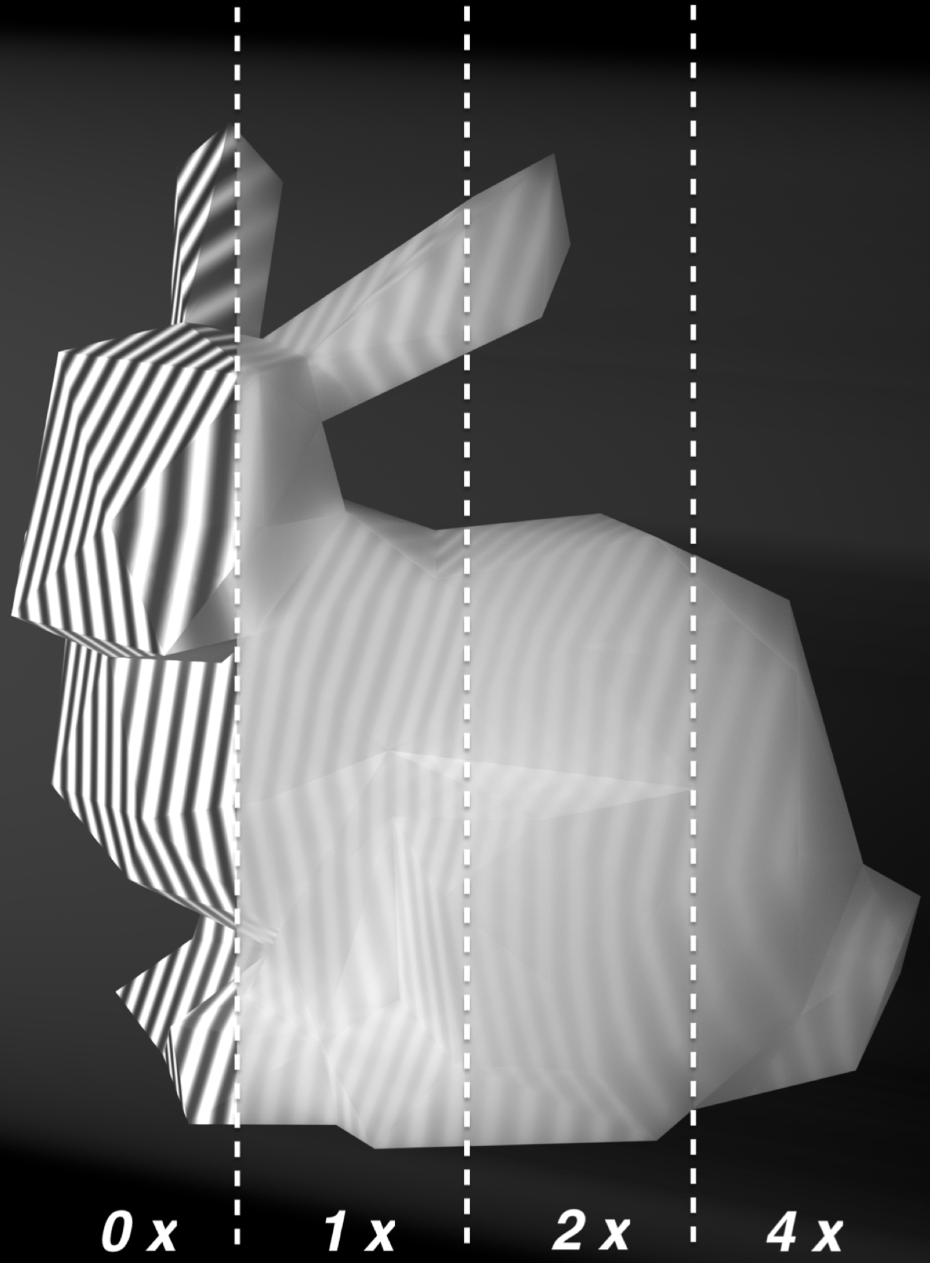
Subsurface scattering correction model

3DV tutorial session, September 2018

Søren K. S. Gregersen



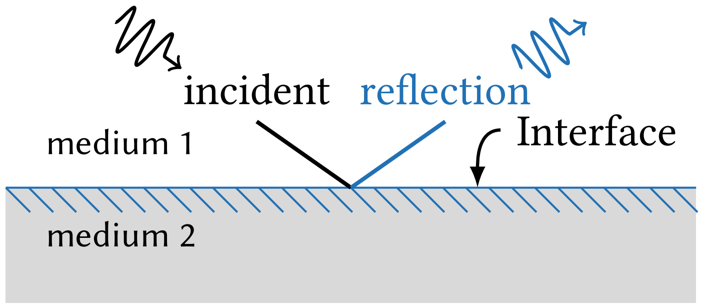
One hour agenda:

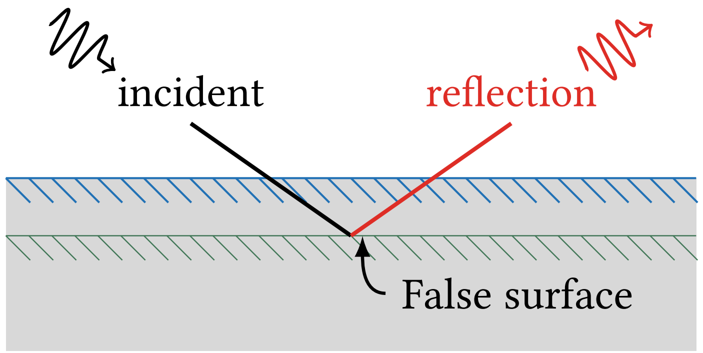
1. Is it possible to scan a translucent object?
2. Stereovision crash-course
3. **Subsurface scattering in structured light**

Ask questions any time!

Opaque and translucent object reflection

The fundamental difference is the **penetration of light**



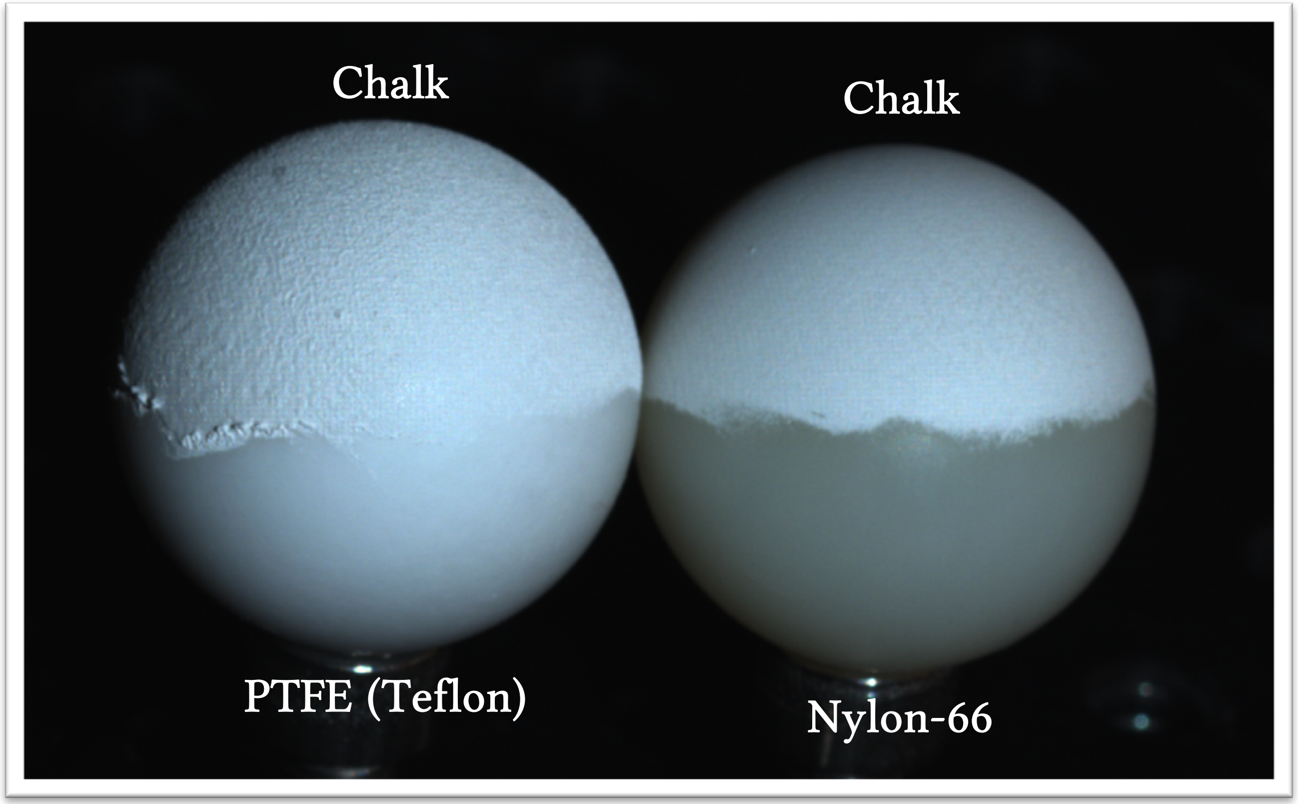


Many relevant translucent objects

Translucency is easily found for, for example, **plastics, marble, and skin**

Subjects of choice—plastic ball bearings

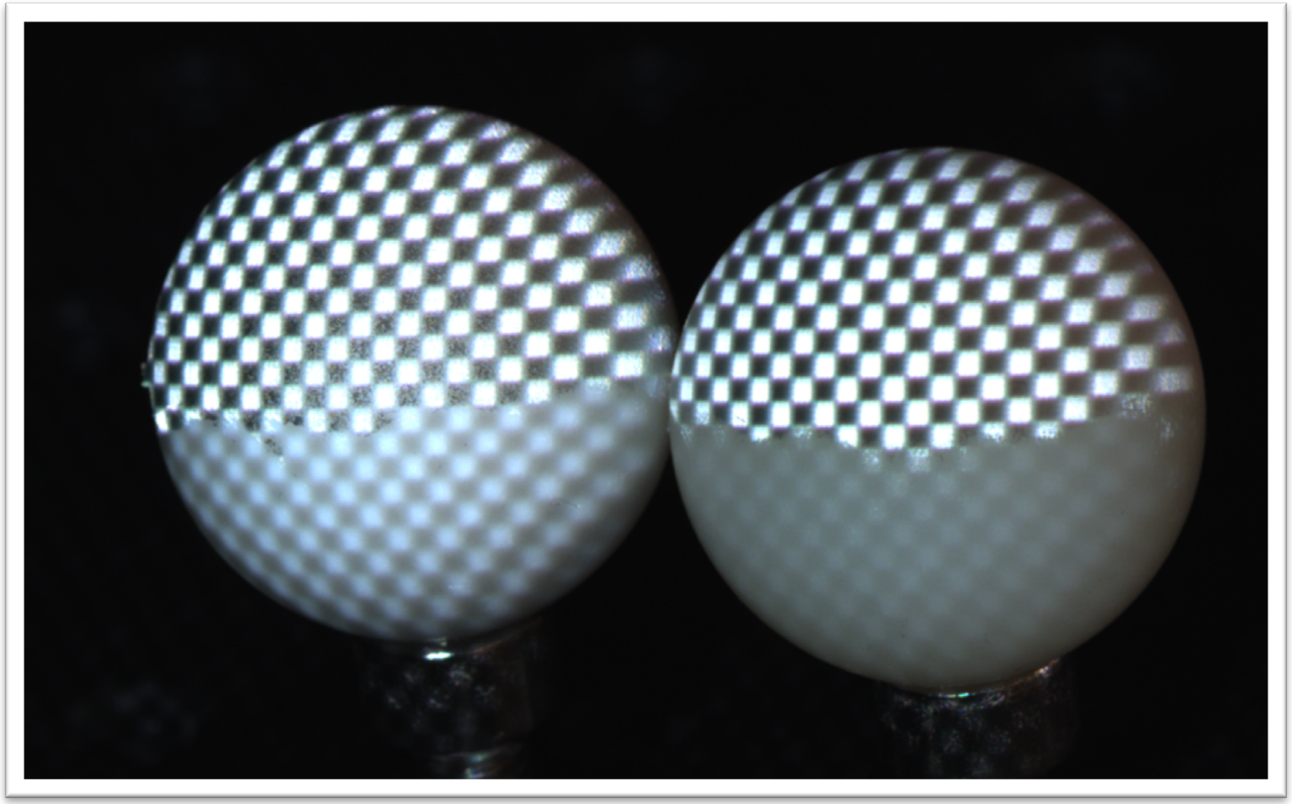
Teflon is somewhat opaque, while colorless **nylon** is translucent



