

DTU



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Deciding a Fragment of (α, β) -Privacy

Automated reasoning about privacy in security protocols

Automated reasoning about privacy in security protocols



Produce a program

Automated reasoning



Produce a program

about privacy



Control information

in security protocols

Automated reasoning



Produce a program

about privacy

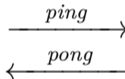


Control information

in security protocols

Alice

Bob



Privacy

Relevant in many fields, a security goal of its own:

- Electronic voting, digital health information, mobile payments...
- Distributed systems in general.
- More than just secrecy.

Privacy

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De facto standard = indistinguishability

- Given two possible worlds, can they be distinguished?
- Automated verification is difficult.
- Specification of goals is not intuitive.
- There is no guarantee that every privacy aspect has been covered.

Novel approach

(α, β) -privacy¹ = logical approach with many advantages:

- declarative and intuitive
- recast privacy as a reachability problem²
- decidable fragments: possibility for automated verification

¹Mödersheim S., Viganò L.: Alpha-Beta Privacy. ACM Trans. Priv. Secur. 22(1), 1–35 (2019).

²Gondron, S., Mödersheim, S., Viganò, L.: Privacy as Reachability. Tech. rep., DTU (2021), <http://www2.compute.dtu.dk/~samo/abg.pdf>



Preliminaries

Table of Contents

Preliminaries

The Fragment

Decision Procedure

Conclusion

Table of Contents

Preliminaries

Herbrand Logic

Frames

(α, β) -Privacy

The Fragment

Decision Procedure

Conclusion

Grammar

$$\langle \textit{Term} \rangle ::= \langle \textit{Variable} \rangle \mid \langle \textit{Function} \rangle(\langle \textit{Term} \rangle, \dots, \langle \textit{Term} \rangle)$$
$$\begin{aligned} \langle \textit{Formula} \rangle ::= & \langle \textit{Term} \rangle = \langle \textit{Term} \rangle \\ & \mid \langle \textit{Relation} \rangle(\langle \textit{Term} \rangle, \dots, \langle \textit{Term} \rangle) \\ & \mid \neg \langle \textit{Formula} \rangle \\ & \mid \langle \textit{Formula} \rangle \wedge \langle \textit{Formula} \rangle \\ & \mid \exists \langle \textit{Variable} \rangle. \langle \textit{Formula} \rangle \end{aligned}$$

Table of Contents

Preliminaries

Herbrand Logic

Frames

(α, β) -Privacy

The Fragment

Decision Procedure

Conclusion

Frame

Frames encode the knowledge of messages based on the protocol specification.

$$F = \{ \{ l_1 \mapsto t_1, \dots, l_k \mapsto t_k \} \}$$

l_i : distinguished constant called *label*

t_i : term that does not contain any l_i

Example: $F = \{ \{ l_1 \mapsto ping, l_2 \mapsto pong \} \}$

Recipes

Frames allow to reason about actions taken and not simply messages themselves.

Set of *recipes* = least set that contains l_1, \dots, l_k and that is closed under cryptographic operators

$F \{ r \}$: application of recipe r to frame F

Static equivalence

F_1 and F_2 with the same domain are *statically equivalent*, written $F_1 \sim F_2$, if the intruder cannot distinguish them.

$$\forall (r_1, r_2), F_1\{r_1\} \approx F_1\{r_2\} \iff F_2\{r_1\} \approx F_2\{r_2\}$$

Static equivalence

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Example:

$$F_1 = \{ l_1 \mapsto \text{scrypt}(k, t_1), l_2 \mapsto k, l_3 \mapsto t_1 \}$$

$$F_2 = \{ l_1 \mapsto \text{scrypt}(k, t_2), l_2 \mapsto k, l_3 \mapsto t_2 \}$$

$F_1 \sim F_2$ because, even though $t_1 \not\approx t_2$, there is no way to distinguish the frames.

Table of Contents

Preliminaries

Herbrand Logic

Frames

(α, β) -Privacy

The Fragment

Decision Procedure

Conclusion

Idea

Formula α : high-level information which is voluntarily disclosed, based on Σ_0

Σ_0 contains only non-technical information

Formula β : includes the technical information, e.g., cryptographic messages exchanged during the execution of the protocol

$$\Sigma_0 \subsetneq \Sigma$$

Violation of privacy: logically deriving information from β that does not follow from α alone



The Fragment

Table of Contents

Preliminaries

The Fragment

Decision Procedure

Conclusion

Message-analysis problem

Substitution θ : a model of α (interpretation of symbols making the formula true)

Example: $\alpha \equiv x, y, z \in \{0, 1\} \wedge x + y + z = 1$ $\theta = [x \mapsto 0, y \mapsto 1, z \mapsto 0]$

Message-analysis problem

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$struct = \{ \{ l_1 \mapsto t_1, \dots, l_k \mapsto t_k \} \}$: the specification of the protocol (structural knowledge)

$concr = \theta(struct)$: one execution of the protocol (concrete knowledge)

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$\beta \equiv MsgAna(\alpha, struct, \theta)$: knowledge of α , $struct$, $concr$ and $struct \sim concr$

Table of Contents

Preliminaries

The Fragment

 Destructor Theories

 Frames with Shorthands

Decision Procedure

Conclusion

The Fragment

Destructors

Destructors are functions used to decrypt (= decompose) terms. Modern cryptographic primitives allow to check if decryption works.

