Uncertainty-aware blob detection in astronomical imaging

Workshop "Imaging with Uncertainty Quantification"

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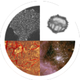

SFB Tomography Across the Scales

Overview of this presentation

1. Short introduction to integrated-light stellar population reconstruction
2. Uncertainty quantification and scale-space aspects
3. Uncertainty-aware blob detection

Short introduction to integrated-light stellar population reconstruction

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Figure: Helmi et al. "The merger that led to the formation of the Milky Way's inner stellar halo and thick disk". Nature (2018). (Artist's impression)

Single-spectrum fitting

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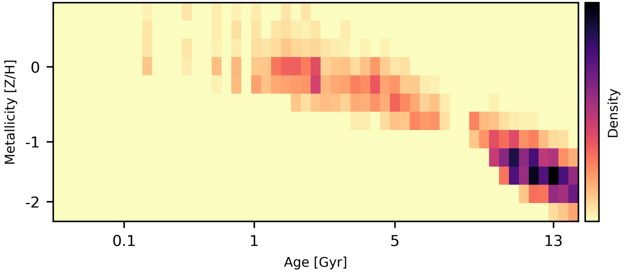

Figure: A stellar distribution function f corresponding to the M54 globular cluster

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S : simple-stellar-populations templates,

L : line-of-sight velocity distribution.

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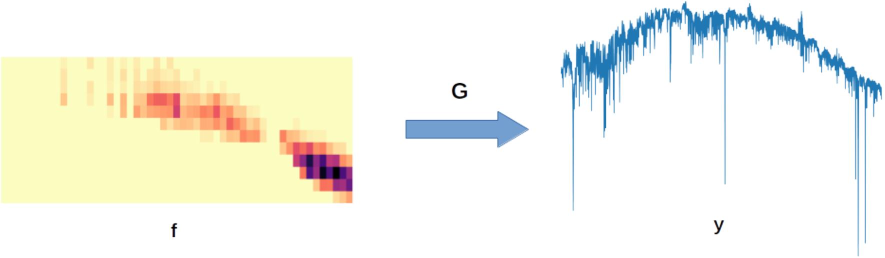

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&\quad \Sigma^{\mathsf{OU}}_{ij,lm} = k(\|\boldsymbol{x}_{ij} - \boldsymbol{x}_{lm}\|), && \boldsymbol{x}_{ij} : \text{position of pixel } (i,j),\\
&\quad k(d) := e^{-d/h}, && h : \text{"correlation length"}.
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- Real data has low signal-to-noise ratio (≈ 100).

Uncertainty quantification and scale-space aspects

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Simultaneous credible intervals

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Simultaneous credible intervals

Given $\alpha \in (0,1)$, $\boldsymbol{f}^{\mathrm{low}}, \boldsymbol{f}^{\mathrm{upp}} \in \mathbb{R}^{T \times Z}$, with $f_{ij}^{\mathrm{low}} \leq f_{ij}^{\mathrm{upp}}$ for all (i,j), the set

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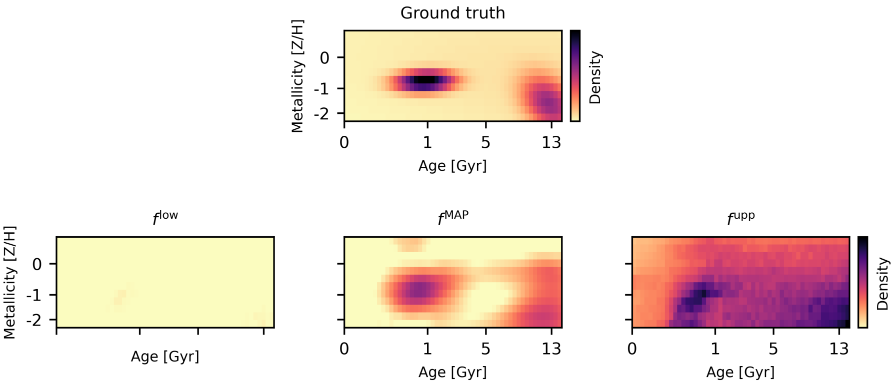
is a simultaneous posterior $(1-\alpha)$-credible interval for $\boldsymbol{f}$ if

$$p(\boldsymbol{f} \in [\boldsymbol{f}^{\text{low}}, \boldsymbol{f}^{\text{upp}}] \mid \boldsymbol{y}) \geq 1 - \alpha.$$