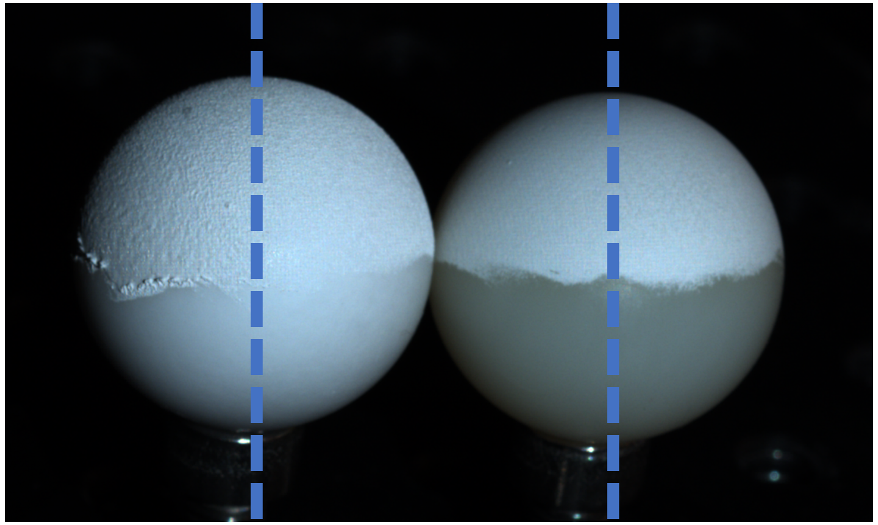
← Common reference point

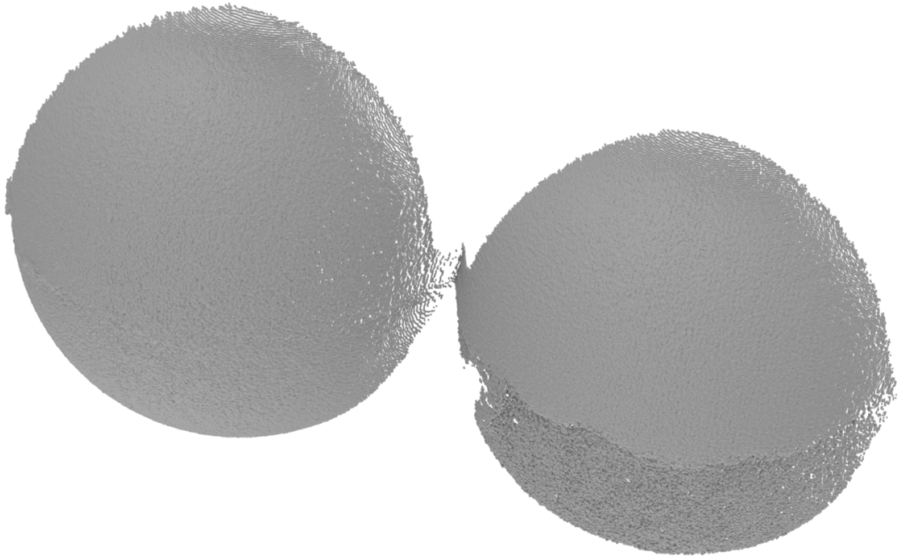
Projected patterns on translucent objects

Projected patterns reveal translucency... they become blurred and shifted!!

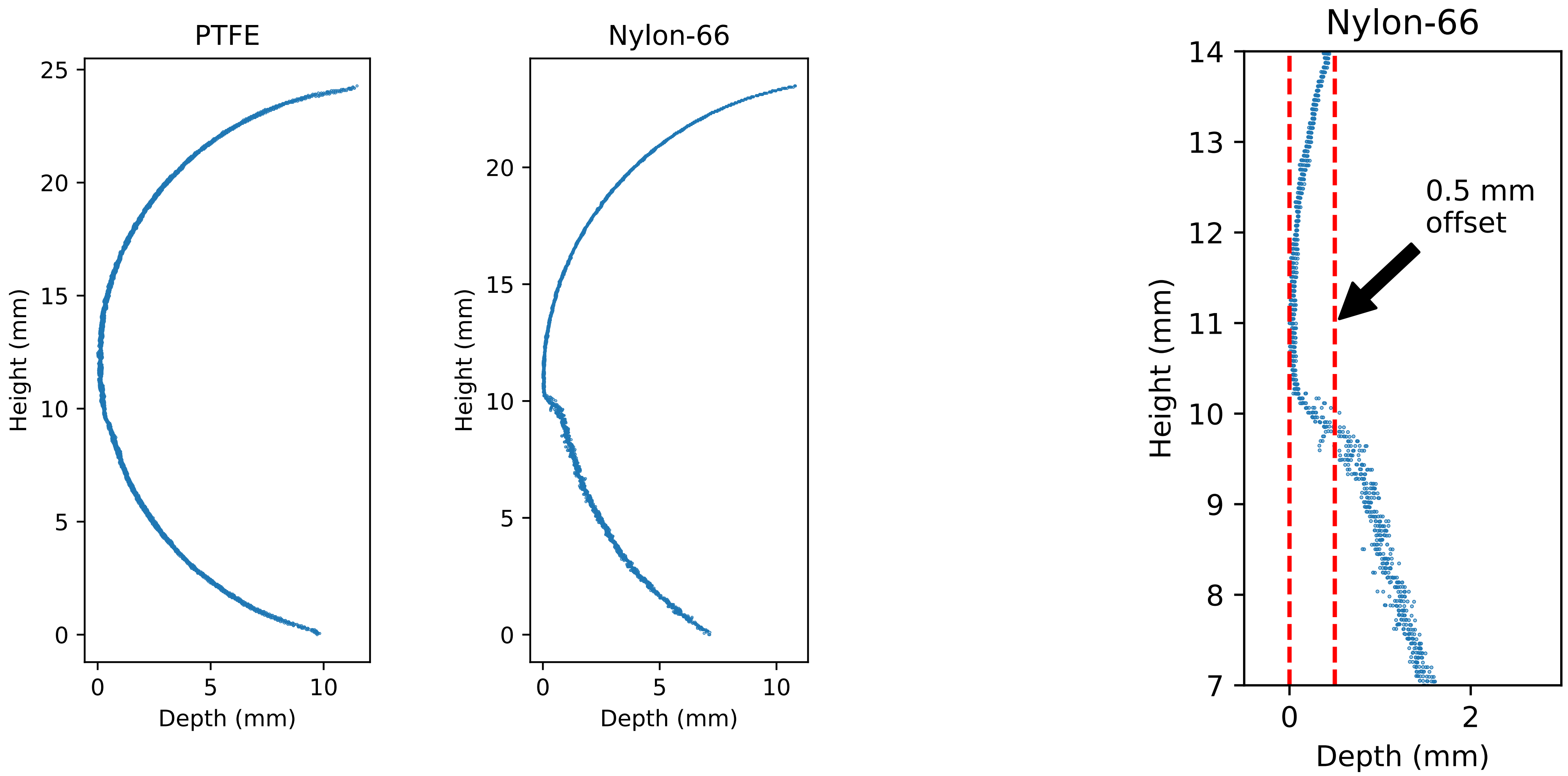


Point cloud reconstruction of ball bearings





Cross sections of ball bearings



Systematic errors in the geometric reconstruction

Stereovision crash-course

Stereo view and surface reconstruction

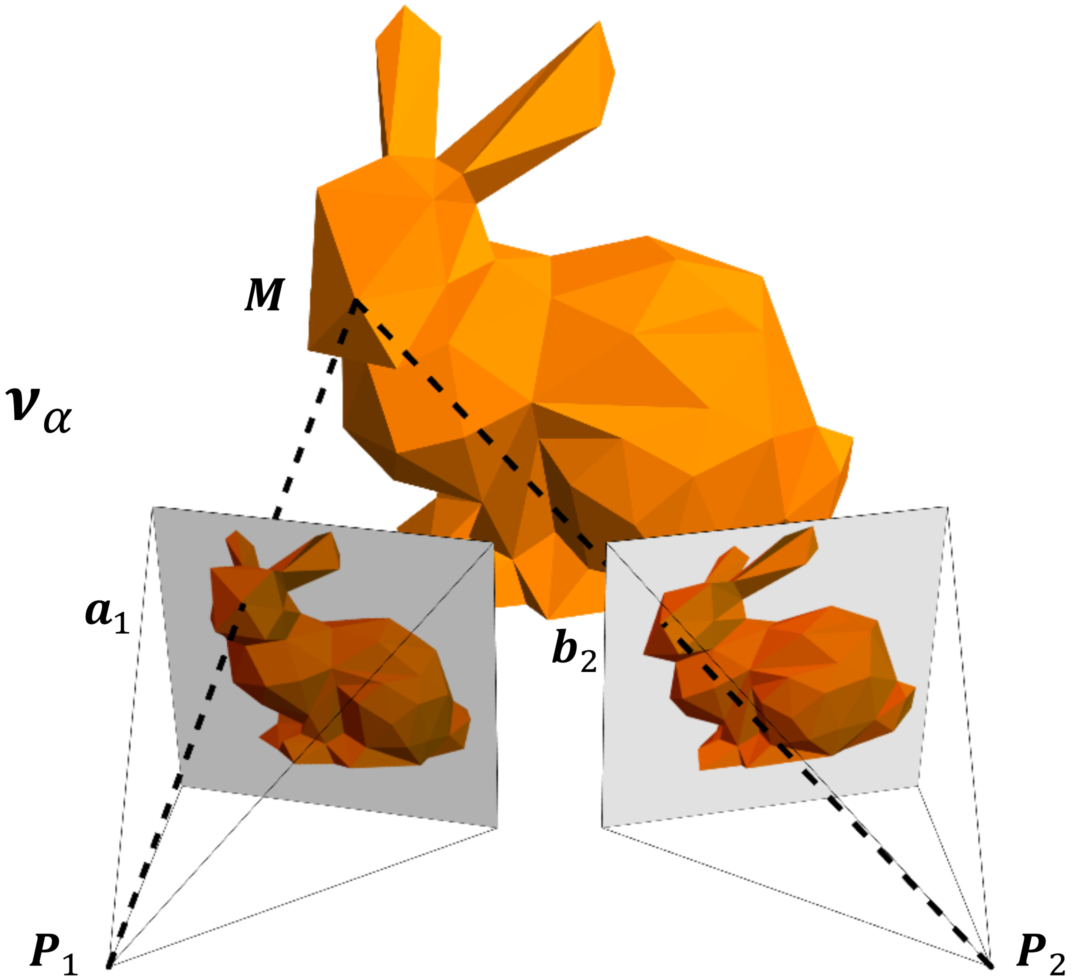
Pinhole camera model
Cameras $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$

Position $\boldsymbol{P}_\alpha$ Rotation $\boldsymbol{R}_\alpha$
Focal length f_α Distortion parameters $\boldsymbol{\nu}_\alpha$
... all learned through calibration

Given pixel set

$$\boldsymbol{a}_1 = \{i, j\}_1 \quad \boldsymbol{b}_2 = \{k, l\}_2,$$

we can locate intersection point $\boldsymbol{M}$
... also known as surface reconstruction!
Requires pixel correspondence!!

