Epistemic and doxastic planning for single- and multi-agent systems

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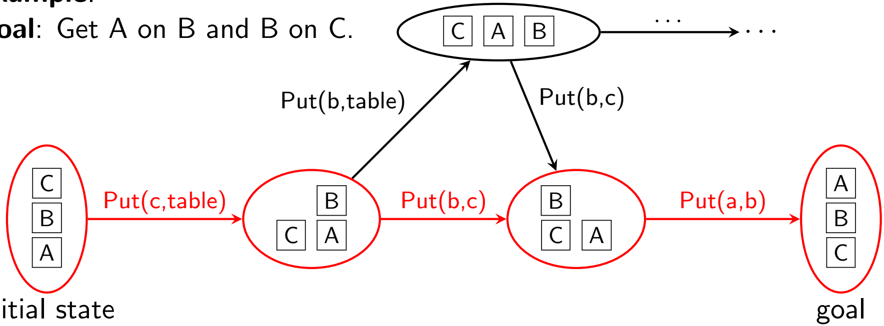
Automated planning

Automated planning (or, simply, **planning**):

- A central subfield of **artificial intelligence** (AI).
- Aims at generating **plans** (**sequences of actions**) leading to desired outcomes.
- More precisely: Given a **goal formula**, an **initial state** and some **possible actions**, an **automated planner** outputs a plan that leads from the initial state to a state satisfying the goal formula.

Example.

Goal: Get A on B and B on C.



Main idea of our work

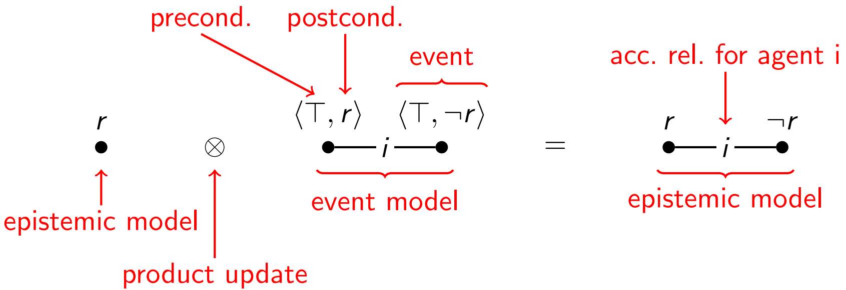
Essentially: A transition from **classical planning** based on propositional logic to planning based on **Dynamic Epistemic Logic** (**DEL**).

	Classical	**DEL-based**
States	models of prop. logic	models of MA epist. logic
Goal formula	formula of prop. logic	formula of MA epist. logic
Actions	induced by action schemas	event models of DEL

Advantages: Generalises classical planning by allowing

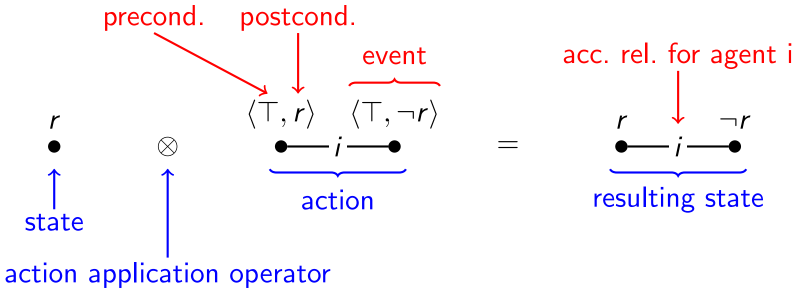
- Planning under partial observability and/or non-determinism with sensing actions.
- Planning including reasoning about other agents (essential to agent communication and collaboration).

DEL by example: Hidden coin toss



- **Epistemic models**: *Finite* multi-agent *S5* models. Reflexive edges omitted. Elements of domain called **worlds**.
- **Event models**: Both pre- and post-conditions as in [van Ditmarsch and Kooi, 2008] (allows ontic actions). Ours differ only in the definition of **postconditions**: conjunctions of propositional literals (as in classical planning). Same expressivity.
- **Product update**: As in [van Ditmarsch and Kooi, 2008].

