European Research Council

Established by the European Commission

UNIVERSITY OF EASTERN FINLAND

Utilising Monte Carlo method for light transport in the inverse problem of quantitative photoacoustic tomography

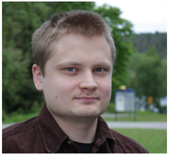
Tanja Tarvainen
Department of Technical Physics, University of Eastern Finland, Kuopio, Finland

Workshop on Inverse problems, Bayes and beyond
DTU, Lyngby, November 22, 2023

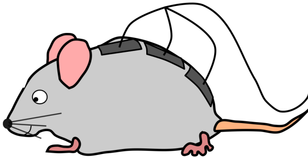
Finnish Centre of Excellence in Inverse Modelling and Imaging 2018-2025

This is joint work with

Niko Hänninen, University of Eastern Finland, Finland
Aleksi Leino, University of Eastern Finland and University of Helsinki, Finland
Aki Pulkkinen, University of Eastern Finland, Finland
Simon Arridge, University College London, United Kingdom

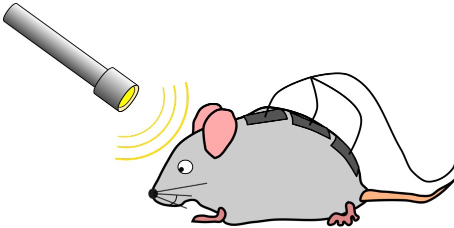
   

Photoacoustic tomography (PAT)



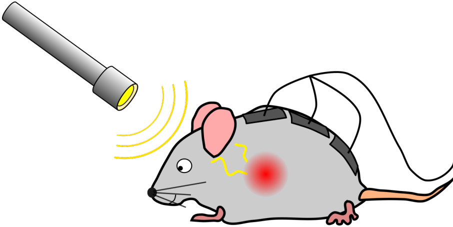
Photoacoustic tomography (PAT)

- Tissue is illuminated with a short pulse of light



Photoacoustic tomography (PAT)

- Tissue is illuminated with a short pulse of light
- As light propagates within the tissue, it is absorbed by chromophores
- The absorbed energy causes pressure rise



Photoacoustic tomography (PAT)

- Tissue is illuminated with a short pulse of light
- As light propagates within the tissue, it is absorbed by chromophores
- The absorbed energy causes pressure rise
- This pressure increase propagates through the tissue as an acoustic wave and can be measured on the boundary of the tissue using ultrasound sensors

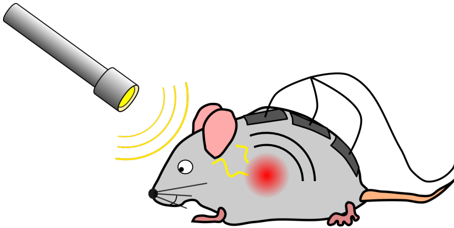
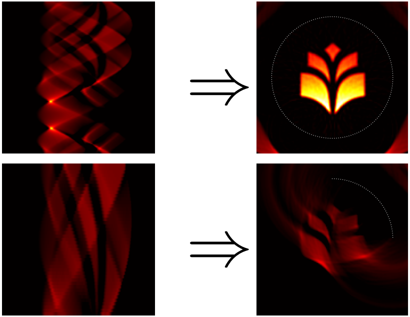
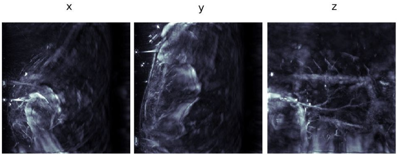


Image reconstruction

- Reconstruct the initial pressure (or absorbed optical energy density) from the photoacoustic wave measured on the boundary of the tissue



- Photoacoustic tomography combines benefits of optical and acoustic methods
- Contrast through optical absorption
  - Tissue chromophores: oxygenated and deoxygenated haemoglobin, water, lipids, melanin
  - Contrast agents
- Resolution by ultrasound
  - Low scattering in soft biological tissue
- Applications in imaging of tissue vasculature, tumours, small animal imaging, etc.