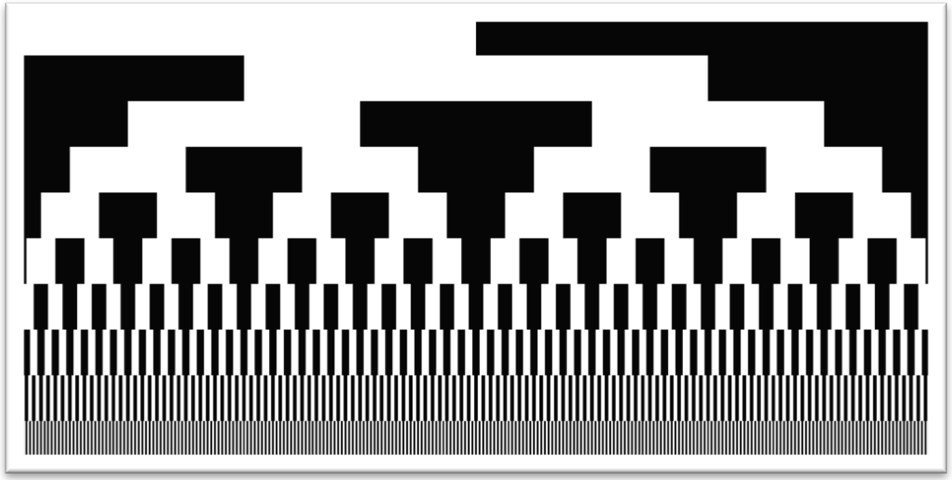


Structured light

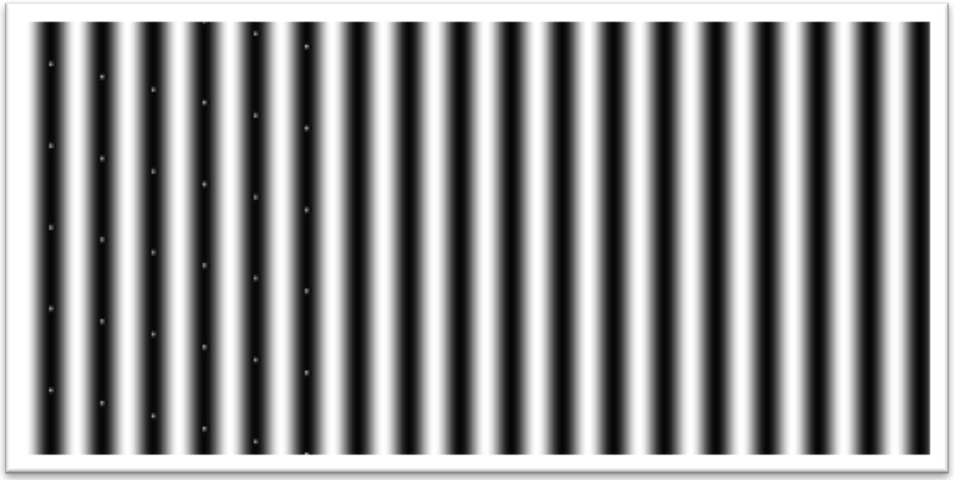
Active patterning enables full image correspondence

Structured light patterns

Two important patterns for "active texturing"



Gray code (binary)

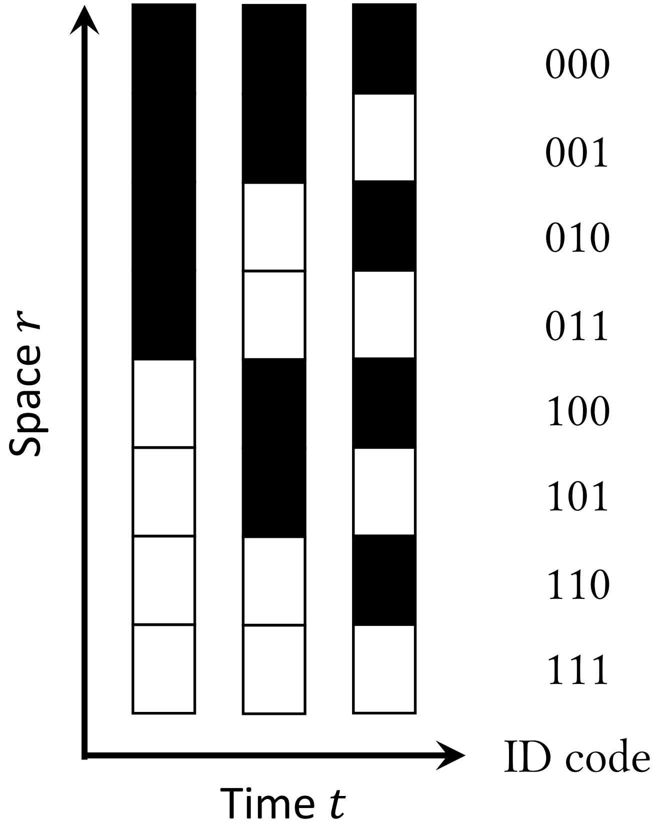


Phase shifting (continuous)

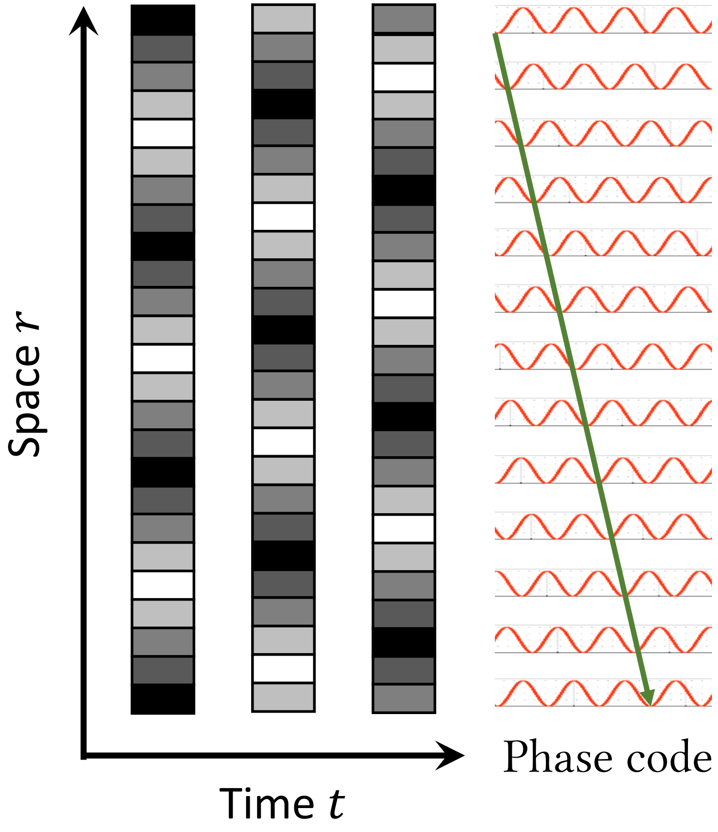
There exist many, many more!!