Planning interpretation of DEL



- **States**: Epistemic models.
- **Actions**: Event models.
- **Result of applying an action in a state**: Product update of state with action.

Epistemic planning problems

Definition. *An* **epistemic planning problem** *consists of:*

- **States** *(including an* **initial state** s_0*): Finite models of multi-agent epistemic logic.*
- *A* **goal formula** ϕ_g*: Formula of multi-agent epistemic logic.*
- *A finite set A of possible* **actions***: Finite event models.*

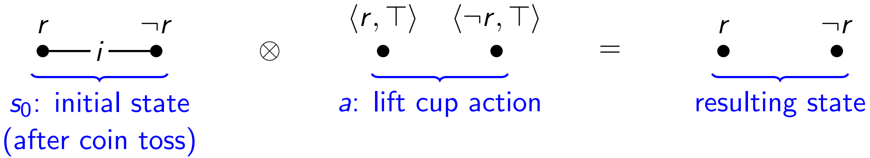
Definition. *A* **solution** *to an epistemic planning problem is a sequence of actions $a_1, \ldots, a_n \in A$ such that*

$$s_0 \otimes a_1 \otimes \cdots \otimes a_n \models \phi_g.$$

We then also say that $a_1, \ldots, a_n$ is a **plan** *for achieving ϕ_g from s_0.*

But wait! In which world(s) is ϕ_g evaluated?...

Planning: hypothesising about the future



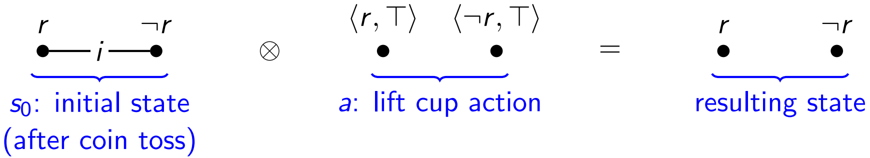
Epistemic planning (and **knowledge-based planning** in general) is about:

hypothesising about the possible outcomes of your actions.

The models (states) represent what the planning agent knows at **plan time** (*a priori*) about the knowledge it will achieve at **run time** (*a posteriori*).

In the example above: The agent will at **run time** (after the action has been performed) **come to know** whether r holds. But at **plan time** (before the action has been performed), it can't point out which of r or $\neg r$ it'll be.

Where is the goal formula evaluated?



Question: So in which world(s) in the resulting state do we evaluate a goal formula?

1st suggestion: Goal formula has to hold globally in the model.

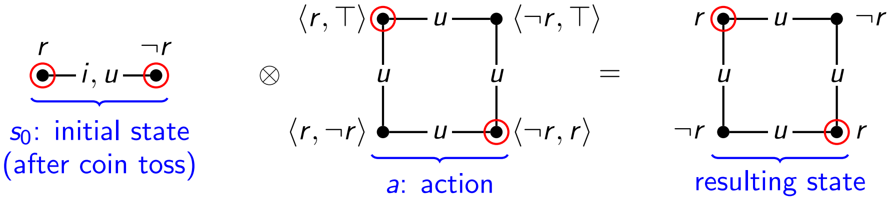
Examples. i is the planning agent.

1. $s_0 \otimes a \models K_i r \vee K_i \neg r$. Thus performing a in s_0 is a plan for achieving knowledge of **whether** r.
2. $s_0 \otimes a \not\models K_i r$. Performing a in s_0 is **not** a plan for achieving the knowledge that r.
3. $s_0 \otimes a \not\models K_i \neg r$. Performing a in s_0 is **not** a plan for achieving the knowledge that $\neg r$.

