

A Logical Approach for Automated Reasoning about Privacy in Security Protocols

Laouen Fernet¹

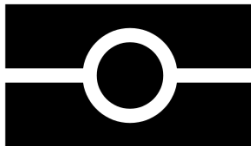
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Take-aways

- ① We should formally verify privacy in many applications
- ② We can define privacy goals in a declarative and intuitive way using (α, β) -privacy
- ③ Automated verification of (α, β) -privacy is practical

(α, β) -privacy

Automated verification

Type-flaw resistance

Composability

Conclusion

(α, β) -privacy

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(α, β) -privacy

Formula α = payload, over alphabet $\Sigma_0 \subset \Sigma$

Formula β = technical information, over alphabet Σ

Violation of privacy = β excludes some models of α

Example: unlinkability



$$\alpha = T_1, T_2 \in \text{Tags}$$

T_1, T_2 *privacy variables*
representing some concrete tag
names

Example of violations:

- $\beta \models T_1 = T_2$
- $\beta \models T_1 = a$
- $\beta \models T_2 \neq b$
- ...

Intruder knowledge

The intruder knows:

- the protocol
- which transaction is being executed
- the payload α of information intentionally disclosed
- the concrete messages sent over the network

Example cryptographic operators

$$\text{dscrypt}(k, \text{scrypt}(k, m, r)) \approx m$$

$$\text{dcrypt}(\text{inv}(k), \text{crypt}(k, m, r)) \approx m$$

$$\text{open}(k, \text{sign}(\text{inv}(k), m)) \approx m$$

$$\text{pubk}(\text{inv}(k)) \approx k$$

$$\text{proj}_1(\text{pair}(m_1, m_2)) \approx m_1$$

$$\text{proj}_2(\text{pair}(m_1, m_2)) \approx m_2$$

We assume failure of decryption can be detected

(α, β) -privacy

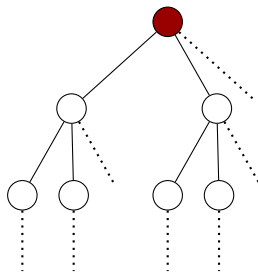
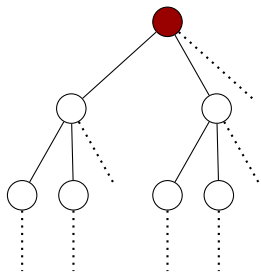
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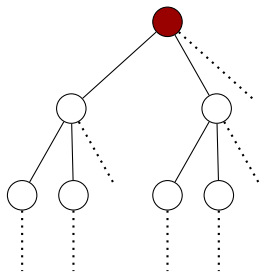
Symbolic representation



Infinite branching: intruder choices of recipes

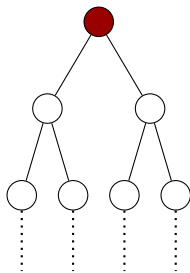
Infinite depth: another transaction can always be executed

Symbolic representation



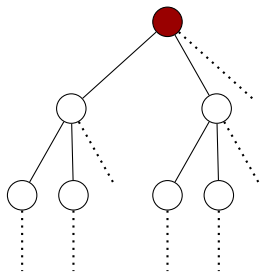
Infinite branching: intruder choices of recipes

Infinite depth: another transaction can always be executed



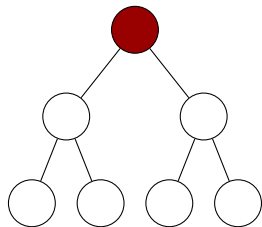
Finite branching: constraint solving (lazy intruder)

Symbolic representation



Infinite branching: intruder choices of recipes

Infinite depth: another transaction can always be executed



Finite branching: constraint solving (lazy intruder)

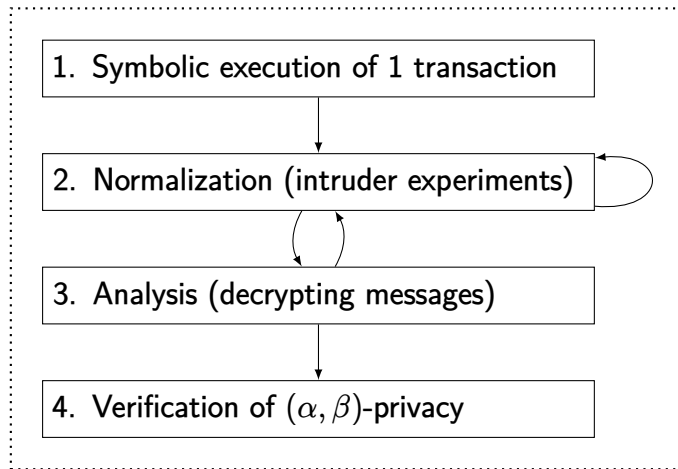
Finite depth: bound on the number of transactions executed