Pattern codes

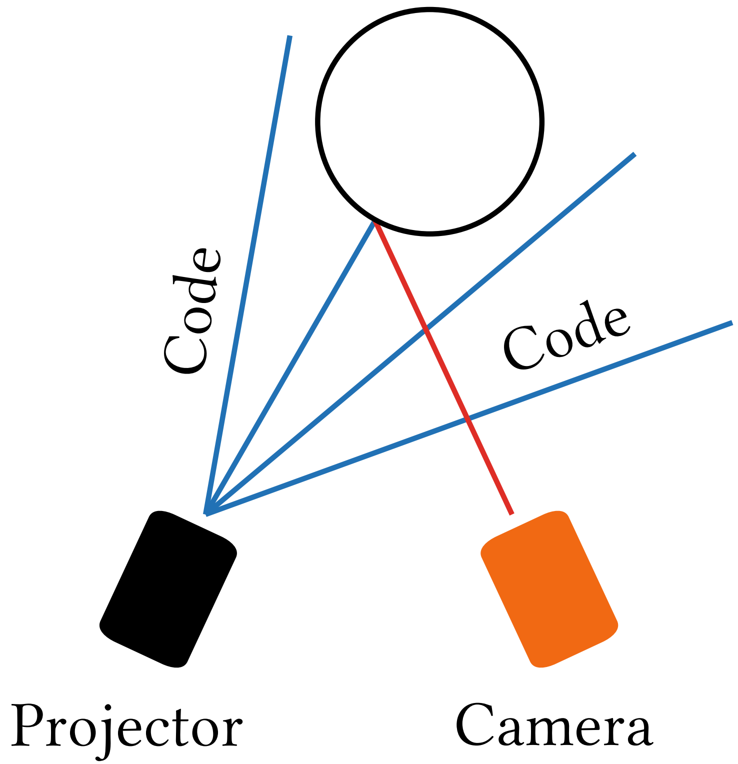
Gray code

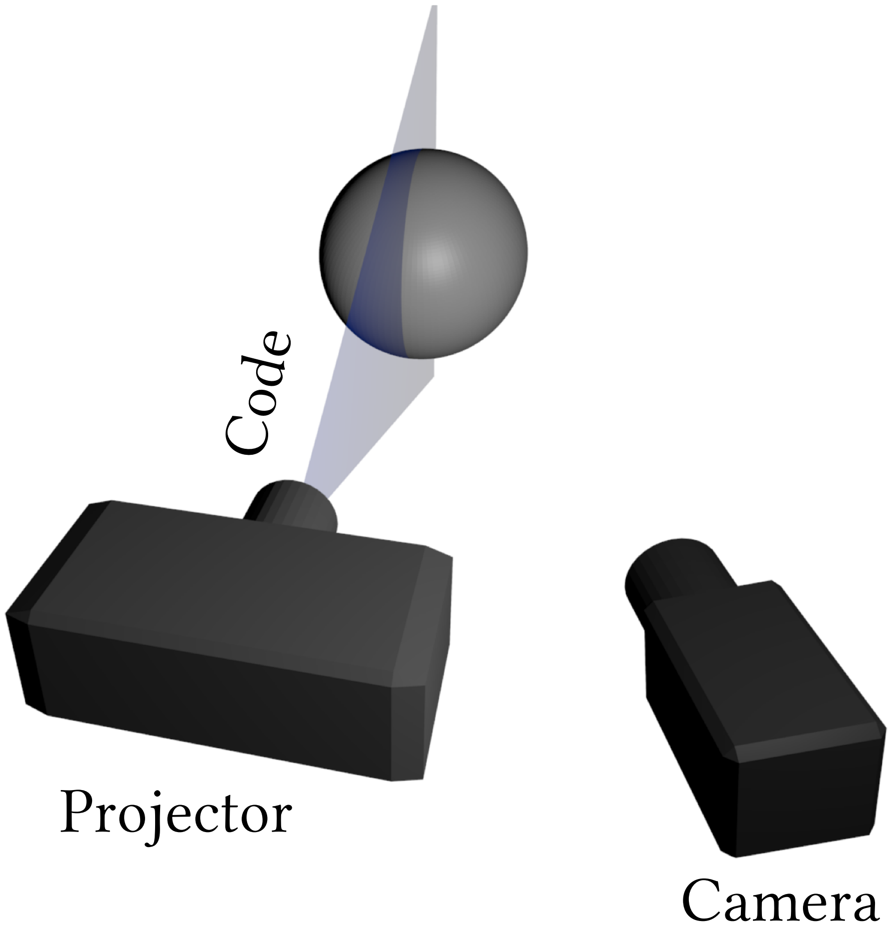


Phase shifting



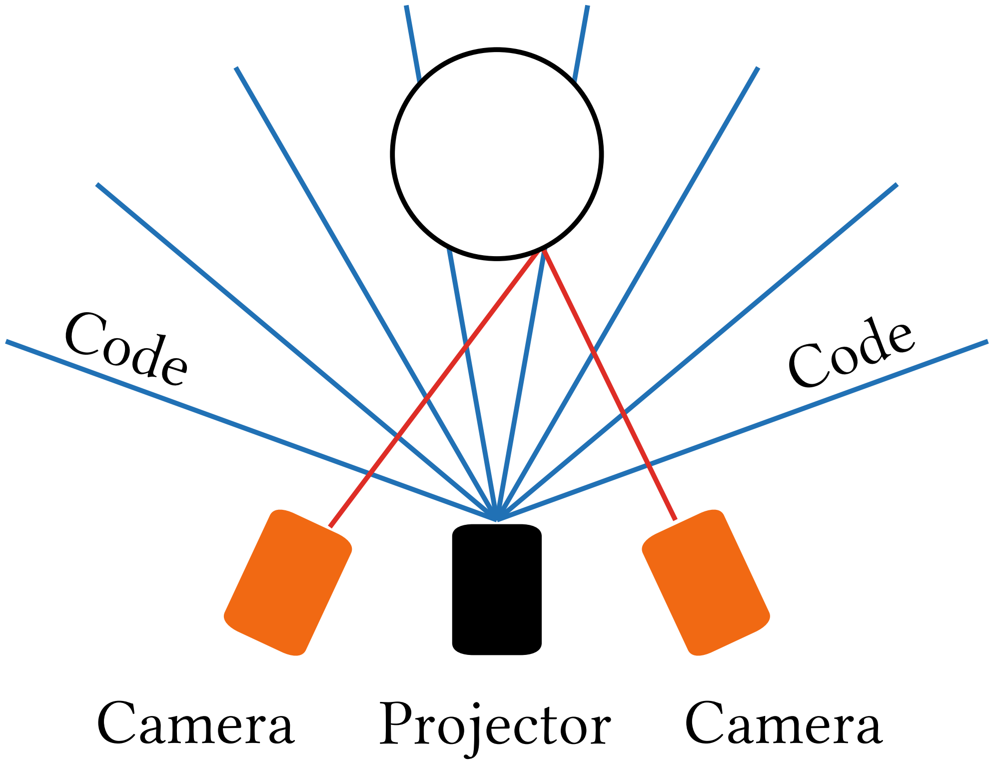
Single camera set up

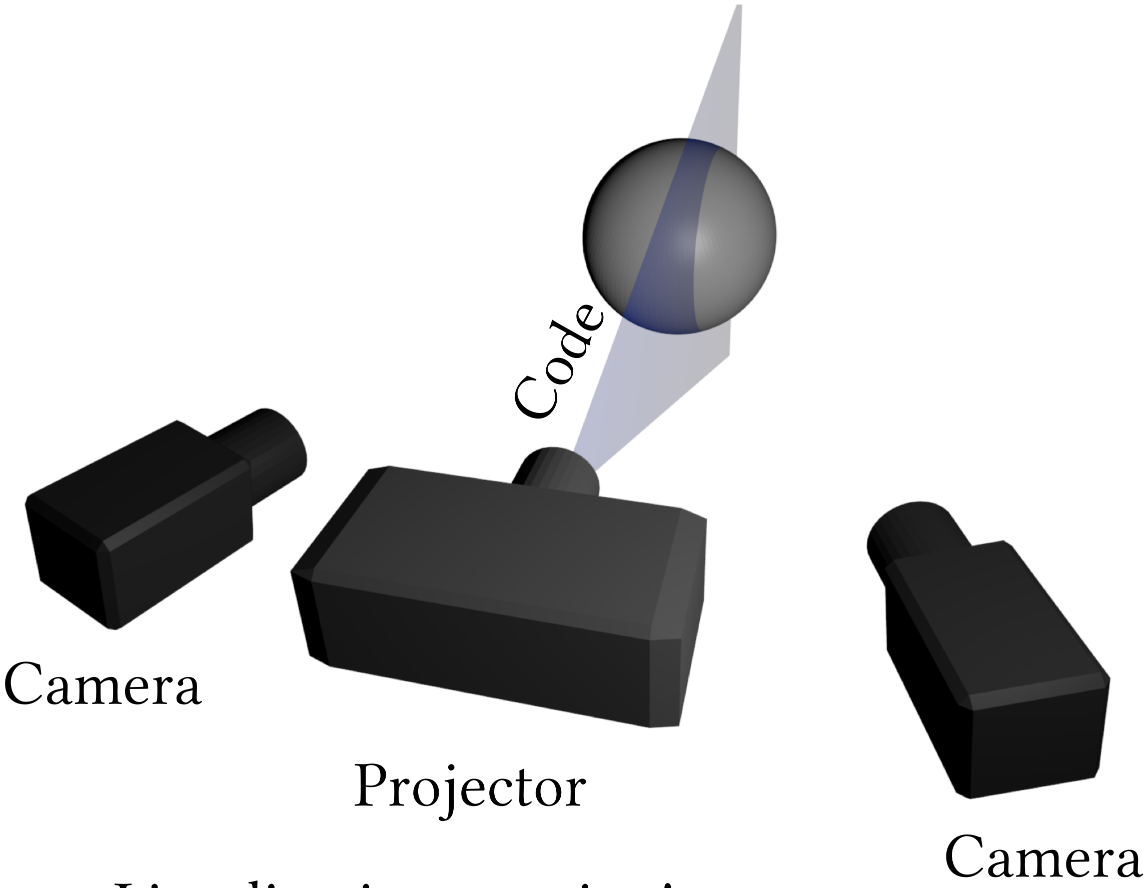




Plane-line intersection!

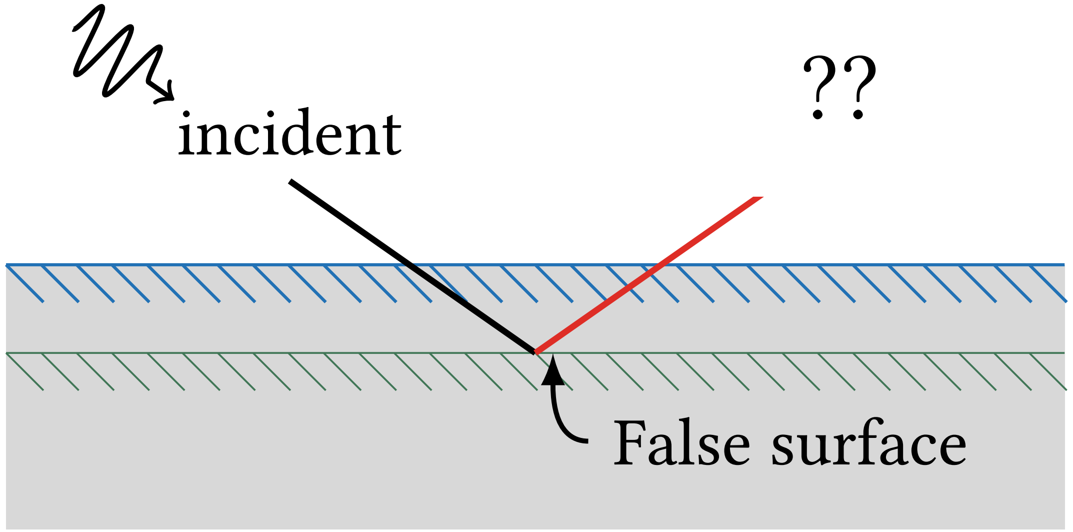
Dual camera set up



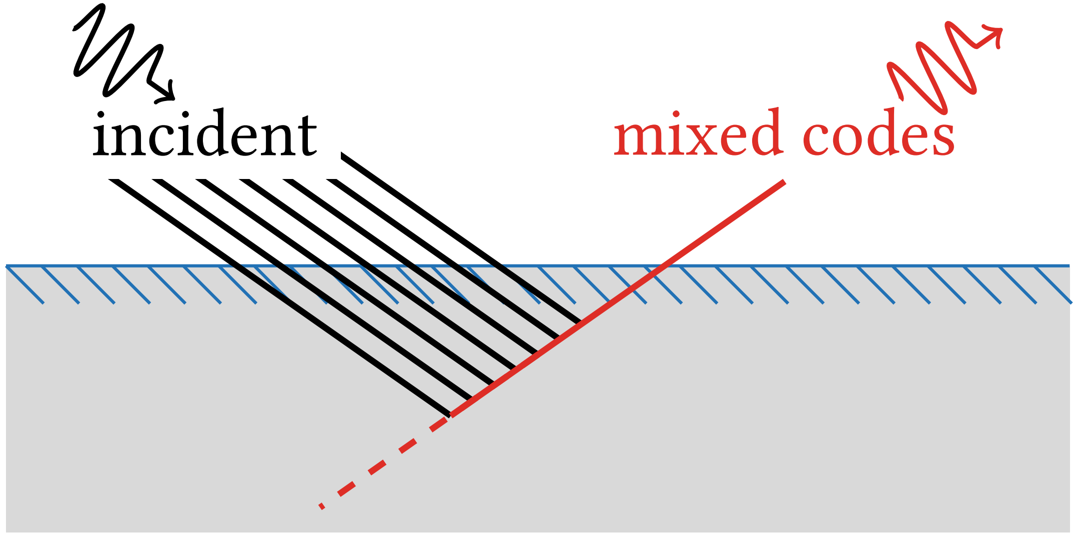


Line-line intersection!

Influence of translucency on projected codes



Short answer: Blur!



Subsurface scattering in structured light

The propagation of light

Direct reflection

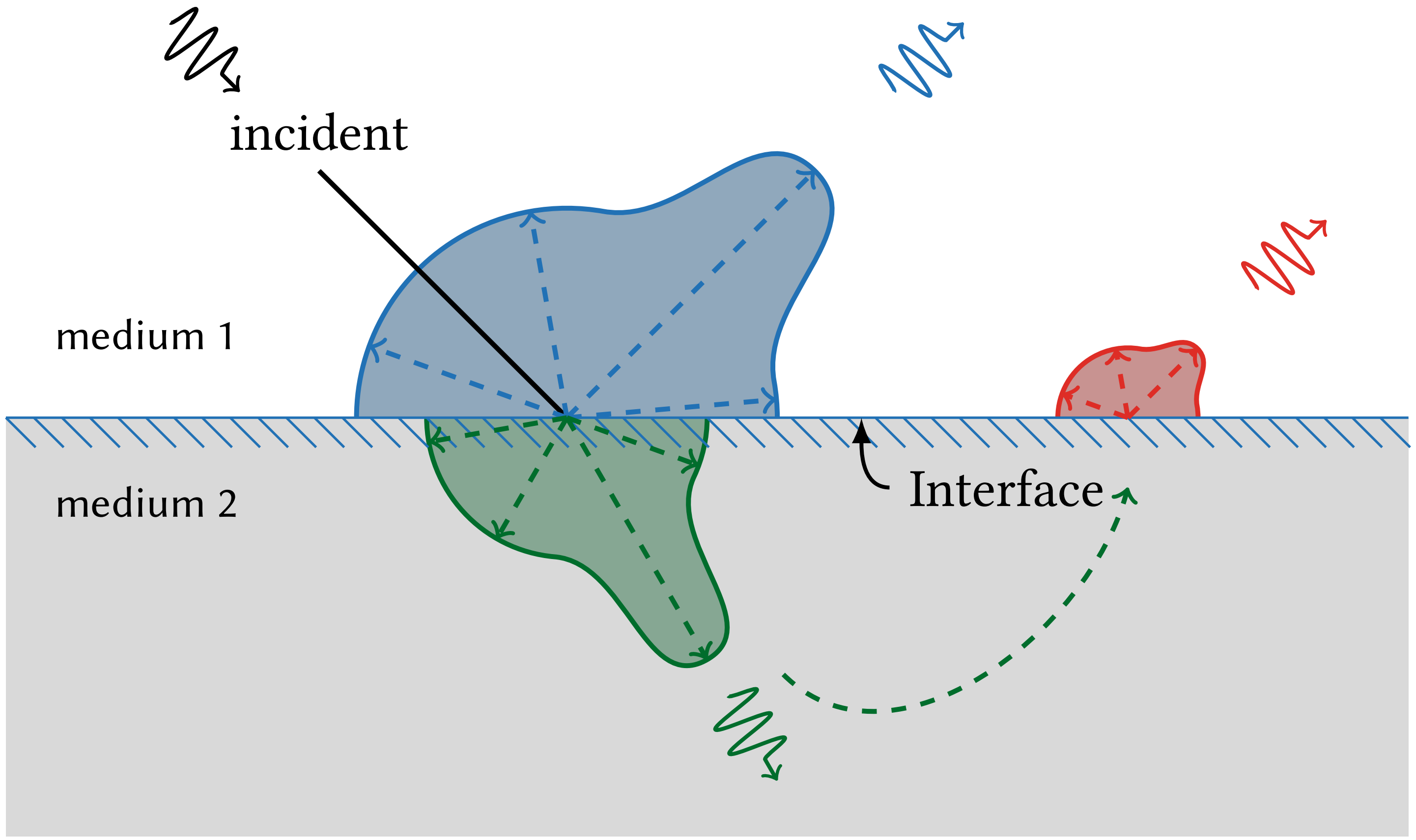
+ scattered reflection

Transmission

+ subsurface scattering

(refraction)