J. Tick et al, Three dimensional photoacoustic tomography in Bayesian framework, *J Acoust Soc Am* 144:2061-2071, 2018

Quantitative photoacoustic tomography (QPAT)

- Aims is to estimate the concentrations of light absorbing molecules
- Two inverse problems:
  - Acoustic inverse problem: estimation of initial pressure from photoacoustic measurements
  - Optical inverse problem: estimation of optical parameters from the initial pressure
- Modelling of light propagation, photoacoustic efficiency and ultrasound propagation are needed

- In this work, we study the optical inverse problem of QPAT
- MAP estimates for absorption and scattering are computed from absorbed optical energy density
- Evaluation of the forward operator is based on Monte Carlo method for light transport, i.e. forward operator is stochastic $\Rightarrow$ The search direction in an optimisation algorithm is also stochastic

Optical forward problem of QPAT

- Radiative transfer equation (RTE)

$$\hat{s} \cdot \nabla \phi(r, \hat{s}) + (\mu_s + \mu_a)\phi(r, \hat{s}) = \mu_s \int_{S^{n-1}} \Theta(\hat{s} \cdot \hat{s}')\phi(r, \hat{s}')\mathrm{d}\hat{s}', \quad r \in \Omega$$

$$\phi(r, \hat{s}) = \begin{cases} \phi_0(r, \hat{s}), & r \in \epsilon_j, \quad \hat{s} \cdot \hat{n} < 0 \\ 0, & r \in \partial\Omega \backslash \epsilon_j, \quad \hat{s} \cdot \hat{n} < 0 \end{cases}$$

where $\phi(r, \hat{s})$ is radiance, μ_a is absorption, μ_s is scattering, $\Theta(\hat{s} \cdot \hat{s}')$ is scattering phase function and $\phi_0(r, \hat{s})$ is light source in position r and direction $\hat{s}$

  - RTE simulates light propagation accurately in a scattering medium
  - It is computationally challenging

- Photon fluence

$$\Phi(r) = \int_{S^{n-1}} \phi(r, \hat{s}) \mathrm{d}\hat{s}$$

- Absorbed optical energy density

$$H(r) = \mu_a(r)\Phi(r)$$

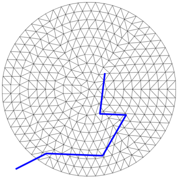
- Initial acoustic pressure

$$p_0(r) = p(r, t = 0) = G(r)H(r) = \frac{\beta c^2}{C_p} H(r)$$

where $G(r)$ is the Grüneisen parameter describing photoacoustic efficiency

Monte Carlo method for light transport

- Simulates same physics as the RTE
- Based on random sampling of photon paths as they undergo scattering and absorption effects in the medium
- In this work, a Monte Carlo software ValoMC[1] is used (https://inverselight.github.io/ValoMC/)



ValoMC
Visible and near infrared light transport on mesh based geometry

[1] A.A. Leino, A. Pulkkinen, T. Tarvainen, ValoMC: a Monte Carlo software and MATLAB toolbox for simulating light transport in biological tissue, *OSA Continuum* 2:957–972, 2019

Photon packet Monte Carlo

- Photon packet with an initial weight w_0 is considered
- The probability for photon absorption in a small length $\mathrm{d}s$ in a propagation direction is $\mu_\mathrm{a}\mathrm{d}s$, and the probability for photon scattering is $\mu_\mathrm{s}\mathrm{d}s$
- Scattering length follows an exponential probability distribution function

$$h(l) = \mu_\mathrm{s}(l) \exp\left[-\int_0^l \mu_\mathrm{s}(l')\mathrm{d}l'\right]$$