Credible intervals for stellar recoveries



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where $\boldsymbol{g}_t$ is a discrete analogue of the Gaussian

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Filtered credible interval

We call $[\boldsymbol{L}_t^{\text{low}}, \boldsymbol{L}_t^{\text{upp}}] \subset \mathbb{R}^{T \times Z}$ a filtered $(1-\alpha)$-credible interval if

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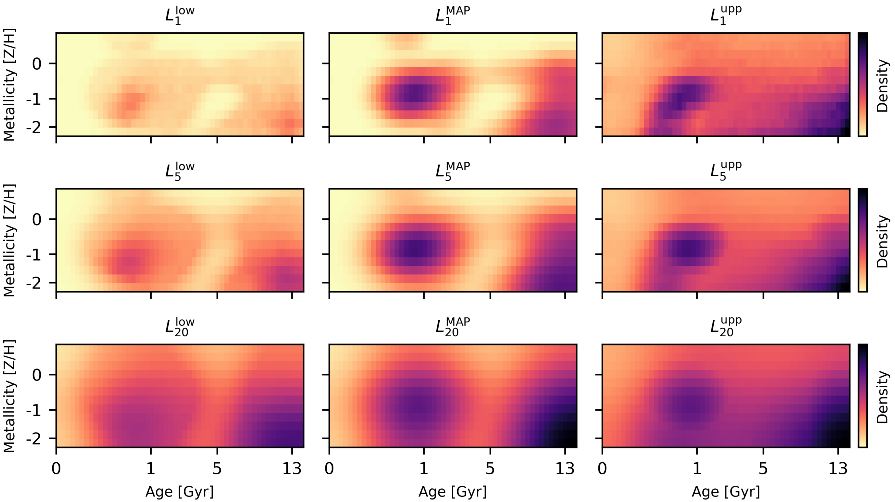
Uncertainty quantification with filtered credible intervals



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- In computer vision, these questions have been adressed by so-called **scale-space methods**.

Scale-space representation: Definition

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Gaussian scale-space representation

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Gaussian scale-space representation

The (Gaussian) scale-space representation of an image $f : \mathbb{R}^2 \to \mathbb{R}$ is defined as $L : \mathbb{R}^2 \times [0, \infty) \to \mathbb{R}$,

$$L(\boldsymbol{x}, t) = f * g_t(\boldsymbol{x}), \qquad x \in \mathbb{R}^2, t \geq 0,$$

where

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- Extrema of scale-normalised derivatives behave "feature-like" (shift-invariance, scale-invariance, etc.).

Example: LoG blob detection applied to MAP estimate

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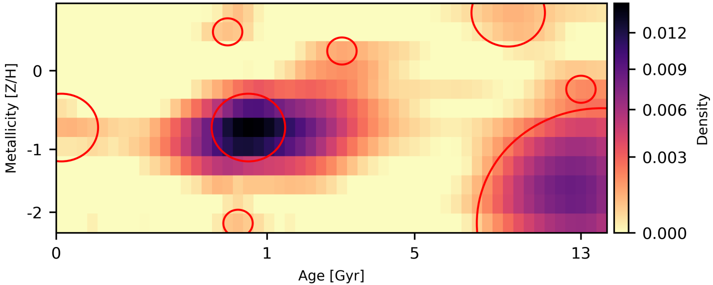

Figure: Results of the Laplacian-of-Gaussians method applied to $\boldsymbol{f}^{\text{MAP}}$. Each scale-space blob $(\boldsymbol{x}, t)$ is visualised as red circle with center $\boldsymbol{x}$ and radius $\sqrt{2t}$.

Uncertainty-aware blob detection

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- Idea: Obtain an uncertainty-aware scale-space representation by minimising a measure of "feature-richness".
- Use this representation in LoG blob detection.