Nonlocal reflection



Total light intensity in a single camera pixel

Relation between incoming and out going light:

Position Direction Position Direction

$$L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \int_A \int_\Omega (\boldsymbol{n} \cdot \boldsymbol{\omega}_i) S(\boldsymbol{x}_i, \boldsymbol{\omega}_i, \boldsymbol{x}_o, \boldsymbol{\omega}_o) L_i(\boldsymbol{x}_i, \boldsymbol{\omega}_i)\ d\,\boldsymbol{\omega}_i\, d\,\boldsymbol{x}_i$$

Outgoing radiance Area All directions Incident radiance

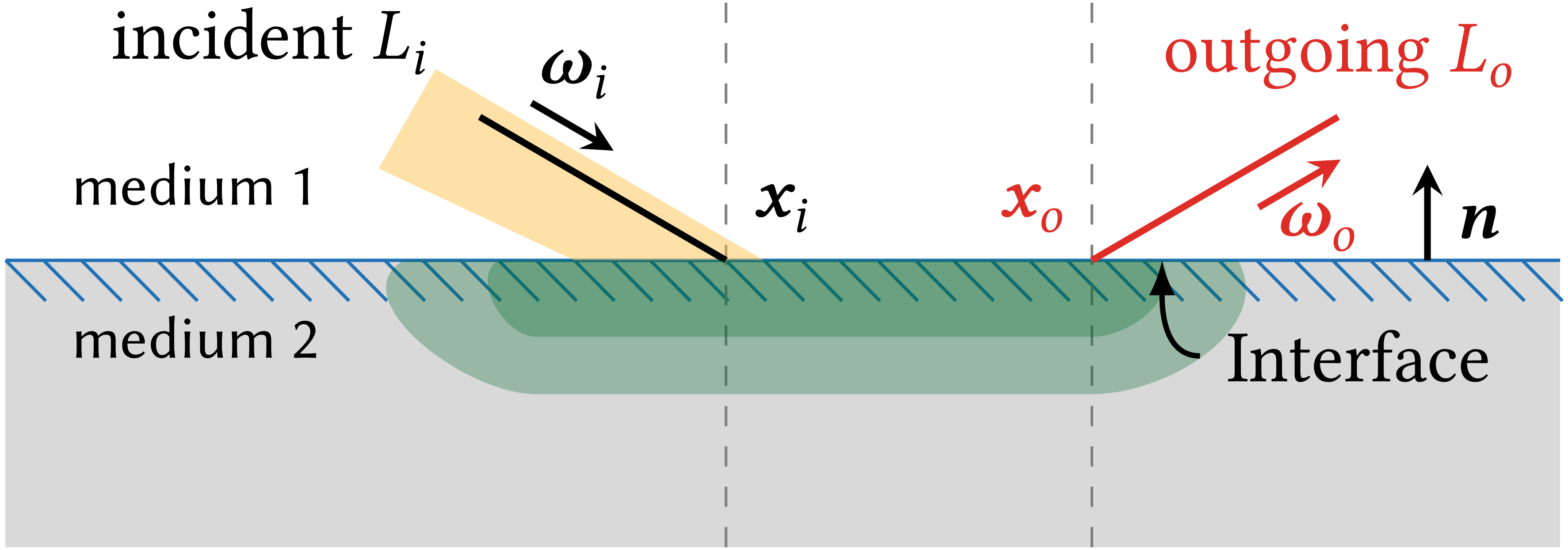
Bidirectional scattering-surface reflectance distribution function

BSSRDF: $S(\boldsymbol{x}_i, \boldsymbol{\omega}_i, \boldsymbol{x}_o, \boldsymbol{\omega}_o) \approx S_d + S_s$

Diffusion Single scattering

Jensen, H. W *et al.* (2001). *Proceedings SIGGRAPH '01* (pp. 511–518).

Diffusive BSSRDF



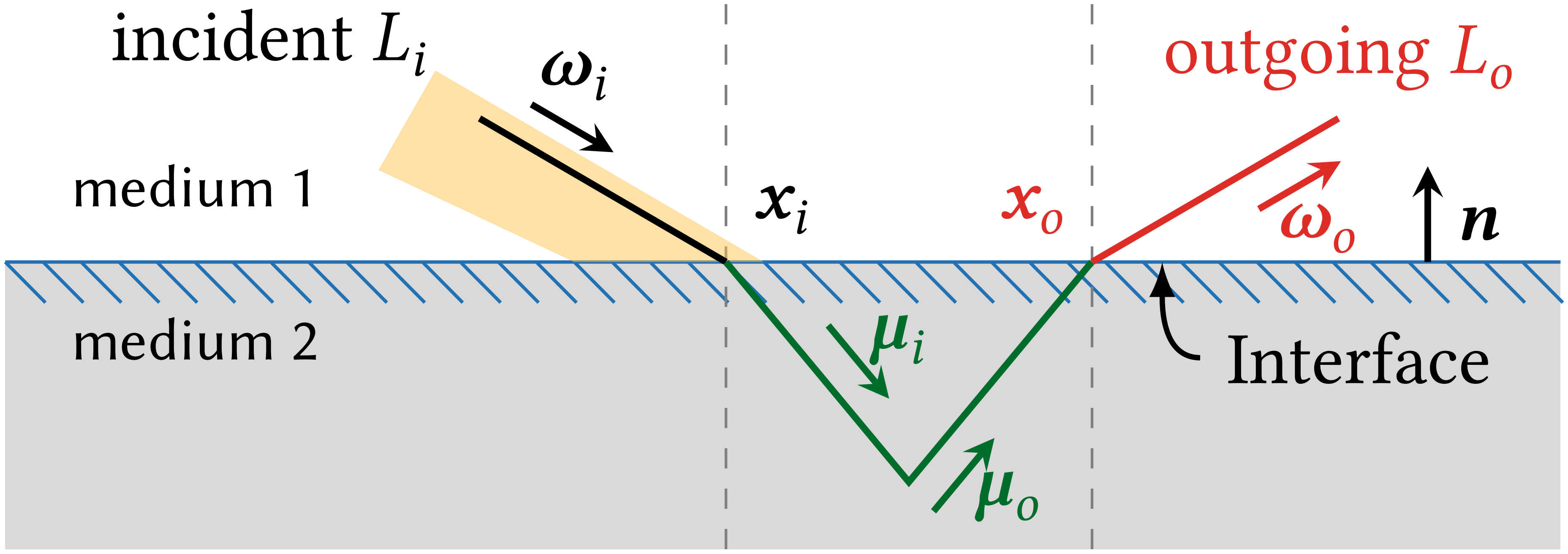
Jensen, H. W *et al.* (2001). *Proceedings SIGGRAPH '01* (pp. 511–518).

Diffusive BSSRDF

$$S_d = \frac{1}{\pi} \underbrace{F(\boldsymbol{\omega}_i)F(\boldsymbol{\omega}_o)}_{\text{Fresnel}} \underbrace{R_d(\boldsymbol{x}_i, \boldsymbol{x}_o)}_{\text{Diffusion}}$$

Jensen, H. W *et al.* (2001). *Proceedings SIGGRAPH '01* (pp. 511–518).

Single scattering BSSRDF



Jensen, H. W *et al.* (2001). *Proceedings SIGGRAPH '01* (pp. 511–518).

Single scattering BSSRDF

Mean free path

Extinction coefficient $\sigma_t = 1/\tau$

$$(\boldsymbol{n} \cdot \boldsymbol{\omega}_i)\, S_s = \sigma_s\, F(\boldsymbol{\omega}_i) F(\boldsymbol{\omega}_o) P(\boldsymbol{\mu}_i, \boldsymbol{\mu}_o) \mathrm{Exp}(-\sigma_t l)$$

Scattering Fresnel Phase Extinction Propagation length

Already assuming **homogeneous material**

Jensen, H. W *et al.* (2001). *Proceedings SIGGRAPH ’01* (pp. 511–518).

Single scattering limit

Assuming only single scattering

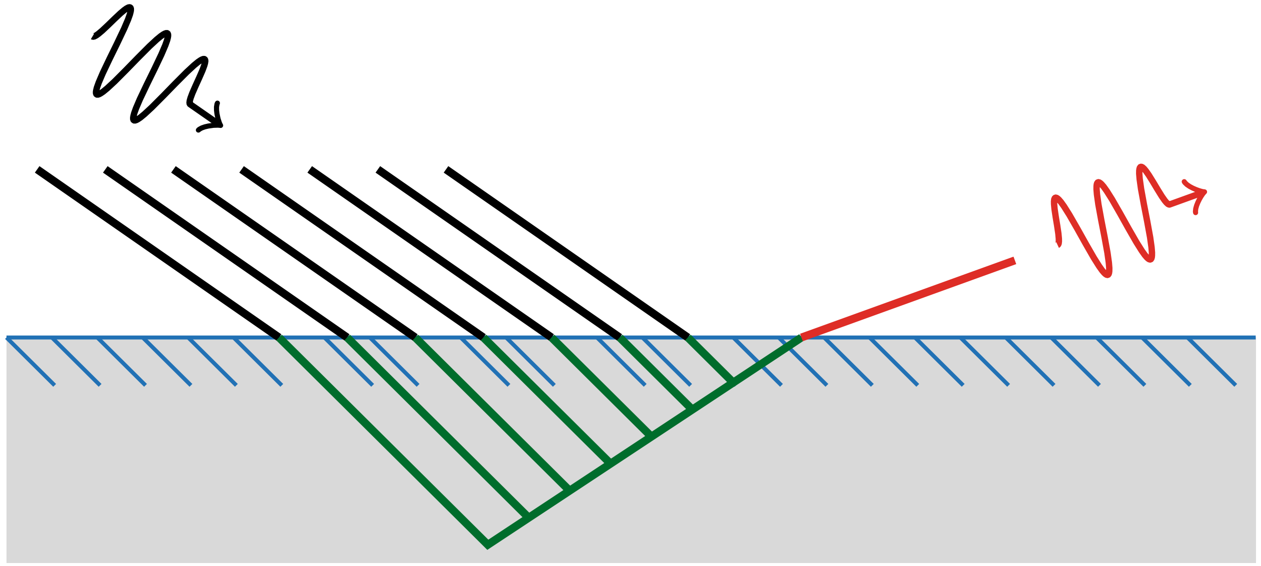
$$S(\boldsymbol{x}_i, \boldsymbol{\omega}_i, \boldsymbol{x}_o, \boldsymbol{\omega}_o) \rightarrow S_s$$

An approximate model, which can explain behavior – a heuristic technique

Enables analytic models later on!