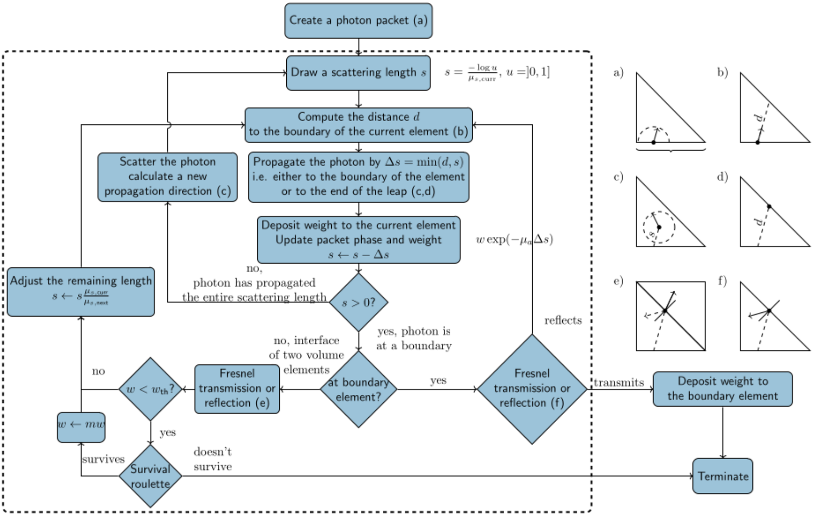
- Scattering angle follows a probability distribution (Henyey-Greenstein phase function)
- Weight of a photon packet along trajectory is reduced according to

$$w(s) = \exp\left[-\int_0^s \mu_\mathrm{a}(s')\mathrm{d}s'\right]$$

- In Monte Carlo simulation for QPAT with piecewise constant optical coefficients $\mu_{\mathrm{a},i}$ and $\mu_{\mathrm{s},i}$, the total absorbed optical energy density H_j deposited to discretisation element j is computed as

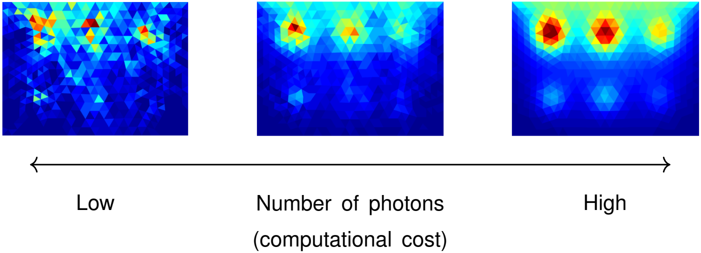
$$H_j = \frac{1}{A_j} \sum_e w_e (1 - \exp[-\mu_{\mathrm{a},j} S_{e,j}])$$

where w_e is the weight of the photon packet before entrance to the j:th element, and $S_{e,j}$ is the distance travelled on each entrance to j, A_j is the area/volume of the element in 2D/3D



Create a photon packet (a)
Draw a scattering length s
$s = \frac{-\log u}{\mu_{s,\mathrm{curr}}}$, $u =]0, 1]$
Compute the distance d to the boundary of the current element (b)
Scatter the photon calculate a new propagation direction (c)
Propagate the photon by $\Delta s = \min(d, s)$ i.e. either to the boundary of the element or to the end of the leap (c,d)
Deposit weight to the current element Update packet phase and weight $s \leftarrow s - \Delta s$
$w \exp(-\mu_a \Delta s)$
Adjust the remaining length $s \leftarrow s \frac{\mu_{s,\mathrm{curr}}}{\mu_{s,\mathrm{next}}}$
no, photon has propagated the entire scattering length
$s > 0?$
yes, photon is at a boundary
reflects
no, interface of two volume elements
at boundary element?
yes
Fresnel transmission or reflection (e)
Fresnel transmission or reflection (f)
transmits
Deposit weight to the boundary element
no
$w < w_{\mathrm{th}}?$
$w \leftarrow mw$
yes
survives
Survival roulette
doesn't survive
Terminate
a)
b)
c)
d)
e)
f)

- Number of photon packets affects on the accuracy of the approximation and the computational cost



Inverse problem

- Optical inverse problem: estimate distribution of absorption and scattering parameters from absorbed optical energy density
- A discrete observation model for QPAT in the presence of additive noise model is

$$y = f(x) + e$$

where $y \in \mathbb{R}^m$ is the data, $x \in \mathbb{R}^n$ are the unknown optical parameters, $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ is the discretised forward model, and $e \in \mathbb{R}^m$ denotes the noise