Summary of the procedure

Until the bound is reached:

- ① Symbolic execution of a given transaction
- ② Normalization via intruder experiments
- ③ Analysis of messages in intruder knowledge
- ④ Verification of (α, β) -privacy

Summary of the procedure



Counts as 1 towards
the bound

1. Execution: Runex

Transaction process

★ $x \in \text{Agent}$.

★ $y \in \{\text{yes}, \text{no}\}$.

$\text{rcv}(M)$.

try $N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M)$ in

if $y = \text{yes}$ then

$\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), R'))$

else

$\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{no}, R'))$

State

$\alpha = \text{true}$

$\mathcal{P} = \{(\star x \in \text{Agent} \dots, \text{true}, \mathcal{A})\}$

$\mathcal{A} = -l_1 \mapsto \text{inv}(\text{pk}(i)). \dots$

1. Execution: Runex

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State

$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$

$\mathcal{P} = \{(\text{rcv} \dots, \text{true}, \mathcal{A})\}$

$\mathcal{A} = -l_1 \mapsto \text{inv}(\text{pk}(i)). \dots$

1. Execution: Runex

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State

$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$

$\mathcal{P} = \{(\text{try} \dots, \text{true}, \mathcal{A})\}$

$\mathcal{A} = \dots . + R \mapsto M$

1. Execution: Runex

Transaction process

★ $x \in \text{Agent}$.

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$\text{rcv}(M)$.

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State

$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$

$\mathcal{P} = \{(\text{try} \dots, \text{true}, \mathcal{A})\}$

$\mathcal{A} = \dots . + R \mapsto M$

Solve $M \stackrel{?}{=} \text{crypt}(\text{pk}(s), M_1, M_2)$
with the lazy intruder

Lazy intruder

Constraint-solving technique: variables are instantiated on demand, only when it is actually needed

Intruder model: the intruder can send their own messages and reuse observed messages (also make guessing attacks, for privacy variables)

1. Execution: Runex

Transaction process

★ $x \in \text{Agent}$.

★ $y \in \{\text{yes}, \text{no}\}$.

$\text{rcv}(M)$.

try $N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M)$ in

if $y = \text{yes}$ then

$\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), R'))$

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$\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{no}, R'))$

State

$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$

$\mathcal{P} = \{(\text{try} \dots, \text{true}, \mathcal{A})\}$

$\mathcal{A} = \dots . + R \mapsto M$

$R \mapsto \text{crypt}(\text{pk}(s), R_1, R_2)$

$M \mapsto \text{crypt}(\text{pk}(s), M_1, M_2)$

$N \mapsto M_1$

1. Execution: Runex

Transaction process

★ $x \in \text{Agent}$.
★ $y \in \{\text{yes}, \text{no}\}$.
 $\text{rcv}(M)$.
 $\text{try } N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M) \text{ in}$
if $y = \text{yes}$ then
 $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, M_1), R'))$
else
 $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{no}, R'))$

State

$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$

$\mathcal{P} = \{(\text{if } \dots, \text{true}, \mathcal{A})\}$

$\mathcal{A} = \dots . + R_1 \mapsto M_1 . + R_2 \mapsto M_2$

1. Execution: Runex

Transaction process

★ $x \in \text{Agent}$.
★ $y \in \{\text{yes}, \text{no}\}$.
 $\text{rcv}(M)$.
 $\text{try } N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M) \text{ in}$
if $y = \text{yes}$ then
 $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, M_1), R'))$
else
 $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{no}, R'))$

State

$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$
 $\mathcal{P} = \{(\text{snd} \dots, y = \text{yes}, \mathcal{A}),$
 $(\text{snd} \dots, y \neq \text{yes}, \mathcal{A})\}$
 $\mathcal{A} = \dots . + R_1 \mapsto M_1 . + R_2 \mapsto M_2$