Single scattering limit on an infinite plane

All **relevant** scattering happens along a single line—the camera pixel direction!



$$\int_A d\,\boldsymbol{x}_i \to \int_0^\infty d\,l$$

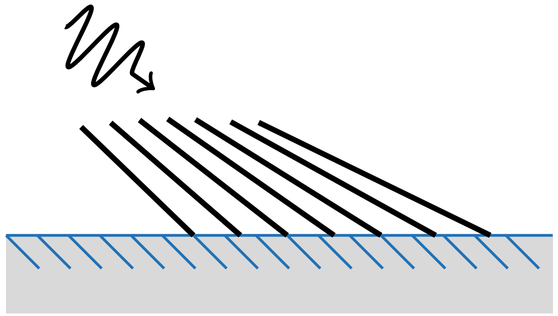
$$\boldsymbol{x}_i \to \boldsymbol{x}_i(l)$$

Local response – parallel rays

Assuming parallel projection and a single camera ray:

$$L_i[\boldsymbol{x}_i(l), \boldsymbol{\omega}_i] = \delta(\boldsymbol{p} - \boldsymbol{\omega}_i) L_i[\boldsymbol{x}_i(l)]$$

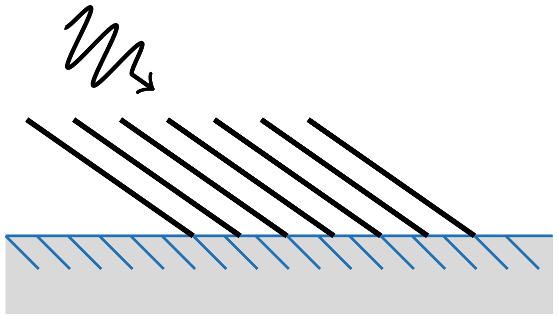
$$\boldsymbol{\omega_i} \to \boldsymbol{c}$$



The out going radiance now reads:

$$L_o(\boldsymbol{x}_o, \boldsymbol{c}) = \Sigma_\mathrm{s} \int_0^\infty \mathrm{Exp}(-\sigma_t l) \overset{\substack{\text{Pattern} \\ \downarrow}}{L_i[\boldsymbol{x}_i(l)]} d\, l$$

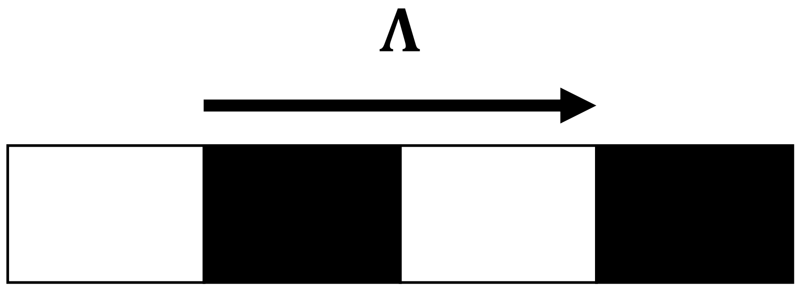
$$\Sigma_\mathrm{s} = \sigma_s F(\boldsymbol{p}, \boldsymbol{c}) P(\boldsymbol{p}', \boldsymbol{c}')$$



$\boldsymbol{p}'$ and $\boldsymbol{c}'$ are the refractive rays of, respectively, $\boldsymbol{p}$ and $\boldsymbol{c}$

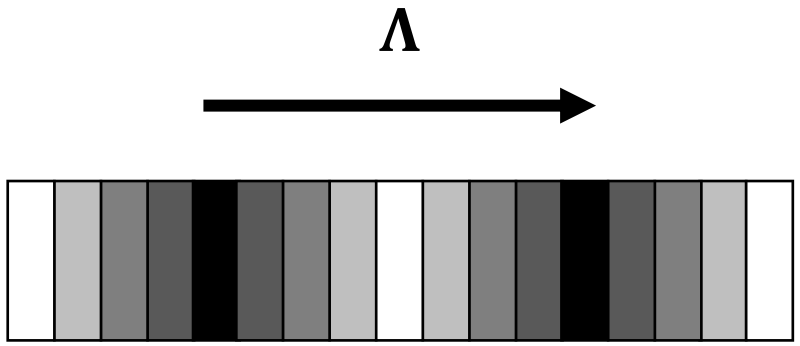
Incident patterns

Gray code: $L_i(\boldsymbol{x}_i) = \begin{cases} 0, & 0 \leq mod(2\,\boldsymbol{x}_i \cdot \boldsymbol{K}, 2) < 1 \\ L_i, & 1 \leq mod(2\,\boldsymbol{x}_i \cdot \boldsymbol{K}, 2) < 2 \end{cases}$



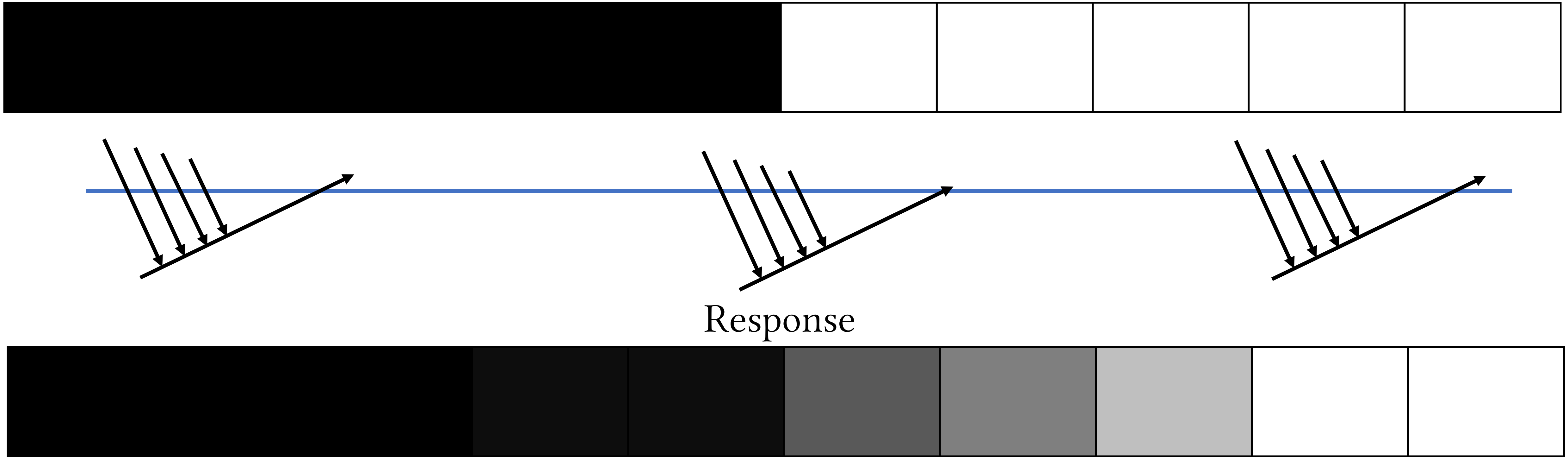
$\boldsymbol{K} = \frac{\boldsymbol{\Lambda}}{|\boldsymbol{\Lambda}|^2}$, where $\boldsymbol{\Lambda}$ is the periodic vector

Phase shifting: $L_i(\boldsymbol{x}_i) = L_i\left[\frac{1}{2} + \frac{1}{2}\cos(\boldsymbol{x}_i \cdot \boldsymbol{K})\right]$



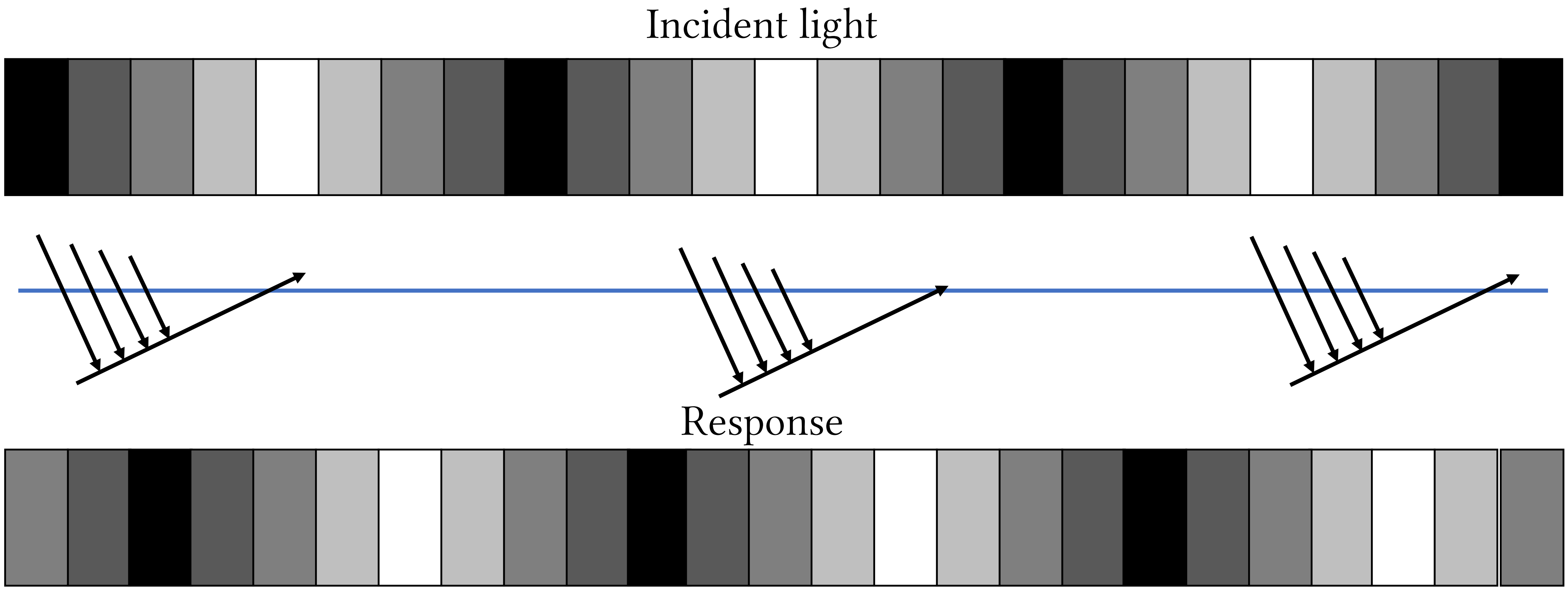
Hand waving derivation – gray code

Incident light



Blurred edges!!

Hand waving derivation – phase shifting



Sum of waves = shifted wave!!

Back to derivation – light path

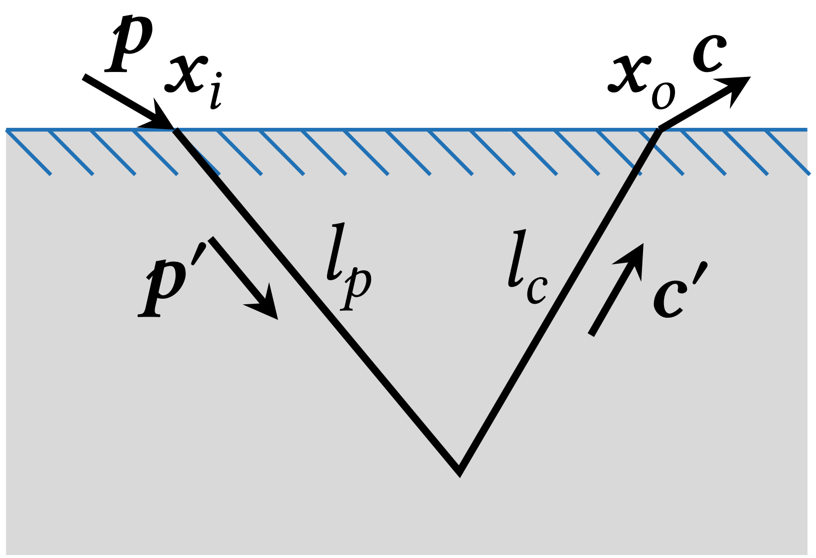
The path from incident to out going is straightforward:

$$\boldsymbol{x}_o = \boldsymbol{x}_i + \boldsymbol{p}' l_p + \boldsymbol{c}' l_c$$

$$\boldsymbol{x}_i = \boldsymbol{x}_o - \boldsymbol{p}' l_p - \boldsymbol{c}' l_c$$

$$(-\boldsymbol{p}' \cdot \boldsymbol{n}) l_p = (\boldsymbol{c}' \cdot \boldsymbol{n}) l_c$$

