Defect Detection in Bayesian CT Imaging of Subsea Pipes

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INTRODUCTION

A non-destructive testing (NDT) application of X-ray computed tomography (CT) is defect detection in multi-layered subsea pipes in operation via 2D cross sectional scans. The reconstructed image contains both large-scale structures (the pipe layers) and small-scale structures (defects) which is challenging to model with a single prior. This motivates our work, where the main purposes are:

1) Separate the reconstruction into a sum of large- and small- scale structures.
2) Impose priors that promote the structures of the pipe and defects.
3) Obtain uncertainty quantification related to the defect reconstruction.

METHODOLOGY

CT forward problem

$$\mathbf{d} = \mathbf{A}(\mathbf{x} + \boldsymbol{\varepsilon}) + \mathbf{e}$$

$\mathbf{d} \in \mathbb{R}^m$: observed projections
$\mathbf{e} \in \mathbb{R}^m$: data noise with variance λ^{-1}
$\mathrm{x} \in \mathbb{R}^n$: pixel representation of the pipe
$\boldsymbol{\varepsilon} \in \mathbb{R}^n$: pixel representation of defects
$\mathbf{A} \in \mathbb{R}^{m\times n}$: CT forward model

Prior distributions

We use a Structural Gaussian Prior [1] to promote the pipe structure in $\mathbf{x}$:

$$\mathbf{x} \sim \mathcal{N}\left(\boldsymbol{\mu}_{SGP}, \left(\mathbf{R}_{SGP}^T\mathbf{R}_{SGP}\right)^{-1}\right).$$

There are few defects, so we enforce sparsity in $\boldsymbol{\varepsilon}$ with a hierarchical prior:

$$\varepsilon_i|\eta_i \sim \mathcal{N}(0,\eta_i), \qquad \eta_i \sim \mathcal{IG}\left(\tfrac{\nu}{2},\tfrac{\nu}{2}\omega^2\right),$$

where $i = 1, \dots, n$ and $\mathcal{IG}(\cdot)$ is the inverse gamma distribution. This is equivalent to the following student's t-distribution:

$$\varepsilon_i \sim t(\nu, 0, \omega),$$

which shows that, lower ν increases sparsity, and lower ω leads to lower variance.

Bayesian inverse problem

From the priors and the likelihood

$$\mathbf{d}|\mathbf{x},\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{A}(\mathbf{x}+\boldsymbol{\varepsilon}), \lambda^{-1}\mathbf{I}),$$

we can derive the posterior distributions from Bayes' Theorem:

$$p(\boldsymbol{x}|\boldsymbol{d},\boldsymbol{\varepsilon}) \propto p(\boldsymbol{d}|\mathbf{x},\boldsymbol{\varepsilon})p(\mathbf{x}),$$
$$p(\boldsymbol{\varepsilon}|\boldsymbol{d},\mathbf{x},\boldsymbol{\eta}) \propto p(\boldsymbol{d}|\mathbf{x},\boldsymbol{\varepsilon}) \textstyle\prod_{i=1}^n p(\varepsilon_i|\eta_i),$$
$$p(\boldsymbol{\eta}|\boldsymbol{\varepsilon}) \propto \textstyle\prod_{i=1}^n p(\varepsilon_i|\eta_i)p(\eta_i).$$

Gibbs sampling

We use CUQIpy to sample the posterior. The Gibbs sampling scheme is given below.
Repeat:
1) One sample from $\boldsymbol{x}|\boldsymbol{d},\boldsymbol{\varepsilon}$ using Linear RTO [2].
2) One sample from $\boldsymbol{\varepsilon}|\boldsymbol{d},\mathbf{x},\boldsymbol{\eta}$ using Linear RTO.
3) One sample from $\boldsymbol{\eta}|\boldsymbol{\varepsilon}$ utilizing conjugacy.