Multiple agents and designated worlds

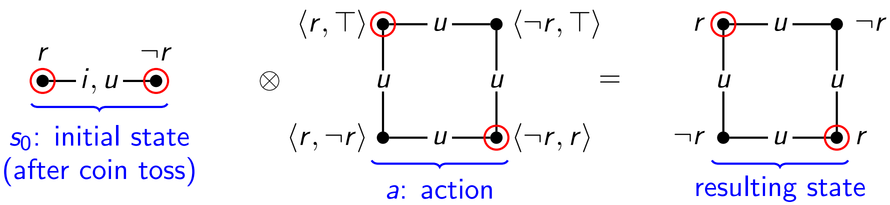
In the multi-agent case things get slightly more complicated.

Let i be I and u be you!



- Action a: I look at the coin and either flip it or not. You see the action, but not the result.
- I might choose to flip iff it's $\neg r$, thereby enforcing r.
- But then afterwards, I'll know r. How do I see this in the resulting state?
- **Solution**: Use **designated** worlds and events: ⊙ (gives **multi-pointed** epistemic models and event models).

Multiple agents and designated worlds (cont'd)



Recall question: In which world(s) in the resulting state do we evaluate a goal formula?

2nd suggestion (final): In the **designated** worlds.

Example. Applying a in s_0 achieves the goal of me knowing r but not you.

Redefinitions.

- **State**: *Multi-pointed epistemic model.*
- **Action**: *Multi-pointed event model.*
- $s \models \phi$ *means* ϕ *holds in all the* **designated** *worlds of the state* s.

Modelling the internal perspective

Multi-pointed models provide an **internal perspective**:

> *The planning agent can not always himself point out the actual world, but can point out the subset of worlds he considers possible.*

A slight generalisation of the standard **external perspective**, where an actual world is always pointed out.

Applicability

Consider this suggested plan for achieving cash:

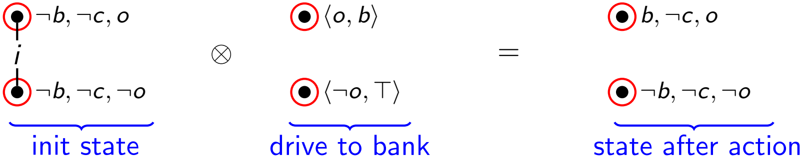
Drive to bank, Get cash at bank.

Drive to bank: $\langle o, b \rangle$; $\langle \neg o, \top \rangle$

b : at bank
c : have cash
o : car OK.

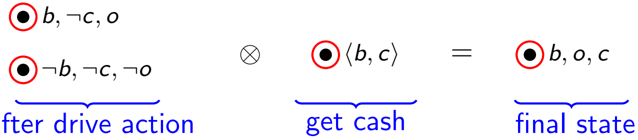
Get cash at bank: $\langle b, c \rangle$

Driving to the bank, initially not being at the bank ($\neg b$), having no cash ($\neg c$) and not knowing whether the car is OK or not (o or $\neg o$):



Applicability (cont'd)

Getting the cash:



Problem: I can now, incorrectly, conclude that after having executed *Drive to bank*, *Get cash from bank*, I **know** I have cash ($K_i c$).

Solution: Concept of applicability.

Definition (Applicability). An action a is said to be **applicable** in a state s if:

> *for each designated world in s there is a designated event in a having its precondition satisfied in the world.*

In other words: For each world the agent considers possible, the action specifies at least one applicable event.

Redefine concept of **solution** accordingly.

Main results

Theorem
Plan existence in single-agent epistemic planning is decidable.

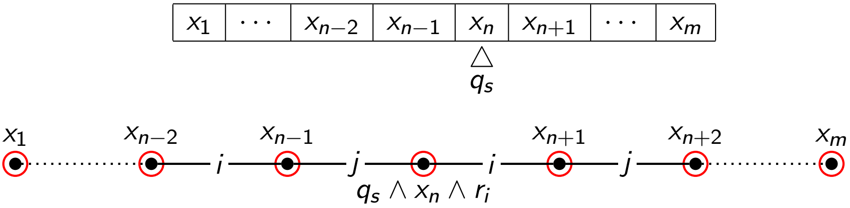
Proof idea: The number of propositional symbols is assumed to be finite. Hence there can only be finitely many distinct single-agent epistemic models (S5 models) up to bisimulation.