$$l = l_p + l_c$$



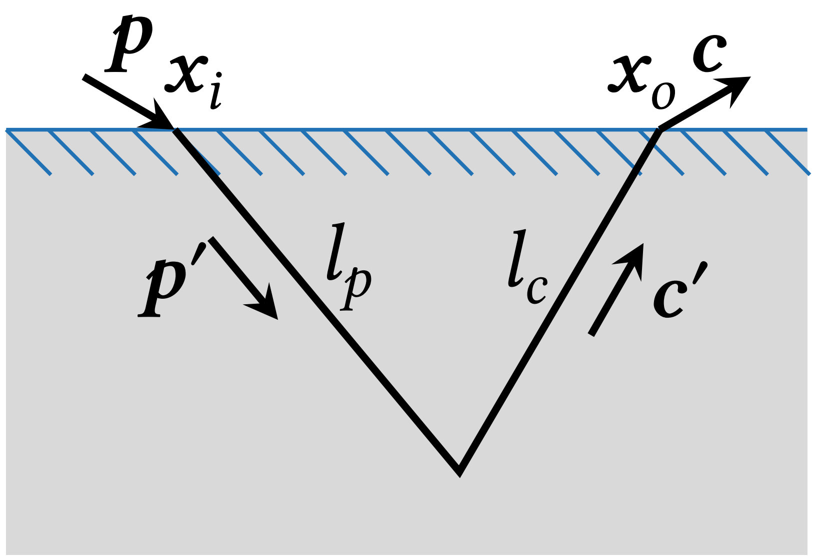
Back to derivation – light path

The path from incident to out going is straightforward:

$$l_p = \frac{-\boldsymbol{c}' \cdot \boldsymbol{n}}{\boldsymbol{p}' \cdot \boldsymbol{n} - \boldsymbol{c}' \cdot \boldsymbol{n}}\, l$$

$$l_c = \frac{\boldsymbol{p}' \cdot \boldsymbol{n}}{\boldsymbol{p}' \cdot \boldsymbol{n} - \boldsymbol{c}' \cdot \boldsymbol{n}}\, l$$

$$\boldsymbol{x}_o = \boldsymbol{x}_i + \frac{\boldsymbol{p}'(\boldsymbol{c}' \cdot \boldsymbol{n}) - \boldsymbol{c}'(\boldsymbol{p}' \cdot \boldsymbol{n})}{\boldsymbol{p}' \cdot \boldsymbol{n} - \boldsymbol{c}' \cdot \boldsymbol{n}}\, l$$



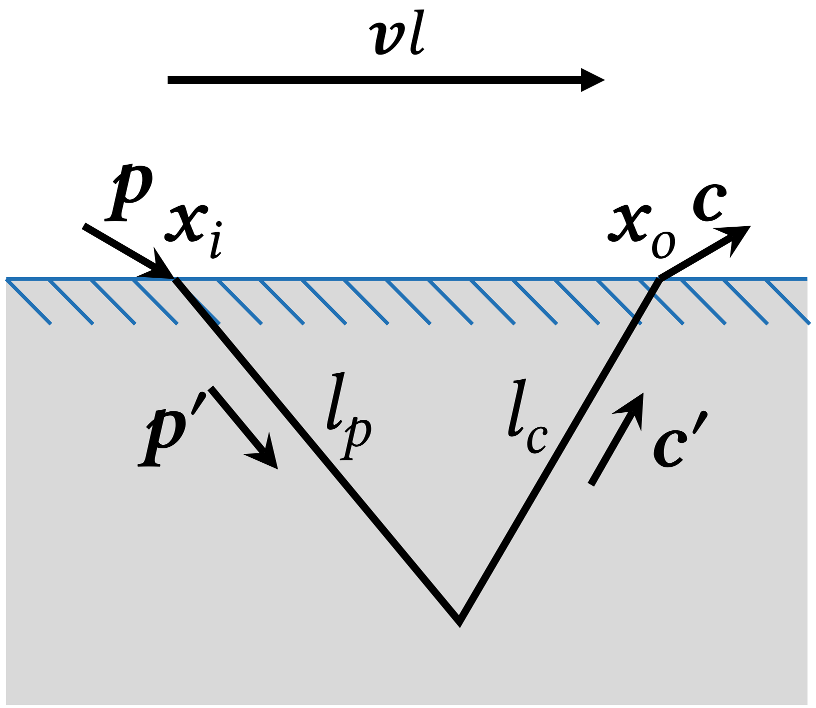
Back to derivation – light path

The path from incident to out going is straightforward:

$$\boldsymbol{v} = \frac{\boldsymbol{p}'(\boldsymbol{c}' \cdot \boldsymbol{n}) - \boldsymbol{c}'(\boldsymbol{p}' \cdot \boldsymbol{n})}{\boldsymbol{p}' \cdot \boldsymbol{n} - \boldsymbol{c}' \cdot \boldsymbol{n}}$$

$$\boldsymbol{x}_o = \boldsymbol{x}_i + \boldsymbol{v}\, l$$

$\boldsymbol{v}$: surface path projection



Light intensity for gray code

Assuming large period $\sigma_t \Lambda \gg 1$, then $L_i \rightarrow \Theta(\dots)$

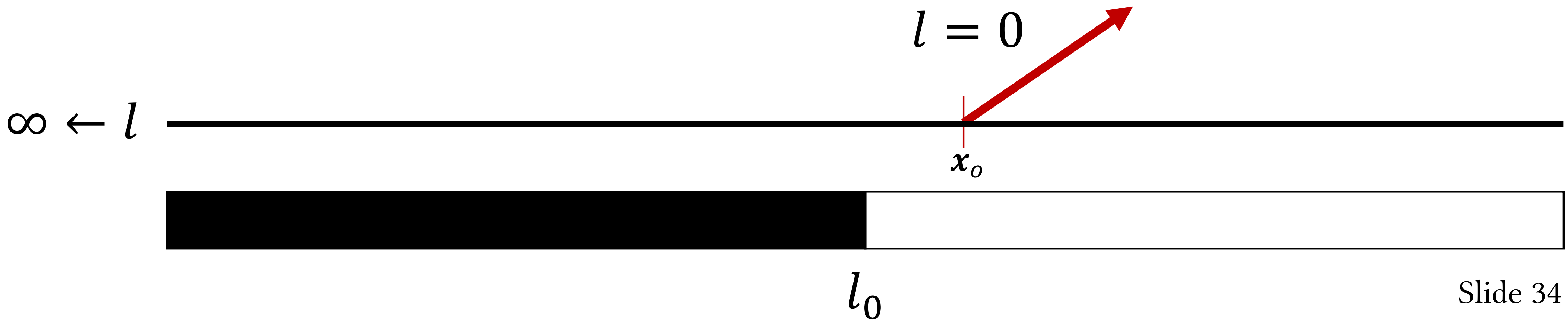
$$L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s L_i \int_a^b \mathrm{Exp}(-\sigma_t l) d\, l$$

$$a = b = 0 \implies L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = 0$$

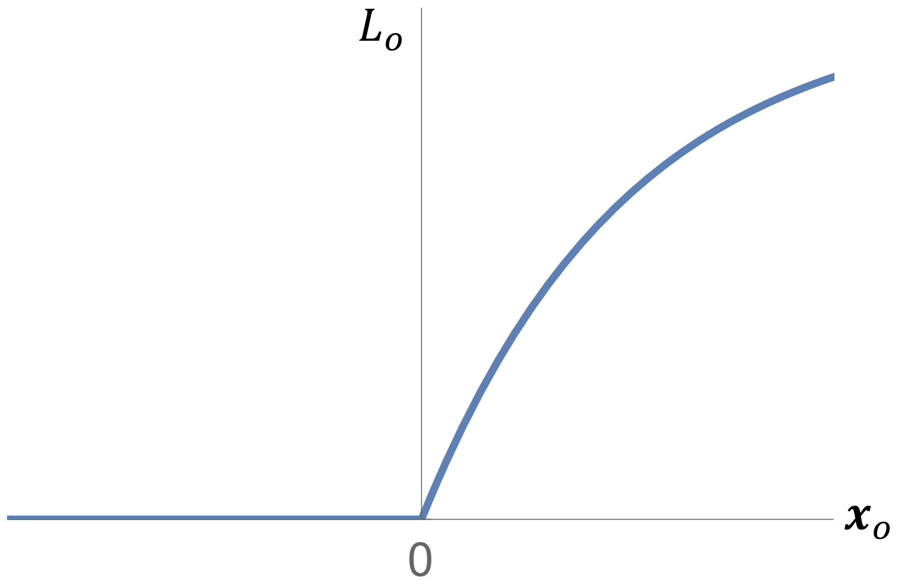
Light intensity for gray code

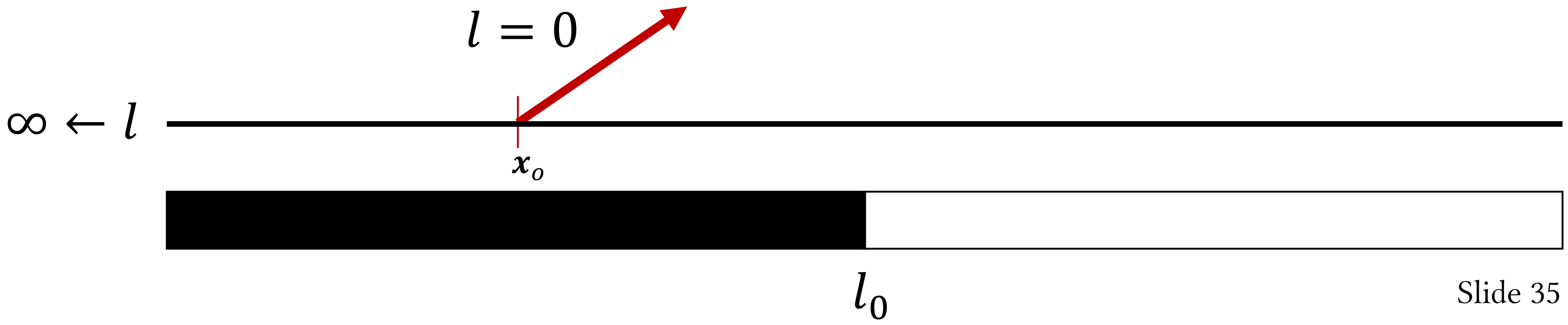
$$L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s L_i \int_a^b \mathrm{Exp}(-\sigma_t l) d\, l$$

$$a = 0 \wedge b = l_0 \implies L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s L_i \frac{1 - e^{-\sigma_t l_0}}{\sigma_t}$$



Light intensity for gray code

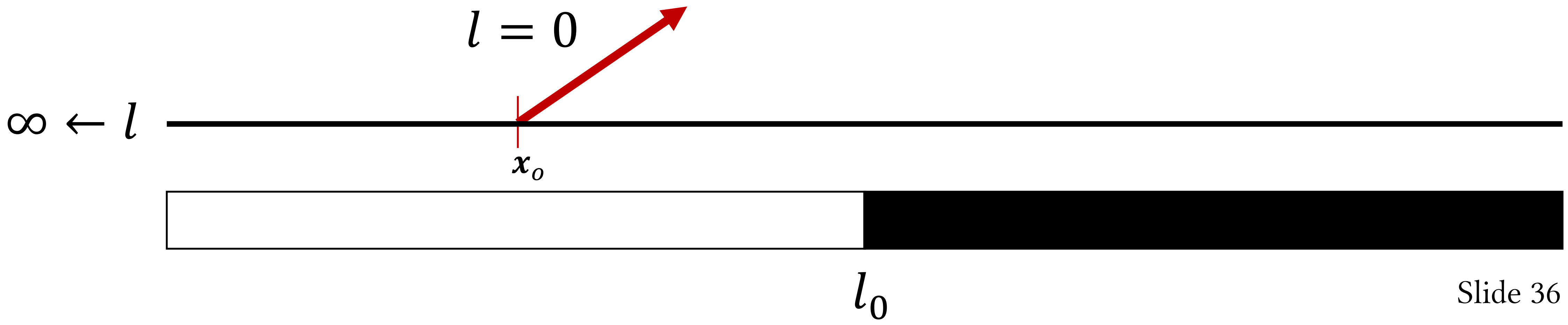




Light intensity for gray code

$$L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s L_i \int_a^b \mathrm{Exp}(-\sigma_t l) d\, l$$