PRELIMINARY RESULTS

Test set-up

- Pipe phantom with layered structure and small defects.
- 360 equidistant view angles.
- Fanbeam measurements.
- 2 % Gaussian measurement noise.
- $\nu = 10, \ \omega = 1/300.$

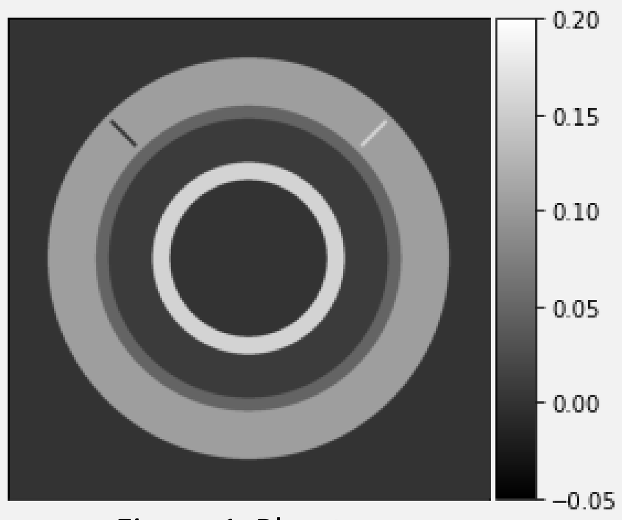
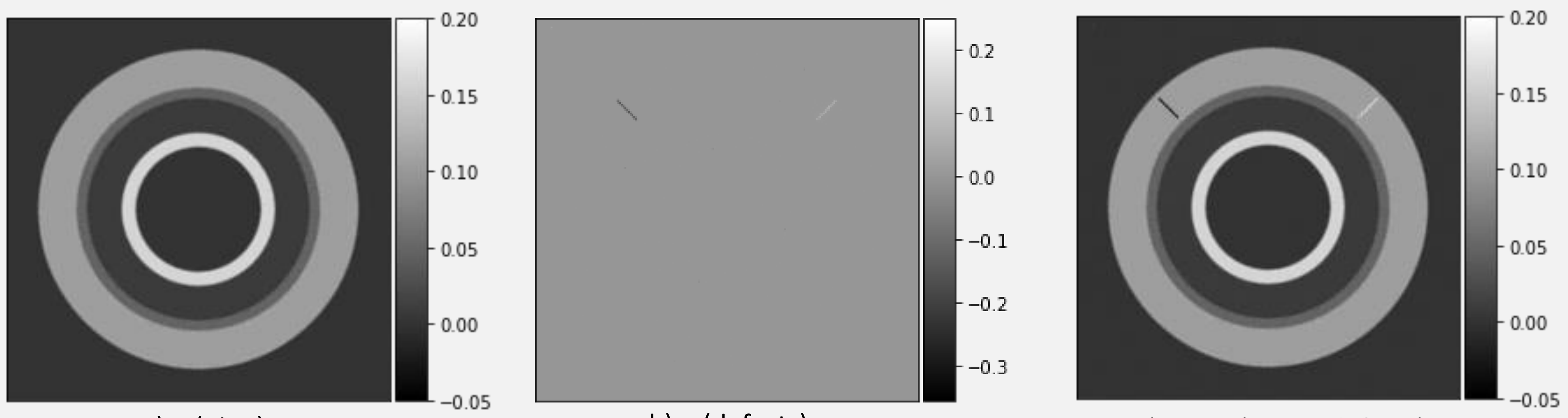


Figure 1: Phantom

Posterior means

The posterior means are computed based on 2000 independent samples after burn-in. The method successfully splits the image into pipe and defect reconstructions.