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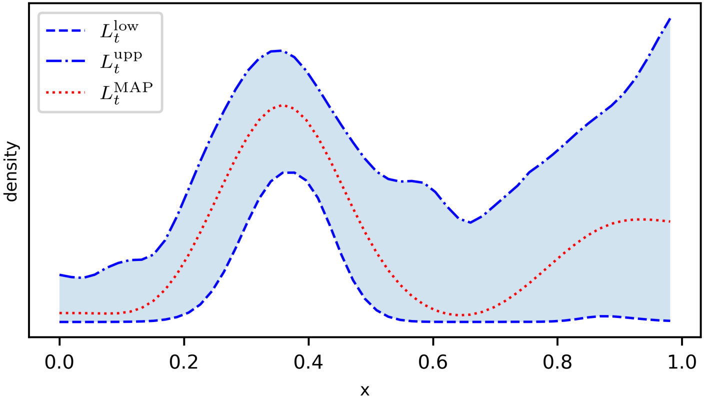

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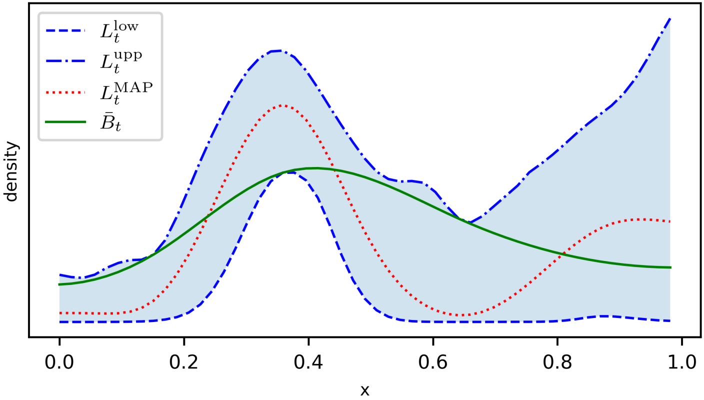

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Given discrete scales $t_1 < \ldots < t_K$, compute $\bar{\boldsymbol{B}} \in \mathbb{R}^{T \times Z \times K}$ by solving

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- Can be separated into K optimisation problems in $\mathbb{R}^{T \times K}$.
- Can think of each $\bar{\boldsymbol{B}}_{t_k}$ as "blanket" spanned between $\boldsymbol{L}_{t_k}^{\text{low}}$ and $\boldsymbol{L}_{t_k}^{\text{upp}}$.

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3. Compute $\bar{\boldsymbol{B}}$ by solving K constrained least-squares problems in $\mathbb{R}^{T \times Z}$.
4. "Significant" blobs are identified as local minimizers of $\Delta^{\text{norm}} \bar{\boldsymbol{B}} \in \mathbb{R}^{T \times Z \times K}$.

ULoG applied to single-spectrum fitting

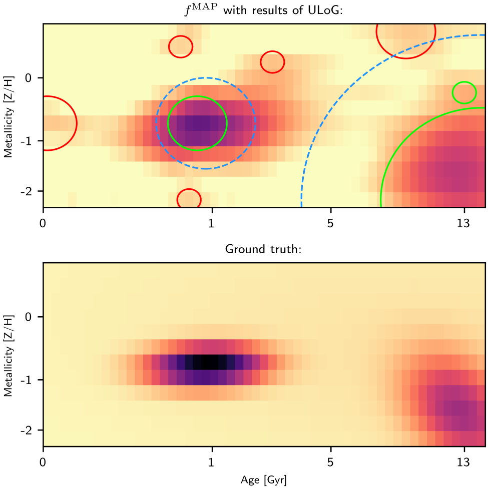

Figure: Upper panel: Results of ULoG. The significant blobs are visualised by dashed blue circles. The green/red circles correspond to the matching/unmatching blobs in $\boldsymbol{f}^{\mathrm{MAP}}$. Lower panel: The ground truth $\boldsymbol{f}^*$.

Summary

- Use **filtered credible intervals** as a way to visualise scale-dependency of uncertainty.
- Minimising a suitable measure of feature-richness then leads to an **uncertainty-aware scale-space representation**.
- This can be used to formulate uncertainty-aware versions of scale-space methods for blob detection (LoG $\rightarrow$ ULoG).