1. Execution: Runex

Transaction process

★ $x \in \text{Agent}$.
★ $y \in \{\text{yes}, \text{no}\}$.
 $\text{rcv}(M)$.
try $N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M)$ in
if $y = \text{yes}$ then
 $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, M_1), R'))$
else
 $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{no}, R'))$

State

$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$
 $\mathcal{P} = \{(\mathbf{0}, y = \text{yes}, \mathcal{A}_1),$
 $(\mathbf{0}, y \neq \text{yes}, \mathcal{A}_2)\}$
 $\mathcal{A}_1 = \cdots .-l_2 \mapsto \text{crypt}(\dots \text{yes} \dots)$
 $\mathcal{A}_2 = \cdots .-l_2 \mapsto \text{crypt}(\dots \text{no} \dots)$

All processes are nil, we are in a
reachable state

2. Normalization via intruder experiments

Suppose encryption is not randomized:

$$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$$

$$\mathcal{P} = \{(0, y = \text{yes}, \mathcal{A}_1), (0, y \neq \text{yes}, \mathcal{A}_2)\}$$

$$\mathcal{A}_1 = \cdots .-l_2 \mapsto \text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, M_1))$$

$$\mathcal{A}_2 = \cdots .-l_2 \mapsto \text{crypt}(\text{pk}(x), \text{no})$$

The intruder can make a guessing attack:

$$l_2 \stackrel{?}{\simeq} \text{crypt}(\text{pk}(a), \text{no}) \quad l_2 \stackrel{?}{\simeq} \text{crypt}(\text{pk}(b), \text{no}) \quad \dots$$

3. Analysis of messages in intruder knowledge

$$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$$

$$\mathcal{P} = \{(0, y = \text{yes}, \mathcal{A}_1), (0, y \neq \text{yes}, \mathcal{A}_2)\}$$

$$\mathcal{A}_1 = -l_1 \mapsto \text{inv}(\text{pk}(i)). \dots . -l_2 \mapsto \text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, M_1), r)$$

$$\mathcal{A}_2 = -l_1 \mapsto \text{inv}(\text{pk}(i)). \dots . -l_2 \mapsto \text{crypt}(\text{pk}(x), \text{no}, r)$$

$$\text{dcrypt}(l_1, l_2) \stackrel{?}{\simeq} \text{fail}$$

2. Normalization via intruder experiments

If $\text{dcrypt}(l_1, l_2) \not\approx \text{fail}$, i.e., $x = i$:

$$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$$

$$\mathcal{P} = \{(0, y = \text{yes} \wedge x = i, \mathcal{A}_1), (0, y \neq \text{yes} \wedge x = i, \mathcal{A}_2)\}$$

$$\mathcal{A}_1 = \cdots .-l_3 \mapsto \text{pair}(\text{yes}, M_1).-l_4 \mapsto \text{inv}(\text{pk}(x))$$

$$\mathcal{A}_2 = \cdots .-l_3 \mapsto \text{no}. -l_4 \mapsto \text{inv}(\text{pk}(x))$$

New experiment: $l_3 \stackrel{?}{\simeq} \text{no}$

4. Verification of (α, β) -privacy

If $l_3 \simeq \text{no}$:

$$\alpha = x \in \text{Agent} \wedge y \in \{\text{yes}, \text{no}\}$$

$$\mathcal{P} = \{(0, y \neq \text{yes} \wedge x = i, \mathcal{A}_2)\}$$

$$\mathcal{A}_2 = \cdots .-l_3 \mapsto \text{no} .-l_4 \mapsto \text{inv}(\text{pk}(x)) .-l_5 \mapsto \text{pk}(x)$$

Intruder learns $y \neq \text{yes} \wedge x = i$ (more than allowed by α)
 $\rightarrow$ violation of privacy

Release: selective disclosure of information

```
★  $x \in \text{Agent}$ .  
★  $y \in \{\text{yes}, \text{no}\}$ .  
rcv( $M$ ).  
try  $N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M)$  in  
if  $x \in \text{Dishonest}$  then ★  $x = \gamma(x) \wedge y = \gamma(y)$   
                        else ★  $x \in \text{Honest}$ .  
if  $y = \text{yes}$  then  
     $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), R'))$ .  
else  
     $\nu R'. \text{snd}(\text{crypt}(\text{pk}(x), \text{no}, R'))$ 
```

Now (α, β) -privacy holds

Case studies

- Basic Hash: provides unlinkability but not forward privacy
- OSK: has linkability flaws
- BAC: has linkability flaws in some implementations
- Private Authentication: several variants with privacy goals beyond unlinkability

(α, β) -privacy

Automated verification

Type-flaw resistance

Composability

Conclusion

Type-flaws

A participant in the protocol expects a message of some type, but receives a message of a different type

E.g., expect a nonce but receive an encrypted message

Type annotations

★ $x \in \text{Agent}$.