- The solution of the inverse problem given by the Bayes' formula (posterior probability distribution)

$$\pi(x|y) \propto \pi(y|x)\pi(x)$$

where $\pi(y|x)$ is the likelihood and $\pi(x)$ is the prior

- Model the unknown x and noise e mutually independent and Gaussian distributed

$$x \sim \mathcal{N}(\eta_x, \Gamma_x), \quad e \sim \mathcal{N}(\eta_e, \Gamma_e)$$

where $\eta_x \in \mathbb{R}^n$ and $\eta_e \in \mathbb{R}^m$ are the means and $\Gamma_x \in \mathbb{R}^{n\times n}$ and $\Gamma_e \in \mathbb{R}^{m\times m}$ are the covariance matrices

- The posterior distribution

$$\pi(x|y) \propto \exp\left\{-\frac{1}{2}\|L_e(y - f(x) - \eta_e)\|^2 - \frac{1}{2}\|L_x(x - \eta_x)\|^2\right\}$$

where $\Gamma_e^{-1} = L_e^T L_e$ and $\Gamma_x^{-1} = L_x^T L_x$

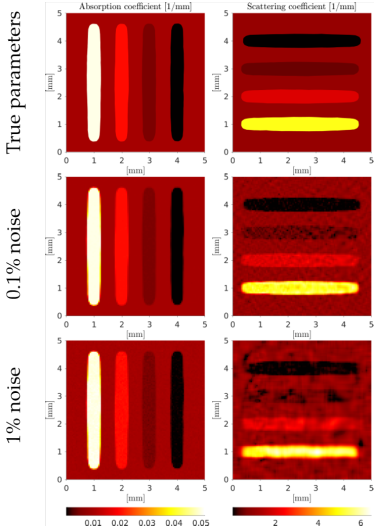
- The maximum a posteriori (MAP) estimate

$$x_{\mathrm{MAP}} = \arg\min_x \left\{\frac{1}{2}\|L_e(y - f(x) - \eta_e)\|^2 + \frac{1}{2}\|L_x(x - \eta_x)\|^2\right\}$$

- MAP estimate can be computed using methods of computational optimisation (in this work Gauss-Newton method)
- For the Gauss-Newton method, Jacobian matrices need to be formed
  - Absorption: derivative can be computed from the exponential decay of the weight of the photon packet
  - Scattering: a so-called perturbation approximation of Monte Carlo is utilised

Simulations

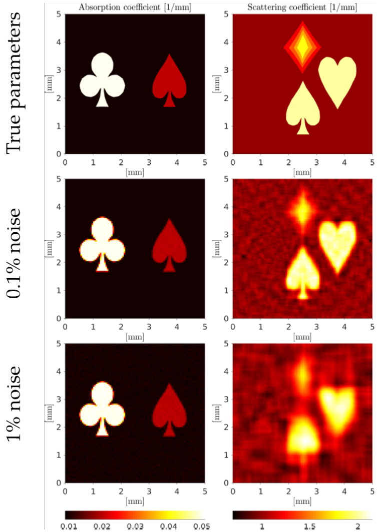
- $5\,\mathrm{mm} \times 5\,\mathrm{mm}$ square computation domain
- μ_a in scale $[0.0001\,,\,0.05]\,\mathrm{mm}^{-1}$, and μ_s in scale $[0.01\,,\,5]\,\mathrm{mm}^{-1}$
- Henyey-Greenstein scattering anisotropy parameter $g = 0.9$
- Four planar illuminations from different sides of the domain
- Two noise levels for the additive noise: 0.1 % and 1 %
- Used 10^9 photon packets in data simulation, and 10^8 in the inversion on each iteration for each illumination



Relative errors of the estimates

$E_{\mu_a} = 0.3\%$ and $E_{\mu_s} = 11\%$

$E_{\mu_a} = 2.2\%$ and $E_{\mu_s} = 20\%$



Relative errors of the estimates

$E_{\mu_\mathrm{a}} = 0.2\%$ and $E_{\mu_\mathrm{s}} = 6.1\%$

$E_{\mu_\mathrm{a}} = 1.9\%$ and $E_{\mu_\mathrm{s}} = 11\%$

Stochastic framework

- Using Monte Carlo, the 'accurate' forward model $f(x)$ or its Jacobian $J(x)$ are unavailable
- Simulating finite number of photon packets P provides only approximations $f_P(x)$ and $J_P(x)$

$$f_P(x) = f(x) + \epsilon_P$$
$$J_P(x) = J(x) + \xi_P$$

  where ϵ_P and ξ_P are (stochastic) errors of the forward model and its Jacobian
- Increasing the number of photon packets decreases the errors, but comes with a higher computational cost