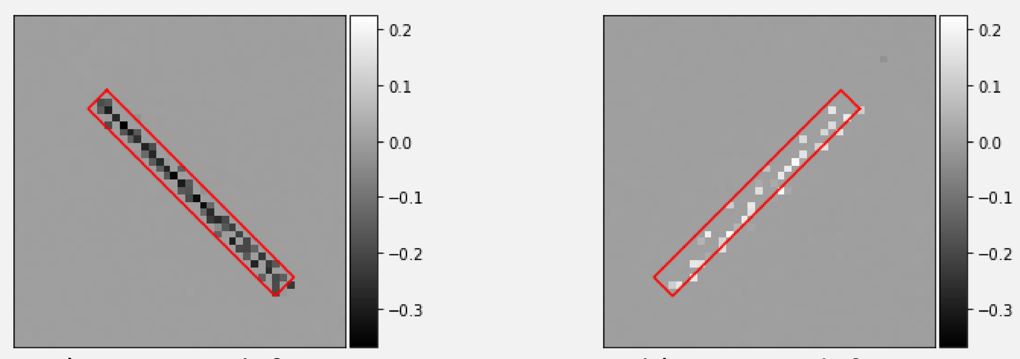


a) $\mathbf{x}$ (pipe) b) $\boldsymbol{\varepsilon}$ (defects) c) $\mathbf{x}+\boldsymbol{\varepsilon}$ (pipe + defects)

Figure 2: Posterior means

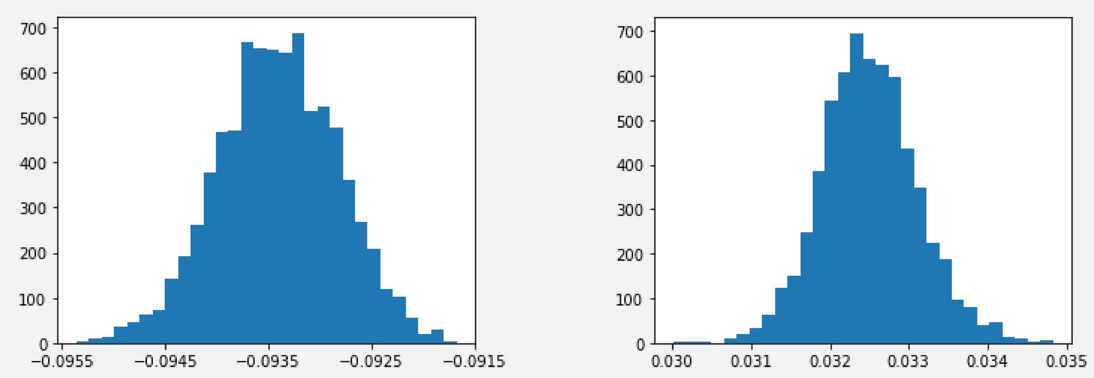
Defect detection

- Defects are clearly detected.
- Absorption coefficients of the defects are estimated, but the histograms do not include the true values.
- Reconstructing the defects are challenging due to their small size (width ~ 3 pixels).



a) Negative defect. b) Positive defect.

Figure 3: Zooms of $\boldsymbol{\varepsilon}$ posterior mean. Red mark the true defect outlines.



a) Negative defect. Truth = -0.11. b) Positive defect. Truth = 0.05.

Figure 4: Histograms showing the mean defect absorption coefficients over all samples.

CONCLUSIONS

1) Successful separate reconstructions of pipe structure and defects.
2) Priors promoting pipe structure and sparsity are appropriate.
   - SGP for the pipe.
   - Hierarchical prior equivalent to a student's t-distribution on the defects.
3) Demonstrated an example of how uncertainty quantification can be used to analyze detected defects.

Future work

- Extent the uncertainty analysis of $\boldsymbol{\varepsilon}$ for instance using hypothesis testing.
- More extensive numerical experiments using other priors on $\mathbf{x}$ and $\boldsymbol{\varepsilon}$.
- More extensive numerical experiments using test cases with less data.
- Apply to real data.

ACKNOWLEDGEMENTS

This work was supported by The Villum Foundation (grant no. 25893). We also thank FORCE Technology for motivating the case.

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