★ $y \in \{\text{yes}, \text{no}\}$.

$\text{rcv}(M : \text{crypt}(\text{pk}(\text{agent}), \text{nonce}, \text{nonce}))$.

try $N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M)$ in

if $y = \text{yes}$ then

$\nu R' : \text{nonce}.$ $\text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), R'))$

else

$\nu R' : \text{nonce}.$ $\text{snd}(\text{crypt}(\text{pk}(x), \text{no}, R'))$

Making sure the outgoing messages have the same type

★ $x \in \text{Agent}$.

★ $y \in \{\text{yes}, \text{no}\}$.

$\text{rcv}(M : \text{crypt}(\text{pk}(\text{agent}), \text{nonce}, \text{nonce}))$.

try $N = \text{dcrypt}(\text{inv}(\text{pk}(s)), M)$ in

if $y = \text{yes}$ then

$\nu R' : \text{nonce. snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), R'))$

else

$\nu R' : \text{nonce. snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{no}, n_0), R'))$

From explicit destructors to pattern matching

★ $x \in \text{Agent}$.

★ $y \in \{\text{yes}, \text{no}\}$.

$\text{rcv}(\text{crypt}(\text{pk}(S : \text{agent}), N : \text{nonce}, R : \text{nonce}))$.

if $S = s$ then

if $y = \text{yes}$ then

$\nu R' : \text{nonce}. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), R'))$

else

$\nu R' : \text{nonce}. \text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{no}, n_0), R'))$

Message patterns = all messages in the transactions after removing destructors

$\text{Agent} \cup \{x, y, \text{yes}, \text{no}, \text{crypt}(\text{pk}(S), N, R), S, s, R', \text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), R'),$
 $\text{crypt}(\text{pk}(x), \text{pair}(\text{no}, n_0), R')\}$

Type-flaw resistance

SMP = all message patterns in protocol and their well-typed instances (including subterms and key terms)

$$\forall s, t \in SMP \setminus \mathcal{V}. (s, t) \text{ unifiable} \longrightarrow type(s) = type(t)$$

Runex does not achieve type-flaw resistance

- $\text{crypt}(\text{pk}(S), N, R) \in \text{SMP} \setminus \mathcal{V}$
- $\text{crypt}(\text{pk}(a), \text{pair}(\text{yes}, n_2), r) \in \text{SMP} \setminus \mathcal{V}$
- These are unifiable, but
$$\text{type}(\text{crypt}(\text{pk}(S), N, R)) \neq \text{type}(\text{crypt}(\text{pk}(a), \text{pair}(\text{yes}, n_2), r))$$
- Because
$$\text{type}(N) = \text{nonce} \neq \text{pair}(\text{decision}, \text{nonce}) = \text{type}(\text{pair}(\text{yes}, n_2))$$

Achieving type-flaw resistance with formats

★ $x \in \text{Agent}$.

★ $y \in \{\text{yes}, \text{no}\}$.

$\text{rcv}(\text{crypt}(\text{pk}(s), f_1(N : \text{nonce}), R : \text{nonce}))$.

if $y = \text{yes}$ then

$\nu R' : \text{nonce. snd}(\text{crypt}(\text{pk}(x), f_2(\text{yes}, N), R'))$

else

$\nu R' : \text{nonce. snd}(\text{crypt}(\text{pk}(x), f_2(\text{no}, n_0), R'))$

Typing result

For type-flaw resistant protocols: **“if there is an attack, then there is a well-typed one”**

Proof argument: the lazy intruder, intruder experiments and analysis are only doing well-typed instantiations

The result holds without any bound on the number of transitions

Case studies

- Basic Hash: type-flaw resistant
- OSK: out of scope
- BAC and Private Authentication: type-flaw resistant (after some adaptations to the models)

(α, β) -privacy

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Composition and modular verification

$$Spec = Spec_1 \parallel Spec_2$$

Can we derive the security of $Spec$ by verifying $Spec_1$ and $Spec_2$ in isolation?

