

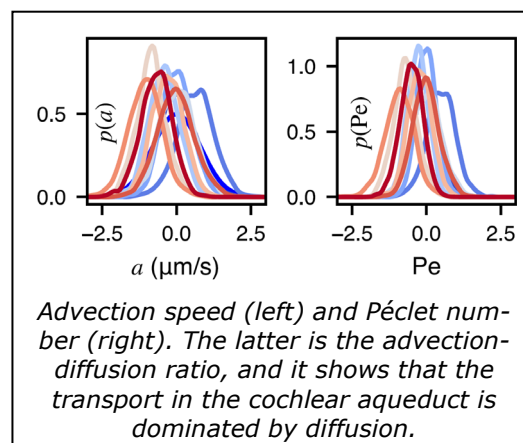
Villum Programme: Villum Investigator
Villum project number: 25893
Project title: CUQI – Computational Uncertainty Quantification for Inverse Problems
Name of PI: Per Christian Hansen

Project summary

Let us define the ingredients of the project title:

- *Inverse problems* involve mathematical models for computing and reconstructing internal (hidden) features from exterior measurements in, e.g., medical imaging/tomography, industrial inspection, baggage scanners, astronomy, materials science, and bio imaging.
- *Uncertainty quantification* (UQ) is a branch of applied mathematics and statistics that allows us to study the sensitivity of a computed result with respect to measurement errors in the data and uncertainties in the mathematical models.
- *Computational UQ* deals with how to perform the UQ analysis efficiently on a computer by means of flexible and efficient computational methods, and it includes elements of scientific computing, numerical methods, and software development.

In the [CUQI project](#) we turned UQ into a practical tool for modelling and analysis of inverse problems. An important part of this work involved the development of a mathematical, statistical and computational foundation and framework that can be used for many different problems (and not just specific applications). For example, our methods for UQ analysis of dynamical systems were used to study glacier ice-sheet flow, fluid flow in the cochlear aqueduct between the inner ear and the brain (see the figure), and instabilities in tokamak fusion plasma. We also developed new theory that gives insight into UQ analysis, leading to new methods and new applications. Our research is documented in a total of 100 publications listed on the [homepage](#), including one book.



We also built a large software package called [CUQIpy](#) that allows scientists and engineers to apply UQ to their inverse problems through a user-interface suited for both experts and non-experts. Our software gives immediate access to the methods and techniques developed in CUQI, as well as many other methods. The novel feature of the software is that, in principle, all types of inverse problems are treated in the same way. Therefore, CUQIpy is also well suited for teaching and training students and practitioners in the use of computational UQ.

During the project we arranged two international workshops, [IUQ22](#) in Helsingør and [UQIP124](#) in Edinburgh, as well as a smaller workshop [CUQI Perspectives](#) towards the end of the project. At DTU we created PhD courses on UQ and Bayesian inverse problems, and we gave several training courses on computational UQ in Denmark and abroad.

In summary, we contributed to advancing the UQ research field, we trained young researchers in data science who now work in both academia and industry. We cemented our group's international position and recognition, and we are invited to international collaborations.

Project highlights

The project was a highly collaborative effort, because many different skills were needed to achieve our goals. The postdocs did excellent cross-disciplinary work that provided an important glue within the project. All project members contributed by sharing results and experience and their interest in working on joint projects, thus creating a unique identity in the project. We are now very well prepared for large collaborative projects in the future.

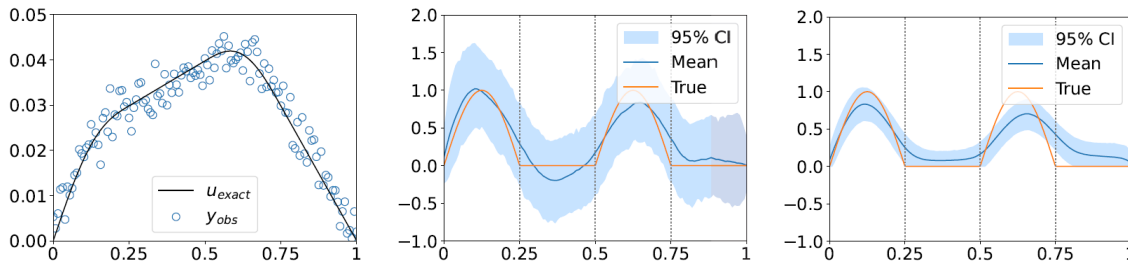
Our main research goal was not to apply UQ to specific applications, but rather to develop a *framework*: a combination of mathematical theory, models, and methods for computational UQ for the general class of Bayesian inverse problems. Challenging applications were included for validation and inspiration. We contributed to the ongoing establishment of a solid mathematical foundation and understanding of the UQ field, and we produced new methods that address challenges associated with complex computations. While CUQI was not a dedicated software project, due to collaborative programming efforts we also produced a large software package.

Our theory development addressed fundamental questions related to UQ analysis such as statistical consistency, i.e., the convergence of the results as the noise approaches zero. Here we mention some highlights. In a series of papers, postdoc Jasper Everink provided new theory that explains how implicit priors (such as non-negativity) can offer practical alternatives to the classical explicit priors in UQ. He was invited to present his work at the Joint Statistical Meeting in Nashville in 2025. Postdoc Babak Afkham developed a theoretical basis for goal-oriented methods where we perform UQ on specific features of a solution, which is much more efficient and powerful than performing a classical full UQ analysis. PhD student Kristoffer Linder-Steinlein developed theory for inverse source problems, establishing results for passive measurements and characterizing how random media affect the information for source reconstruction.

