Epistemic Planning: Semantic Approach

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http://www2.compute.dtu.dk/~tobo/children_cabinet_cropped.mp4
Epistemic planning = automated planning + Theory of Mind reasoning

**Aim:** To compute plans that can take the mental states of other agents into account.

**Essentially:** (Decentralised) **multi-agent planning** in environments with (potentially higher-order) **information asymmetry**.
Syntactic vs semantic, explicit vs implicit

When moving from standard propositional states to states including a Theory of Mind, there are two distinct paths one might take.

- **Syntactic approach**: States are (sets of) formulas (e.g. formulas of S5 epistemic logic)
- **Semantic approach**: States are semantic models (e.g. epistemic models = Kripke models).

Note: For propositional planning under full observability, the approaches are trivially equivalent.

Furthermore, for the semantic approach, there is a choice between:

- **Explicit approach**: Full state space is assumed given, and solution concept is defined directly in terms of this. E.g. logics like ATEL and CSL. [van der Hoek and Wooldridge, 2002, Jamroga and Aagotnes, 2007]
- **Implicit approach**: State space is induced by initial state and action library (as in classical STRIPS/PDDL planning).

DEL-based epistemic planning is *implicit* and *semantic*. [Bolander and Andersen, 2011]
**Epistemic states**: Multi-pointed epistemic models of multi-agent S5. Nodes are worlds. **Designated worlds**: □ (those considered possible by planning agent).
The coordinated attack problem in dynamic epistemic logic (DEL)

Two generals (agents), $a$ and $b$. They want to coordinate an attack, and only win if they attack simultaneously.

$d$: “general $a$ will attack at dawn”.

$m_i$: the messenger is at general $i$ (for $i = a, b$).

Initial epistemic state:

$$s_0 = w_1 \xrightarrow{d, m_a} b \xrightarrow{m_a} w_2$$

Nodes are **worlds**, edges are **indistinguishability edges** (reflexive loops not shown).
The coordinated attack problem in dynamic epistemic logic (DEL)

Recall: \(d\) means “\(a\) attacks at dawn”; \(m_i\) means messenger is at general \(i\).

Available **epistemic actions** (aka **action models** aka **event models**):

\[
a: \text{send} = \begin{array}{c}
\text{pre: } d \land m_a \\
\text{post: } m_b \land \neg m_a
\end{array}
\]

\[
\begin{array}{c}
\text{pre: } \top \\
\text{post: } \neg m_a \land \neg m_b
\end{array}
\]

And symmetrically an epistemic action \(b: \text{send}\). We read \(i: \alpha\) as “agent \(i\) does \(\alpha\)”.

Nodes are **events**, and each event has a **precondition** and a **postcondition** (effect). The precondition is an epistemic formula and the postcondition is a conjunction of literals.

[Baltag et al., 1998, van Ditmarsch and Kooi, 2008]
The product update in dynamic epistemic logic

\[ s_0 = d, m_a \quad b \quad m_a \]

\[ s_0 \models K_a d \land \neg K_b d \]

\[ a:\text{send} = \]

\[
\begin{array}{c}
\text{pre: } d \land m_a \\
\text{post: } m_b \land \neg m_a
\end{array}
\]

\[ a \quad \]

\[
\begin{array}{c}
\text{pre: } \top \\
\text{post: } \neg m_a \land \neg m_b
\end{array}
\]

\[ a:\text{send} \]

\[ s_0 \otimes a:\text{send} = d, m_b \quad a \quad d \quad b \]

\[ s_0 \otimes a:\text{send} \models K_a d \land K_b d \land \neg K_a K_b d \]
\[ s_0 = d, m_a \]

\[ s_1 = s_0 \otimes a:send = d, m_b \]

\[ s_2 = s_1 \otimes b:send = d, m_a \]

\[ s_3 = d, m_a \]
**Epistemic planning tasks**

**Definition.** An epistemic planning task (or simply a planning task) $T = (s_0, A, \gamma)$ consists of an epistemic state $s_0$ called the initial state; a finite set of epistemic actions $A$; and a goal formula $\gamma$ of the epistemic language.

**Definition.** A (sequential) solution to a planning task $T = (s_0, A, \gamma)$ is a sequence of actions $\alpha_1, \alpha_2, \ldots, \alpha_n$ from $A$ such that for all $1 \leq i \leq n$, $\alpha_i$ is applicable in $s_0 \otimes \alpha_1 \otimes \cdots \otimes \alpha_{i-1}$ and

$$s_0 \otimes \alpha_1 \otimes \alpha_2 \otimes \cdots \otimes \alpha_n \models \gamma.$$ 

**Example.** Let $s_0$ be the initial state of the coordinated attack problem. Let $A = \{a:send, b:send\}$. Then the following are planning tasks:

1. $T = (s_0, A, Cd)$, where $C$ denotes common knowledge. It has no solution.
2. $T = (s_0, A, E^n d)$, where $E$ denotes “everybody knows” and $n \geq 1$. It has a solution of length $n$.

[Bolander et al., 2020]
Epistemic planning example: Get the cube

- **Objects**: \( \mathcal{O} = \{b_1, b_2, c\} \), two boxes \( b_1 \) and \( b_2 \), and a cube \( c \).

- **Agents**: \( \mathcal{A} = \{h, a\} \), a human \( h \) and a robot \( r \). The robot is the planning agent.