Stochastic Gauss-Newton method (SGN)

- The estimates are updated by

$$x_{i+1} = x_i + \alpha_i \delta_{P_i}(x_i),$$

  where α_i is the step size parameter and $\delta_{P_i}(x_i)$ is solved from

$$(J_{P_i}(x_i)^{\mathrm{T}} \Gamma_e^{-1} J_{P_i}(x_i) + \Gamma_x^{-1}) \delta_{P_i}(x_i) = J_{P_i}(x_i)^{\mathrm{T}} \Gamma_e^{-1} (y - f_{P_i}(x_i)) - \Gamma_x^{-1} (x_i - \eta_x),$$

  where $f_{P_i}(x_i)$ is the forward model and $J_{P_i}(x_i)$ its Jacobian, computed with P_i photon packets at point x_i
- With a fixed number of photon packets, the effect of stochastic noise increases when residual $\|y - f_{P_i}(x)\|$ decreases

Adaptive number of photon packets

- The number of photon packets is adjusted during iteration
- Norm test approach[2,3]
  - The expected relative error of the SGN search direction is controlled

$$V_{P_i}(x_i)^2 := \frac{\mathbb{E}[||\delta(x_i) - \delta_{P_i}(x_i)||^2]}{||\delta(x_i)||^2} < \gamma^2,$$

    where $\delta(x_i)$ is the reference SGN direction at point x_i and γ is a threshold parameter describing accepted error
  - If the norm test fails, the number of photon packets is increased

$$P_i \leftarrow \frac{V_{P_i}(x_i)^2}{\gamma^2} P_i,$$

---

[2] R.G. Carter. On the global convergence of trust region algorithms using inexact gradient information. *SIAM J Numer Anal* 28:251-265, 1991

[3] C.M. Macdonald, S. Arridge, S. Powell. Efficient inversion strategies for estimating optical properties with Monte Carlo radiative transport models. *J Biomed Opt* 24:1-13, 2019

Algorithm 1 Adaptive stochastic Gauss–Newton

Set the initial number of photon packets P_1, number of samples in the norm test L, initial value $\mu_a^{(1)}$ and $i \leftarrow 1$;

Repeat

Compute a set of approximative solutions $\{H_{P_i}^{(\ell)}(\mu_a^{(i)})\}$ and Jacobians $\{J_{P_i}^{(\ell)}(\mu_a^{(i)})\}$, $\ell = 1, \ldots, L$;

Compute a set of approximate GN directions $\{\delta_{P_i}^{(\ell)}(\mu_a^{(i)})\}$ from $\{H_{P_i}^{(\ell)}(\mu_a^{(i)})\}$ and $\{J_{P_i}^{(\ell)}(\mu_a^{(i)})\}$;

Compute an approximation of the accurate GN direction $\delta(\mu_a^{(i)})$ using means of $\{H_{P_i}^{(\ell)}(\mu_a^{(i)})\}$ and $\{J_{P_i}^{(\ell)}(\mu_a^{(i)})\}$;

Compute $V_{P_i}(\mu_a^{(i)})^2$ from Eq. (21);

if $V_{P_i}(\mu_a^{(i)})^2 > \gamma^2$ **then**

if $\frac{V_{P_i}(\mu_a^{(i)})^2}{\gamma^2} > L$ **then**

Set $P_i \leftarrow \frac{V_{P_i}(\mu_a^{(i)})^2}{\gamma^2} P_i$;

Compute GN direction $\delta_{P_i}(\mu_a^{(i)})$ using P_i photon packets;

Set $\xi_i \leftarrow \delta_{P_i}(\mu_a^{(i)})$;

Else

Set $P_i \leftarrow \frac{V_{P_i}(\mu_a^{(i)})^2}{\gamma^2} P_i$;

Set $\xi_i \leftarrow \delta(\mu_a^{(i)})$;

Else

Set $\xi_i \leftarrow \delta(\mu_a^{(i)})$;