Theorem

Plan existence in multi-agent epistemic planning is undecidable in each of the following cases:

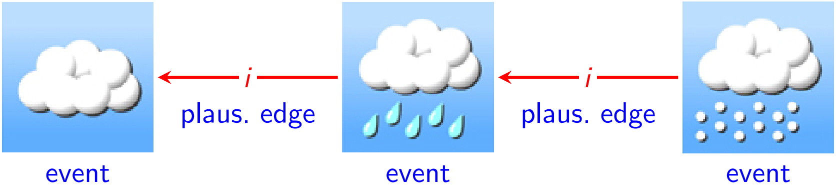
- *There are at least 3 agents.*
- *There are at least 2 agents, and the epistemic language includes the common knowledge modality.*
- *There is at least 1 agent, and we allow arbitrary frames (not only S5).*

Proof idea: Reduction to Halting problem. States (epistemic models) encode IDs of TM, actions (event models) encode transitions of TM.



Generalising to plausibility models

Example: Boots or shoes?

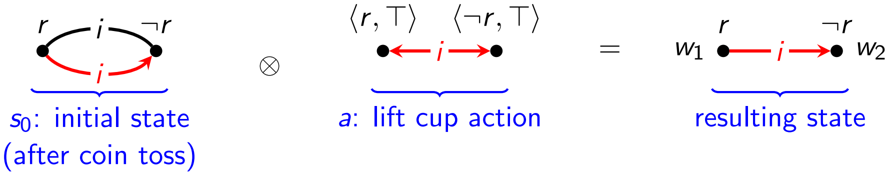


Essentially: A transition from DEL-based planning to planning based on **epistemic plausibility models** [Baltag and Smets, 2006].

Advantage: Agents can do **plausibility planning** where only the n most plausible layers of plausibility are taken into account in the planning phase. (Defeasible planning).

Plausibility planning example

Example. Tossing a biased coin.

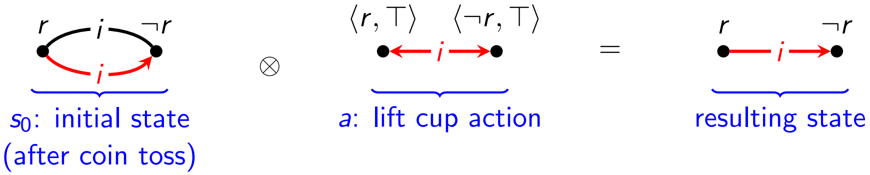


The resulting state represents the agents **plan time** knowledge about the possible outcomes of executing the plan:

- There are two possible outcomes, r and $\neg r$.
- When the plan has been executed, it will be known which it is (no epistemic link between the two).
- Currently (at plan time), it is considered most plausible that it will be $\neg r$.

Note: Our plausibility relation is the **a priori plausibility relation** ("beliefs about some virtual state"), not the **local plausibility relation** ("beliefs about the actual, current state") [Baltag and Smets, 2006].

Example cont'd



- *Toss coin* is a plan for achieving $B_i \neg r$, but not for achieving any knowledge.
- *Toss coin, lift cup* is a 1-strong plausibility plan for achieving $K_i \neg r$.
- *Toss coin, lift cup* is not a 2-strong plausibility plan for $K_i \neg r$.

Summing up

- Presented a **planning framework based on DEL** (with ontic actions): partial observability, non-determinism, multiple agents.
- Single agent planning is **decidable**, multi-agent planning is **undecidable**.
- The framework is currently generalised to **epistemic plausibility models**: different degrees of plausibility planning.

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