- **Atomic propositions**: \( \text{ln}(x, y) \) means \( x \) is in \( y \), where \( x, y \in \mathcal{O} \cup \mathcal{A} \) (when \( y \in \mathcal{A} \), it means \( y \) is holding \( x \)).

Initial epistemic state:

\[
\begin{array}{c}
\text{s}_0 = \quad \text{ln}(c, b_1) \quad \overset{h}{\longrightarrow} \quad \text{ln}(c, b_2)
\end{array}
\]

The goal is for the human to hold the red cube, \( \text{ln}(r, h) \).
Actions specialised for the case of $O = \{b_1, b_2, c\}$.

Agent $i$ (semi-privately) **peeks** into box $x$:

$i$:peek($x$) =

| pre: $\text{ln}(c, x)$ | $\mathcal{A} - \{i\}$ | pre: $\neg\text{ln}(c, x)$ |

Agent $i$ (publicly) **picks up** object $x$ from $y$:

$i$:pickup($x, y$) =

| pre: $\text{ln}(x, y)$ | post: $\text{ln}(x, i) \land \neg\text{ln}(x, y)$ |

Agent $i$ (publicly) **puts** object $x$ in $y$:

$i$:putdown($x, y$) =

| pre: $\text{ln}(x, i)$ | post: $\text{ln}(x, y) \land \neg\text{ln}(x, i)$ |

Agent $i$ (publicly) **announces** that formula $\varphi$ is true:

$i$:ann($\varphi$) =

| pre: $\varphi$ |
Get the cube: Planning task and solutions

The planning task $T$ has the actions of the previous slide and initial state $s_0$ and goal $\gamma$ given by:

$$
\begin{align*}
  s_0 &= \text{In}(c, b_1) \otimes \text{In}(c, b_2) \\
  \gamma &= \text{In}(r, h)
\end{align*}
$$

Solution to $T$, by robot $R$:

$$
\begin{align*}
  s_0 &= \text{In}(c, b_1) \otimes \text{In}(c, b_2) \\
  s_1 &= s_0 \otimes r:\text{pickup}(c, b_1) = \text{In}(c, r) \\
  s_2 &= s_1 \otimes r:\text{putdown}(c, h) = \text{In}(c, h)
\end{align*}
$$
Applicability, perspective shifts, implicit coordination

Seemingly simpler solution: \( h:\text{pickup}(c, b_1) \). But intuitively, this shouldn’t work, since the human doesn’t know the cube is in box 1...

**Applicability:** An action \( \alpha \) is **applicable** in a state \( s \) if for each designated world \( w \) of \( s \) there is a designated event \( e \) of \( \alpha \) with \( w \models \text{pre}(e) \).

**Perspective shift:** The **perspective shift** of state \( s \) to agent \( i \), denoted \( s^i \), is achieved by closing under the indistinguishability relation of \( i \). We call \( s^i \) the **perspective** of agent \( i \) on state \( s \).

\[
\begin{align*}
  s_0 &= \text{In}(c, b_1) \xrightarrow{h} \text{In}(c, b_2) \\
  s_0^h &= \text{In}(c, b_1) \xrightarrow{h} \text{In}(c, b_2)
\end{align*}
\]

**Example.** \( h:\text{pickup}(c, b_1) \) is not applicable in \( s_0 \) from \( h \)'s perspective.

**Implicitly coordinated solution to planning task:** Each action has to be applicable from the perspective of the acting agent; and the product update \( s \otimes i:\alpha \) is replaced by \( s^i \otimes i:\alpha \).
Get the cube: Implicit coordination

Joint solution to $T$, by robot $R$, implicitly coordinated:

\[
\begin{align*}
    s_0 &= \ln(c, b_1) \\
    s_1 &= s_0 \otimes r:ann(ln(c, b_1)) = \ln(c, b_1) \\
    s_2 &= s_1 \otimes h:pickup(c, b_1) = \ln(c, h)
\end{align*}
\]

If purely epistemic actions (announcements) have a lower cost than ontic actions (moving things around), the solution above is the only optimal one.
Undecidability: lengthening and shortening chains

Consider a chain produced by the coordinated attack problem:

Using preconditions of modal depth 1 we can shorten the chain by 1:

We can now both lengthen (by send) and shorten chains (by shorten), and this allows us to encode two-counter machines ⇒ undecidability of the plan existence problem!

Undecidability holds even with preconditions of modal depth 1, and for purely epistemic planning (no postconditions) even for modal depth 2. [Bolander and Andersen, 2011, Charrier et al., 2016, Bolander et al., 2020]
Some of the current challenges in epistemic planning

- **Undecidability issues**: open complexity problems. [Bolander et al., 2020]

- **State size explosion problems**: find compact state representations. [Charrier and Schwarzentruber, 2017, van Benthem et al., 2018]

- **The belief-revision problem in DEL**: How to recover from false beliefs without an underlying epistemic relation. Relates to the state size explosion problem.

- **Heuristics for epistemic planning**: to reduce all of the above mentioned complexity and scalability issues

- **Languages**: syntactic languages for describing actions. [Baral et al., 2012, Baral et al., 2013]

This, and much more, is discussed in the “Epistemic Planning” special issue of AIJ currently being finalised.
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