Update estimate $\mu_a^{(i+1)} = \mu_a^{(i)} + \xi_i$;

Set $P_{i+1} \leftarrow P_i$;

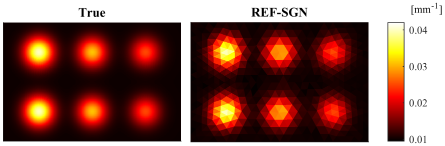
Set $i \leftarrow i + 1$

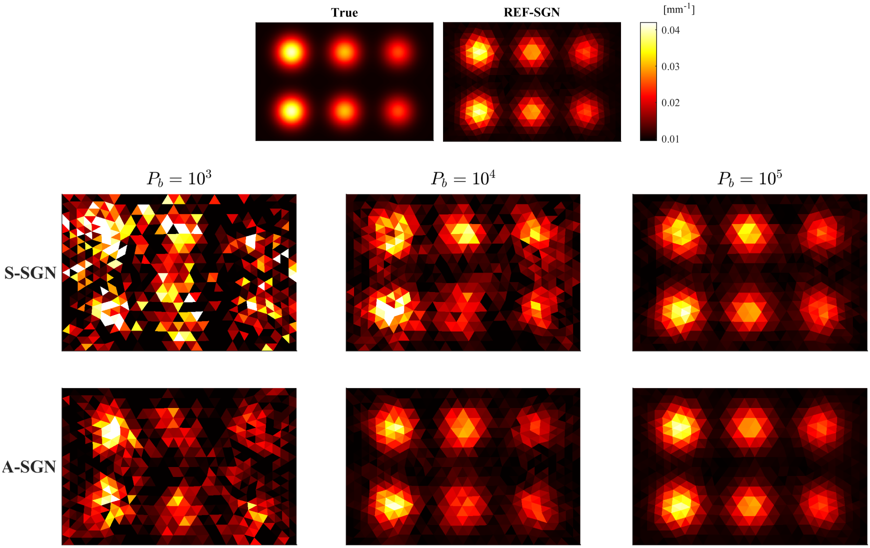
until *a convergence criterion is fulfilled*;

Simulations

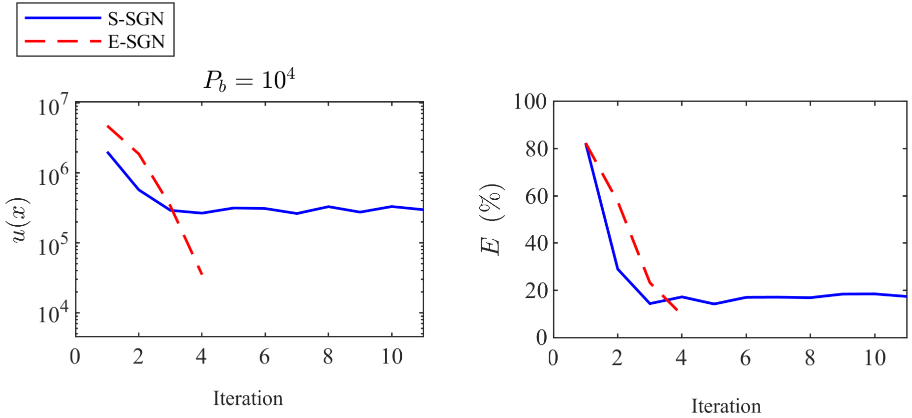
- $15\,\mathrm{mm} \times 10\,\mathrm{mm}$ rectangular domain
- Henyey-Greenstein scattering anisotropy parameter $g = 0.8$
- Four planar illuminations from different sides of the domain
- Data was simulated using 10^9 photon packets for each illumination
- Noise level 1 % for the additive noise
- Three MAP estimates for absorption were computed (scattering assumed to be known):
  - Simple stochastic Gauss-Newton approach (S-SGN): number of photon packets is fixed
  - Adaptive stochastic Gauss-Newton approach (A-SGN): number of photon packets determined by the norm test
  - Reference estimate (REF-SGN) with very large number of photon packets (10^8 per iteration)