Example: $t = \text{sCrypt}(k, x)$

→ can decrypt with k

→ cannot decrypt with k'

Destructor theory

- $\Sigma_{pub} \subseteq \Sigma_f$: public functions
- E : algebraic equations of the form $\text{destr}(k, \text{constr}(t_1, \dots, t_n)) = t_i$ (where $i \in \{1, \dots, n\}$, $fv(k) \subseteq fv(t_1, \dots, t_n)$ and symbols in E are disjoint from Σ_0)

Example cryptographic operators

Constructors	Destructors	Properties
pub, priv		
crypt	dcrypt	$\text{dcrypt}(\text{priv}(s), \text{crypt}(\text{pub}(s), r, t)) = t$
sign	retrieve	$\text{retrieve}(\text{pub}(s), \text{sign}(\text{priv}(s), t)) = t$
scrypt	dscrypt	$\text{dscrypt}(k, \text{scrypt}(k, t)) = t$
pair	proj ₁ , proj ₂	$\text{proj}_1(\text{pair}(t_1, t_2)) = t_1$ $\text{proj}_2(\text{pair}(t_1, t_2)) = t_2$
h		

Table: Example set Σ_{op}

Destructor theory

Example: $t = \text{scrypt}(k, x)$

→ $\text{dscrypt}(k, \text{scrypt}(k, x)) \approx x$

→ $\text{dscrypt}(k', \text{scrypt}(k, x)) \approx \text{error}$

Table of Contents

Preliminaries

The Fragment

Destructor Theories

Frames with Shorthands

Decision Procedure

Conclusion

Frame with shorthands

Frames with shorthands extend the previous definition of frames.

$$F' = \{ \{ l_1 \mapsto t_1, \dots, l_k \mapsto t_k \}, m_1 \mapsto s_1, \dots, m_n \mapsto s_n \}$$

- $F = \{ \{ l_1 \mapsto t_1, \dots, l_k \mapsto t_k \} \}$: frame
- m_j : recipes over the l_i
- $F \{ m_j \} \approx s_j$
- $m_1 \mapsto s_1, \dots, m_n \mapsto s_n$: shorthands

Frame with shorthands

Example:

$$F = \{ l_1 \mapsto \text{sencrypt}(k, t), l_2 \mapsto k \}$$

$$F' = \{ l_1 \mapsto \text{sencrypt}(k, t), l_2 \mapsto k, m_1 \mapsto t \}$$

$$m_1 = \text{dencrypt}(l_2, l_1)$$

Frame with shorthands

Example:

$$F = \{ l_1 \mapsto \text{scrypt}(k, t), l_2 \mapsto k \}$$

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$$m_1 = \text{dscrypt}(l_2, l_1)$$

$$F \{ m_1 \} = F \{ \text{dscrypt}(l_2, l_1) \} = \text{dscrypt}(k, \text{scrypt}(k, t))$$

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Decision Procedure

Table of Contents

Preliminaries

The Fragment

Decision Procedure

Conclusion

Illustration

Example:

$$\theta = [x \mapsto 0, y \mapsto 1, z \mapsto 0]$$

$$struct = \{ \{ l_1 \mapsto \text{scrypt}(k, x), l_2 \mapsto \text{scrypt}(k, y), l_3 \mapsto \text{scrypt}(k, z) \} \}$$

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$$\alpha \equiv x, y, z \in \{0, 1\} \wedge x + y + z = 1 \quad \beta \equiv \text{MsgAna}(\alpha, struct, \theta)$$

Intruder deduction:

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Intruder deduction:

- 1 $concr\{ l_1 \} \approx concr\{ l_3 \}$: two messages are equal at the concrete level
- 2 $struct\{ l_1 \} \approx struct\{ l_3 \}$: they must also be equal at the structural level
- 3 $x = z$: violation of privacy!

Decision Procedure

Table of Contents

Preliminaries

The Fragment

Decision Procedure
Composition
Analysis
Intruder Findings

Conclusion

Composition in a structural frame

Three methods to compose a term:

- 1 Try to use labels with a corresponding substitution.
- 2 If the term is a variable, use the true value $\theta(x)$ (a constant) as a recipe with the substitution $[x \mapsto \theta(x)]$.
- 3 If the top-level is a public function, try to compose all arguments (and combine the substitutions).