Priors play a central role in UQ analysis, since they express our knowledge and assumptions about the solution. Of particular interest is the formulation of hyperpriors that characterize the parameters of our priors, and a highlight here is our work on priors/hyperpriors for reconstruction of edges and contours in images from noisy data. PhD student Andreas Horst developed Besov priors tailored at representing both smooth and rough components in the solution.

We also mention highlights related to new modeling of priors that led to improved UQ analysis. In the framework of goal-oriented UQ we developed new methods for shape priors for the roughness of a surface, e.g., for identification and classification of tumors in medical imaging and for characterizing the seabed which is important for offshore activities.

By developing favorable covariance structures, PhD student Aksel K. Rasmussen contributed new insight about the reliability of UQ for nonlinear problems. PhD student Silja L. Christensen developed structural priors that can incorporate the specific geometry of objects in CT imaging.



The importance of using the nonnegativity prior in a Bayesian deconvolution problem. Left to right: the noisy data, the conventional UQ result, and the much-improved result with our nonnegativity prior where the solution is indeed nonnegative and has less uncertainty.

Furthermore, we have developed new methods to efficiently handle high-dimensional UQ problems. Postdoc Felipe Uribe demonstrated how to formulate the extended horseshoe prior as an efficient alternative to standard heavy-tail priors, and PhD student Rafael Flock developed new dimension reduction methods to alleviate the work with computationally demanding priors.

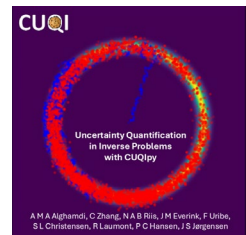
We considered many interesting applications. Highlights are a novel study of the fluid flow in the aqueduct between the inner ear and the brain (performed by postdoc Amal Aghamdi jointly with Copenhagen Univ.), studies of glacier ice-sheet flow (with Univ. of Cambridge), CT scanning for detection of defects in subsea pipes (with FORCE Technology and Heriot-Watt Univ.), and phase-space tomography for detecting instability in tokamak plasma (with DTU Physics).

Open-source software has always been an ideal way to disseminate new results to researchers and engineers, as well as a vehicle for training and teaching activities. The same is true for our CUQIpy software which provides a unique platform for performing practical UQ analysis. From the outset, we designed the software and its interface such that it is easy to use for non-experts while, at the same time, it gives UQ experts full control of the computational details.

A big advantage of our approach is that we separate statistical modeling and UQ logic from the numerical realization of the forward problem. CUQIpy is a modeling environment in which posterior inference, diagnostics, and visualization are coupled to the structure of inverse problems. The design supports a uniform user interface across linear and nonlinear inverse problems (deconvolution, integral equations, PDEs, etc.), and it provides hierarchical models in a common formulation and framework. This means that as soon as one is familiar with the software and its interface, it is easy to apply different UQ methods to different applications.

The software provided the computational backbone of our courses on UQ at DTU, and it was an invaluable tool in our training activities and short courses at international meetings, thus contributing to the visibility of the CUQI project. Users of the software now contribute examples and [showcases](#) that, after review, become available as part of the package.

A final highlight that should be emphasized is our [online book](#) that uses interactive demonstrations and examples to carefully explain the ingredients and capabilities of CUQIpy. The scope of this book is unique, and it was created by collaborative efforts from many team members.



The project was beneficial for the career of many team members:

- Associate professor Kim Knudsen is now professor at DTU Compute.
- Senior researcher Jakob Sauer Jørgensen is now associate professor and scientific director of the imaging center [QIM](#) at DTU Compute.
- Postdoc Babak Afkham is assistant professor at Univ. of Oulu, Finland with his own 5-year grant from Research Council of Finland.
- Postdoc Amal Alghamdi founded the computational consultancy [Impact Alif](#) and is assistant professor at King Fahd University of Petroleum and Minerals in Saudia Arabia.
- Postdoc Richard Huber is postdoc at IDea Lab at the University of Graz, Austria.
- PhD student, later postdoc Jasper Everink is postdoc at the University of Eastern Finland.
- Postdoc Chao Zhang is now continuing as postdoc in the DUDE project (see below).
- The remaining 8 PhD students and 3 postdocs now hold positions in industry

I am proud that the team, together with our international collaborators and co-supervisors, produced strong research that is well received and published in the best scientific journals.

Challenges in the grant period

As always in an ambitious and large project, unexpected and interesting directions presented themselves; they were handled through mutual discussions and agreements with the team.

Future plans

Running a research project of this size was a first for me, and I consider it a highlight of my career. I learned a lot about research management which I can pass on to my younger colleagues, and I am highly motivated to address new projects in applied mathematics and data science. I know that UQ can, and should, be an integral part of any data analysis effort.

During the final stages of the project, I obtained two research grants related to UQ:

1. The project [DUDE](#) (Data Uncertainty and Design/reconstruction Errors) funded by DFF extends some of the CUQI research to the field of iterative reconstruction methods with applications in imaging and design.
2. An industrial postdoc project joint with [3Shape A/S](#) carries this research into advanced techniques for dental imaging.

Both projects will utilize and extend the CUQIpy software.

An application for an NNF Data Science Collaborative Grant has reached the second stage; the topic is phase-space tomographic imaging with UQ for studying the dynamics of tokamak fusion plasma. In this project, UQ is central for the validation of our new methods.

I consider writing a second ERC Advanced Grant proposal. Unfortunately, it is difficult to receive a second such grant and the ERC mathematics panel awards very few grants to applied mathematics (both confirmed by Prof. Jan Philip Solovej).