- First comparison: Same computational cost
  - Fixed the total number of photon packets (photon budget P_b) used in the solution of the inverse problem
  - Terminated SGN iteration when the budget was used
  - S-SGN: 10 iterations with $P_i = \frac{1}{10} P_b$
  - A-SGN: P_i determined by the norm test

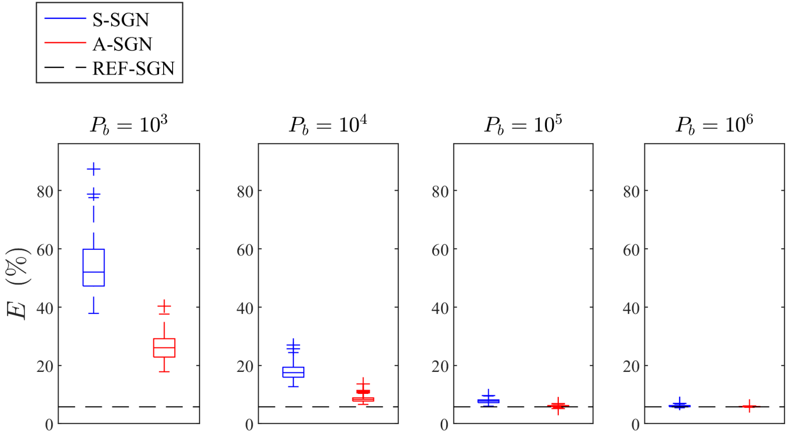

True
REF-SGN
[mm-1]
0.04
0.03
0.02
0.01


True
REF-SGN
[mm-1]
0.04
0.03
0.02
0.01
$P_b = 10^3$
$P_b = 10^4$
$P_b = 10^5$
S-SGN
A-SGN

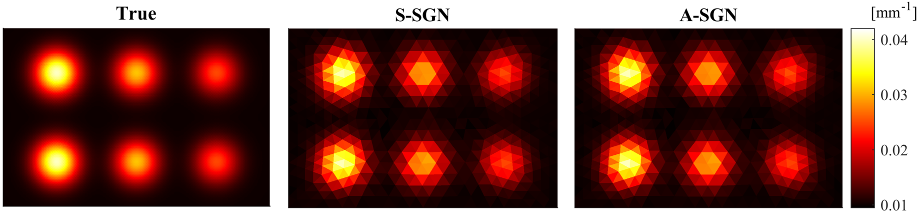
Value of the objective function (left) and relative error of estimates (right) against SGN iteration



Relative error of absorption estimates for 100 simulations



- Second comparison: Same convergence criterion
  - Terminated the SGN iteration when a sufficient convergence was achieved (sufficiently small relative change in three consecutive estimates)
  - S-SGN: P_i is the number of photon packets in the last A-SGN iteration (ensures similar convergence)



Total number of photon packets: 3×10^7 8.6×10^6

Summary

- Monte Carlo method for light transport can be utilised in the inverse problem of QPAT
- Adjusting the number of photons during iteration improves the efficiency of Gauss-Newton iteration
- Adaptive approach and the norm test require some parameters to be predefined
- Simulating a large number of photon packets requires computational resources (methodology is time-consuming)
- Future work:
  - Simultaneous estimation of absorption and scattering
  - Evaluating the reliability of the estimates
  - Modelling of the stochastic noise

Thank you for your attention!

- Leino A, Lunttila T, Mozumder M, Pulkkinen A, Tarvainen T, Perturbation Monte Carlo method for quantitative photoacoustic tomography, *IEEE Transactions on Medical Imaging*, 39(19):2985-2995, 2020
- Hänninen N, Pulkkinen A, Arridge S, Tarvainen T, Adaptive stochastic Gauss-Newton method with optical Monte Carlo for quantitative photoacoustic tomography, *Journal of Biomedical Optics*, 22(8):083013, 2022
- Hänninen N, Pulkkinen A, Arridge SR, Tarvainen T, Estimating absorption and scattering coefficients in quantitative photoacoustic tomography with an adaptive Monte Carlo method for light transport, Submitted

JANE AND AATOS ERKKO FOUNDATION

CENTRES OF EXCELLENCE IN RESEARCH

PREIN

European Research Council
Established by the European Commission