Composition in a structural frame

Example:

$$\theta = [x \mapsto 0, y \mapsto 1, z \mapsto 0]$$

$$struct = \{ \{ l_1 \mapsto \text{script}(k, x), l_2 \mapsto \text{script}(k, y), l_3 \mapsto \text{script}(k, z) \} \}$$

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$$composeUnder(\theta, struct, \text{scrypt}(k, x)) = \{ (l_1, \varepsilon), (l_2, [x \mapsto y]), (l_3, [x \mapsto z]) \}$$

The intruder knows three ways to compose $\text{scrypt}(k, x)$ (substitution = constraints for the recipe to work).

Recipes may generate different terms in $concr = \theta(struct)$!

Decision Procedure

Table of Contents

Preliminaries

The Fragment

Decision Procedure

Composition

Analysis

Intruder Findings

Conclusion

Idea

- We know how to find recipes with *composition only*.
- We want to *all* generable terms using only composition.

—→ Thus we need to perform *analysis steps*: decrypt messages, open signed messages, deserialize etc.

Analysis of a structural frame

Analysis steps: we check if we can decrypt terms in the frame.

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- If the decryption fails in *concr*, i.e., the key cannot be composed, no new terms can be added. But if the key can be composed in *struct*, we can exclude some models.

Analysis of a structural frame

Analysis steps: we check if we can decrypt terms in the frame.

- If the decryption fails in *concr*, i.e., the key cannot be composed, no new terms can be added. But if the key can be composed in *struct*, we can exclude some models.
- If the decryption is successful in *concr*, then it is also successful in *struct* and we can define recipes for the new terms.
→ We add shorthands!

Analysis of a structural frame

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- If the decryption fails in *concr*, i.e., the key cannot be composed, no new terms can be added. But if the key can be composed in *struct*, we can exclude some models.
- If the decryption is successful in *concr*, then it is also successful in *struct* and we can define recipes for the new terms.
→ We add shorthands!
- Repeat until no more new terms can be added.

Analysis of a structural frame

Example:

$$\theta = [x \mapsto k_1, y \mapsto a, z \mapsto k_1]$$
$$struct = \{ \{ l_1 \mapsto \text{script}(x, y), l_2 \mapsto z \} \}$$

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The analysis adds the shorthand $\text{dscrypt}(l_2, l_1) \mapsto y$ because the decryption is successful in *concr.* $\longrightarrow x = z$.

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For the model $\theta' = [x \mapsto k_1, y \mapsto a, z \mapsto k_2]$, the decryption fails. $\longrightarrow x \neq z$.



Decision Procedure

Table of Contents

Preliminaries

The Fragment

Decision Procedure

Composition

Analysis

Intruder Findings

Conclusion

Relations between variables

- 1 Try to compose terms in *concr* in different ways:
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- 2 Try to compose terms in *struct* in different ways:
 - Check pairs (label, recipe).
 - If they generate the same term in *concr*, nothing to deduce (it comes from $concr \sim struct$ and has been found previously).
 - If they generate different terms in *concr*, some models can be excluded.
 \rightarrow Find inequalities ($x \neq t \vee y \neq 0 \dots$)

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 - If they generate different terms in *concr*, some models can be excluded.
 \rightarrow Find inequalities ($x \neq t \vee y \neq 0 \dots$)
- 3 $\phi \equiv$ conjunction of equalities and inequalities (= relations between variables)

Relations between variables

Recall the illustration example:

$$\theta = [x \mapsto 0, y \mapsto 1, z \mapsto 0]$$

$$struct = \{ \{ l_1 \mapsto \text{scrypt}(k, x), l_2 \mapsto \text{scrypt}(k, y), l_3 \mapsto \text{scrypt}(k, z) \} \}$$

$$concr = \{ \{ l_1 \mapsto \text{scrypt}(k, 0), l_2 \mapsto \text{scrypt}(k, 1), l_3 \mapsto \text{scrypt}(k, 0) \} \}$$

$$\alpha \equiv x, y, z \in \{0, 1\} \wedge x + y + z = 1 \quad \beta \equiv \text{MsgAna}(\alpha, struct, \theta)$$

With our decision procedure:

- 1 We analyze *struct*.
- 2 We generate $\phi \equiv x = z \wedge x \neq y$.
- 3 We find that $\alpha \not\models \phi$: violation of privacy!

Relations between variables

- ϕ is enough to decide privacy: we only need to check whether $\alpha \models \phi$.
- If $\alpha \models \phi$: the protocol respects privacy and we have a proof.
- If $\alpha \not\models \phi$: the protocol is not secure and we have a witness.



Conclusion

Table of Contents

Preliminaries

The Fragment

Decision Procedure

Conclusion

Current research

Focus on automation:

- Design a decision procedure to verify privacy goals for decidable fragments.³
- Develop a proof-of-concept tool in Haskell.
- Support more general theories (e.g., commutativity, exponentiation).
- Model and verify real-world protocols.
- Improve tool support.

³Fernet L., Mödersheim S.: Deciding a Fragment of (α, β) -Privacy. STM 2021, LNCS.