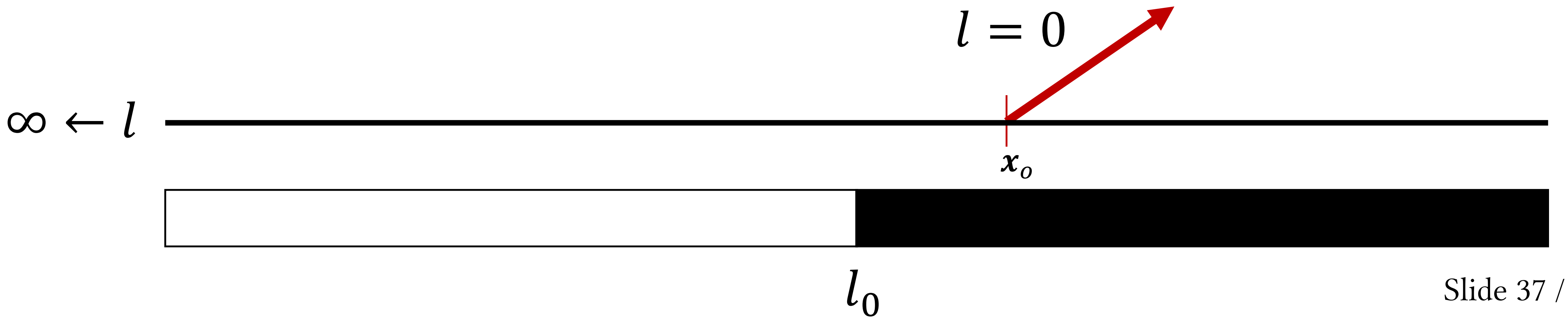
$$a = 0 \wedge b = \infty \implies L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s L_i \frac{1}{\sigma_t}$$



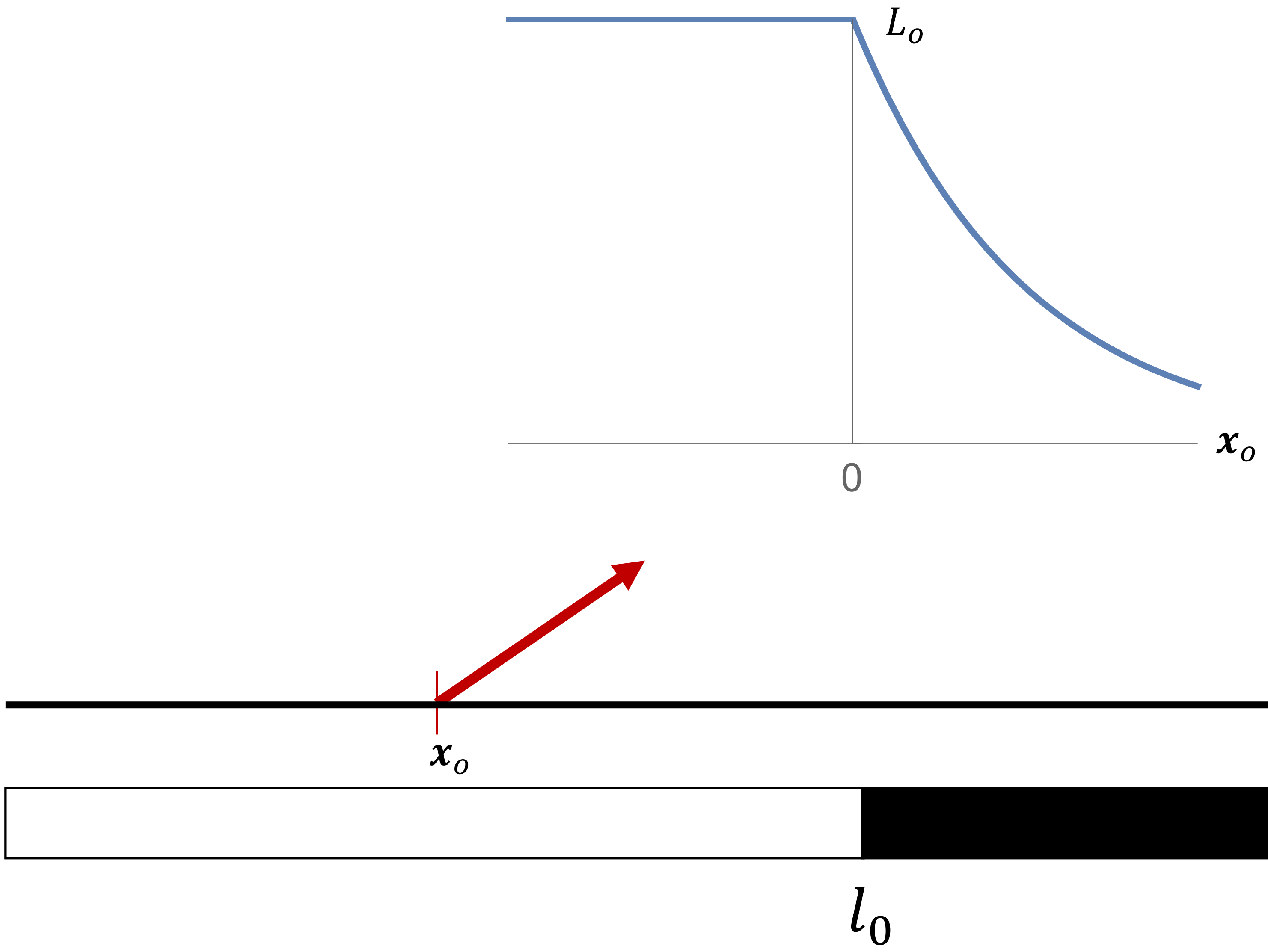
Light intensity for gray code

$$L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s L_i \int_a^b \mathrm{Exp}(-\sigma_t l) d\, l$$

$$a = l_0 \wedge b = \infty \implies L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s L_i \frac{e^{-\sigma_t l_0}}{\sigma_t}$$



Light intensity for gray code



Gray code summery

$\sigma_t \Lambda \gg 1$, then $L_i \to \Theta(\dots)$

- Exponential decay near boundaries: $\mathrm{Exp}(-\sigma_t l)$
- Blurred edges!

If $\sigma_t \Lambda \lesssim 1$

- Wavelike pattern instead… very blurry image!!

- Smaller Λ gives higher resolution
- Smaller Λ gives worse signal

Light intensity for phase shifting

$$L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = \Sigma_s \int_a^b \mathrm{Exp}(-\sigma_t l) L_i \left(\frac{1}{2} + \frac{1}{2}\cos[(\boldsymbol{x}_i + \boldsymbol{v}\, l) \cdot \boldsymbol{K}]\right) d\, l$$

$$L_o(\boldsymbol{x}_o, \boldsymbol{\omega}_o) = C \left[\frac{1}{2} + \underbrace{\frac{1}{2A}}_{\text{Blurring}} \cos(\theta_0 + \underbrace{\Delta\theta}_{\text{Shift}})\right]$$

where

$$C = \frac{\Sigma_s L_i}{\sigma_t} \quad \text{and} \quad A = \sqrt{\frac{(\boldsymbol{v} \cdot \boldsymbol{K})^2}{\sigma_t^2} + 1}$$

$$\theta_0 = \boldsymbol{x}_i \cdot \boldsymbol{K} \quad \text{and} \quad \Delta\theta = \arctan\left(\frac{\boldsymbol{v} \cdot \boldsymbol{K}}{\sigma_t}\right)$$

Light intensity for phase shifting

Phase shift: $\Delta\theta = \arctan\left(\frac{\boldsymbol{v}\cdot\boldsymbol{K}}{\sigma_t}\right)$

- Bounded between $-\frac{\pi}{2} \leq \Delta\theta \leq \frac{\pi}{2}$
- I.e. lateral error bounded between $-\frac{\Lambda}{2} \leq Error \leq \frac{\Lambda}{2}$
- Smaller Λ gives smaller errors!!

Blurriness: $\mathrm{A} = \sqrt{\frac{(\boldsymbol{v}\cdot\boldsymbol{K})^2}{\sigma_t^2} + 1}$

- $\mathrm{A} \to \infty$ as $\Lambda \to 0$
- I.e. Smaller Λ gives worse signal

From mathematical model to “real life”

Apparent material parameters:

- Extinction coefficient $\sigma_t = 1/\tau$
    - τ is the propagation length of light
- Refraction index η, which determines $\boldsymbol{v}$

Geometric parameters:

- Projector and camera rays $\boldsymbol{p}$ and $\boldsymbol{c}$ ⟵ Learned from calibration
- Surface normals $\boldsymbol{n}$ ⟵ Learned from reconstructed surface
- Periodic vectors $\boldsymbol{K}$ ⟵ Learned from calibration

  ↖ Learned from scan gradients with reconstructed surface

Modeling summery

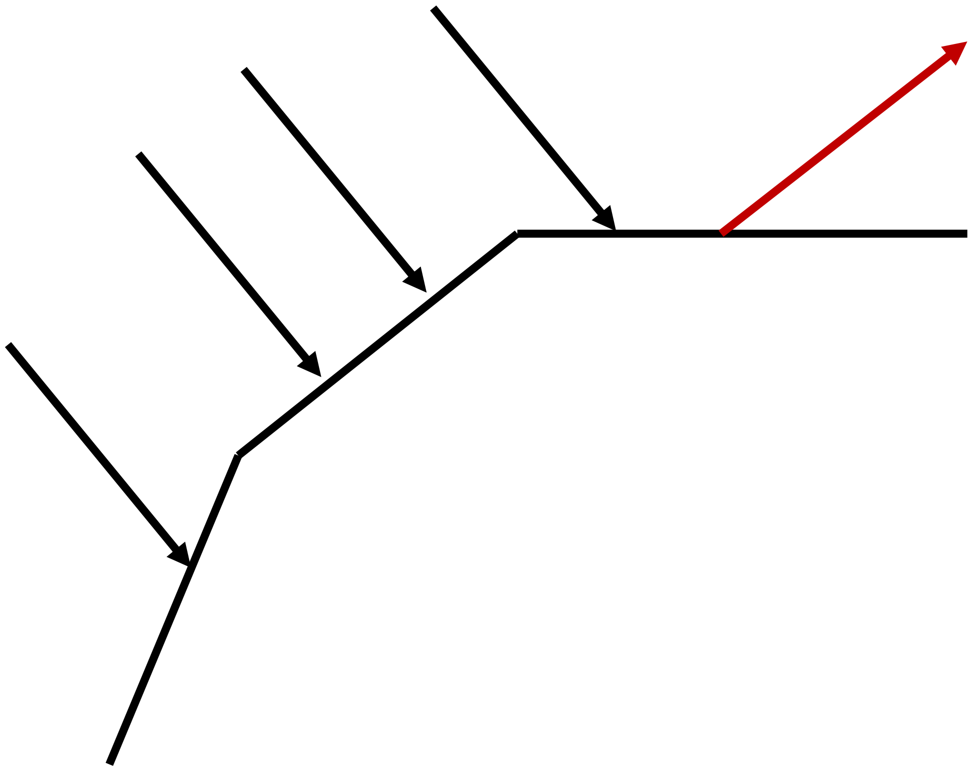
The reconstruction depends on the correction (pattern shift)

The correction depends on the ***true*** surface geometry

Correction model is an optimization scheme:

1. Find the surface that recreates the observed scans given a known material
2. Find the material that recreates the observed scans given a known surface

Open question:
How to model non-planar geometries?



Further reading

Jensen, Henrik Wann, et al. "A practical model for subsurface light transport." *Proceedings SIGGRAPH '01*. ACM, 2001.

- BSSRDF model

Holroyd, Michael, and Jason Lawrence. "An analysis of using high-frequency sinusoidal illumination to measure the 3d shape of translucent objects." *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*. IEEE, 2011.

- Alternative and very similar model

Rao, Li, and Feipeng Da. "Local blur analysis and phase error correction method for fringe projection profilometry systems." *Applied optics* 57.15 (2018): 4267-4276.

- A BSSRDF diffusion approximation