Needham-Schroeder-Lowe with lookup for public keys

Role *Initiator* :

★ $x_A \in \text{Honest}$.

★ $x_B \in \text{Agent}$.

$PKB := \text{lookup}(x_A, x_B)$

;

$\nu N_A : \text{nonce}, R : \text{nonce}$.

$\text{snd}(\text{crypt}(PKB, f_1(N_A, x_A), R))$.

...

Role *Responder* :

$\text{rcv}(\text{crypt}(\text{pk}(B : \text{agent}),$
 $f_1(N_A : \text{nonce}, A : \text{agent}), _ : \text{nonce}))$.

if $B \notin \text{Honest}$ then

 stop

;

$PKA := \text{lookup}(B, A)$

...

Needham-Schroeder-Lowe with lookup for public keys

```
Procedure lookup( $A$  : agent,  $B$  : agent) :  
   $\nu N$  : nonce,  $R$  : nonce.  
  snd(scrypt(sk( $A$ , s), req( $B$ ,  $N$ ),  $R$ ))  
  ;  
  rcv(scrypt(sk( $A$ , s), resp( $B$ ,  $PKB$  : pk(agent),  $N$ ),  $\_$  : nonce)).  
  assert( $PKB = \text{pk}(B)$ ).  
  return(pk( $B$ ))
```

Needham-Schroeder-Lowe with lookup for public keys

Role *Server* :

$\text{rcv}(\text{scrypt}(\text{sk}(A : \text{agent}, s), \text{req}(B : \text{agent}, N : \text{nonce}), _ : \text{nonce})).$

$\nu R : \text{nonce}.$

$\text{snd}(\text{scrypt}(\text{sk}(A, s), \text{resp}(B, \text{pk}(B), N), R))$

Interface between protocols

Specify what each component must at least know about the other components

- All choices of privacy variables and releases
- Procedure calls between components
- Shared messages

Composability requirements

- Operations on *shared* messages, memory cells and procedures must be part of the interface
- All shared messages must be public or declared as secrets (secrets may be declassified)
- ...

Compositionality result

For composable protocols: **“if each component is secure, then their composition is secure”**

Proof argument: any attack trace on the composition can be mapped to an attack trace on one component

secure = (α, β) -privacy goals + all assertions hold + no secret leaked

Examples

- NSL with lookup
- Basic model of TLS handshake

(α, β) -privacy

Automated verification

Type-flaw resistance

Composability

Conclusion

Main contributions

- ① A decision procedure for (α, β) -privacy for a bounded number of transitions, with proofs of soundness and completeness
- ② Prototype tool called *noname* implementing the procedure
- ③ For type-flaw resistant protocols: “if there is an attack, then there is a well-typed one”
- ④ For composable protocols: “if each component is secure, then their composition is secure”

References

- [1] L. Fernet, S. Mödersheim, and L. Viganò. “A Decision Procedure for Alpha-Beta Privacy for a Bounded Number of Transitions”. In: *CSF 2024*. Extended version at <https://people.compute.dtu.dk/samo/alphabeta>. IEEE, 2024, pp. 159–174. DOI: 10.1109/CSF61375.2024.00011.
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- [3] L. Fernet and S. Mödersheim. “Private Authentication with Alpha-Beta-Privacy”. In: *OID 2023*. LNI. GI, 2023. DOI: 10.18420/OID2023_05. URL: <https://people.compute.dtu.dk/samo/alphabeta>.
- [4] L. Fernet, S. Mödersheim, and L. Viganò. “A decision procedure and typing result for alpha-beta privacy”. In: *J. Comput. Secur.* (2024). Submitted for review.
- [5] L. Fernet, S. Mödersheim, and L. Viganò. *A Compositionality Result for Alpha-Beta Privacy*. Tech. rep. DTU, Denmark; King’s, United Kingdom, 2024. URL: <https://people.compute.dtu.dk/samo>.

Ideas for future work

- Implement checks for type-flaw resistance and composability requirements
- Support more algebraic theories (Diffie-Hellman)
- Procedure for unbounded case?
- Investigate relation to approaches based on process equivalences
- Design a procedure for probabilistic (α, β) -privacy

Take-aways

- ① We should formally verify privacy in many applications
- ② We can define privacy goals in a declarative and intuitive way using (α, β) -privacy
- ③ Automated verification of (α, β) -privacy is practical